Pre Calculus -- Ch8 Strategies

<u>Solving a system of equations</u>:

Method 1 - By Gauss-Jordan Elimination.

- *"Use Gauss-Jordan elimination"*
- *"Use an augmented matrix"*

Manually

$$\begin{cases}-3x+2y=0\\ x-y=-1\end{cases}$$

$$\left[\begin{array}{cc|c}-3 & 2 & 0\\ 1 & -1 & -1\end{array}\right]$$

$$\begin{array}{c}R2\\ R1\end{array}\left[\begin{array}{cc|c}1 & -1 & -1\\ -3 & 2 & 0\end{array}\right]$$

$$\begin{array}{c}\\ 3R1+R2\end{array}\left[\begin{array}{cc|c}1 & -1 & -1\\ 0 & -1 & -3\end{array}\right]$$

$$\begin{cases}4x+3y=5\\ 3x+2y=4\end{cases}$$

$$\left[\begin{array}{cc|c}4 & 3 & 5\\ 3 & 2 & 4\end{array}\right]$$

$$\begin{array}{c}3R1\\ -4R2\end{array}\left[\begin{array}{cc|c}12 & 9 & 15\\ -12 & -8 & -16\end{array}\right]$$

$$\begin{array}{c}\\ R1+R2\end{array}\left[\begin{array}{cc|c}12 & 9 & 15\\ 0 & 1 & -1\end{array}\right]$$

In calculator

$$\begin{cases}-3x+2y=0\\ x-y=-1\end{cases}$$

$$A=\left[\begin{array}{cc|c}-3 & 2 & 0\\ 1 & -1 & -1\end{array}\right]$$

$MATRIX, MATH, rref([A])$

Method 2 – By Using an Inverse Matrix.

- *"Using matrix inverse"*
- *"Solve for X if AX=B"*

In calculator with Equation solved manually first:

Write this.... *....enter A and B in calculator*

$$\begin{cases}4x+3y=5\\ 3x+2y=4\end{cases}$$

and multiply $A^{-1}B$

$$\begin{bmatrix}4 & 3\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 4\end{bmatrix}$$

$$A \quad X \;=\; B$$

$$X=A^{-1}B$$

Method3 – By Using Cramer's Rule.

- *"Using Cramer's Rule"*

Determinants done in calculator with Cramer's Rule shown manually:

Write this.... *Write out all determinants, but compute in calculator : MATRIX, Math,* $\det([A])$

$$\begin{cases}4x+3y=5\\ 3x+2y=4\end{cases}$$

$$|A|=\begin{vmatrix}4 & 3\\ 3 & 2\end{vmatrix}=-1 \qquad |A_x|=\begin{vmatrix}5 & 3\\ 4 & 2\end{vmatrix}=-2 \qquad |A_y|=\begin{vmatrix}4 & 5\\ 3 & 4\end{vmatrix}=1$$

$$\begin{bmatrix}4 & 3\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 4\end{bmatrix}$$

Write variables solved using fractions of determinants

$$A \quad X \;=\; B$$

$$x=\frac{|A_x|}{|A|}=\frac{-2}{-1}=2 \qquad y=\frac{|A_y|}{|A|}=\frac{1}{-1}=-1$$

Solving a matrix equation that doesn't contain matrix multiplication:

1. Solve for X algebraically first.
2. Then plug matrices in and compute the answer.

$$Let\ A=\begin{bmatrix}2 & 3\\ -1 & 1\\ 0 & 2\end{bmatrix}\ and\ B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 0\end{bmatrix}$$

Solve for X if $3X-2A=B$

$$3X=2A+B$$

$$X=\frac{1}{3}(2A+B)$$

$$X=\frac{1}{3}\left(2\begin{bmatrix}2 & 3\\ -1 & 1\\ 0 & 2\end{bmatrix}+\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 0\end{bmatrix}\right)$$

steps in PEMDAS order...

$$X=\begin{bmatrix}5/3 & 5/3\\ -2/3 & 4/3\\ 2/3 & 4/3\end{bmatrix}$$

Finding the inverse of a matrix

Manually:

$$If\ A=\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix}\ find\ A^{-1}$$

$$\begin{bmatrix}2 & 1 & | & 1 & 0\\ -1 & 3 & | & 0 & 1\end{bmatrix}$$

(row operations until)

$$\begin{bmatrix}1 & 0 & | & 3/7 & -1/7\\ 0 & 1 & | & 1/7 & 2/7\end{bmatrix}$$

Check in calculator:
MATRIX, A on command line,
then press X^{-1} *button, enter*

Multiplying two matrices

- *AB*
- *Two matrices written next to each other*

$$\begin{bmatrix}\boxed{-2 \quad -1 \quad 0}\\ x \quad 3 \quad 2\end{bmatrix}\begin{bmatrix}\boxed{\begin{matrix}1\\0\\3\end{matrix}} & \begin{matrix}-4\\2\\1\end{matrix}\end{bmatrix}=\begin{bmatrix}-2 & \\ & \end{bmatrix}\quad(-2)(1)+(-1)(0)+(0)(3)=-2$$

.....

$$\begin{bmatrix}-2 \quad -1 \quad 0\\ \boxed{x \quad 3 \quad 2}\end{bmatrix}\begin{bmatrix}\boxed{\begin{matrix}1\\0\\3\end{matrix}} & \begin{matrix}-4\\2\\1\end{matrix}\end{bmatrix}=\begin{bmatrix}-2 & 6\\ x+6 & \end{bmatrix}\quad(x)(1)+(3)(0)+(2)(3)=x+6$$

Finding the determinant of a matrix

- *det(A) or |A|*
- *"evaluate using the method of expansion by minors and cofactors"*

$$\begin{vmatrix}4 & 3\\ -2 & 5\end{vmatrix}$$

$$(4)(5)-(3)(-2)$$

$$20+6$$

$$26$$

$$\begin{vmatrix}-1 & -3 & 2\\ 2 & 0 & -3\\ 3 & 1 & 4\end{vmatrix}$$

$$\begin{vmatrix}\boxed{-1 \quad -3 \quad 2}\\ 2 \quad 0 \quad -3\\ 3 \quad 1 \quad 4\end{vmatrix}(+)\ (-)\ (+)$$

$$(+)(-1)\begin{vmatrix}0 & -3\\ 1 & 4\end{vmatrix}(-)(-3)\begin{vmatrix}2 & -3\\ 3 & 4\end{vmatrix}(+)(2)\begin{vmatrix}2 & 0\\ 3 & 1\end{vmatrix}$$

$$-1(0-(-3))+3(8-(-9))+2(2-0)$$

$$-3+51+4=52$$

Using a determinant to find area, x for a given area, or if points are collinear

$$Area\ of\ triangle = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\ (throw\ away - if\ negative)$$

$$If\ \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,\ then\ points\ are\ collinear$$

Add a generic point (x, y) *and make* $\det = 0$ *to find equation of a line through* 2 *points*:

$$\begin{vmatrix} x & y & 1 \\ 2 & 5 & 1 \\ 1 & 7 & 1 \end{vmatrix} = 0$$