Compound Interest and Natural Base e

Principal – The initial (starting) amount of an investment or a loan.

Interest rate – An extra amount that is added to the principal each year. For an investment, it is extra money in the account at the end of the year. For a loan, it is extra money you pay each year. Interest rate is usually specified as a percentage annually. (e.g. 5% annual interest rate.)

Compounding – After a period of time (the compounding period) the amount is adjusted and interest is added to the principal. A new compounding period starts, and the end amount becomes the new start amount for the next compounding period.

Example: If you invest $100 in a bank account with a 3% annual interest rate, and the account compounds annually (once at the end of each year):

t, in years	Amount at start of year	Interest earned that year (3%)	Amount at end of year	
1	$100.00	$3.00	$103.00	$= P + rP = P(1+r)$
2	$103.00	$3.09	$106.09	$= [P(1+r)](1+r) = P(1+r)^2$
3	$106.09	$3.18	$109.27	$= \left[P(1+r)^2\right](1+r) = P(1+r)^3$

Compounding annually: $A = P(1+r)^t$

What if we compound each month instead of only at the end of the year?

$$A = P\left(1+\frac{r}{12}\right)^{12t}$$

For compounding 'n' times per year:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

compound annually:	n = 1
compound quarterly:	n = 4
compound monthly:	n=12
compound daily:	n = 365

What if compounded every 'instant'...compounded 'continuously'?

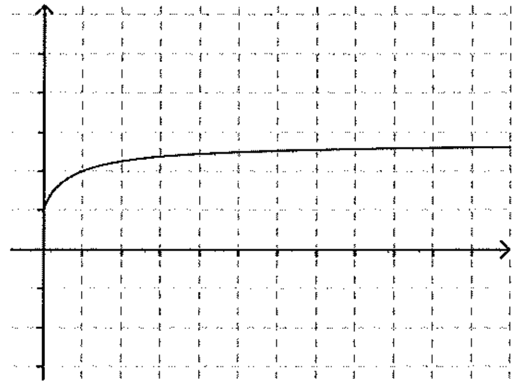
$A = P\left(1+\frac{r}{n}\right)^{nt}$ define $m = \frac{n}{r}$ then, $\frac{r}{n} = \frac{1}{m}$ and $n = rm$

substituting these into the equation:

$A = P\left(1+\frac{1}{m}\right)^{rmt}$ and $A = P\left[\left(1+\frac{1}{m}\right)^{m}\right]^{rt}$

When $n \to \infty, m \to \infty$, so what does the expression in the square brackets do as we compound continuously (as $m \to \infty$)?

m	$\left(1+\frac{1}{m}\right)^m$
1	2.0000
10	2.5937...
100	2.7048...
1000	2.7169...
10000	2.7181...



As m increases, the expression in the brackets approaches a number. That number is called 'e'

$$e = 2.718281828459....$$

e is called the 'natural base' and is an irrational number, like π.

We can then rewrite our compounding equation for the 'continuous compounding' case:

For continuous compounding: $A = Pe^{rt}$