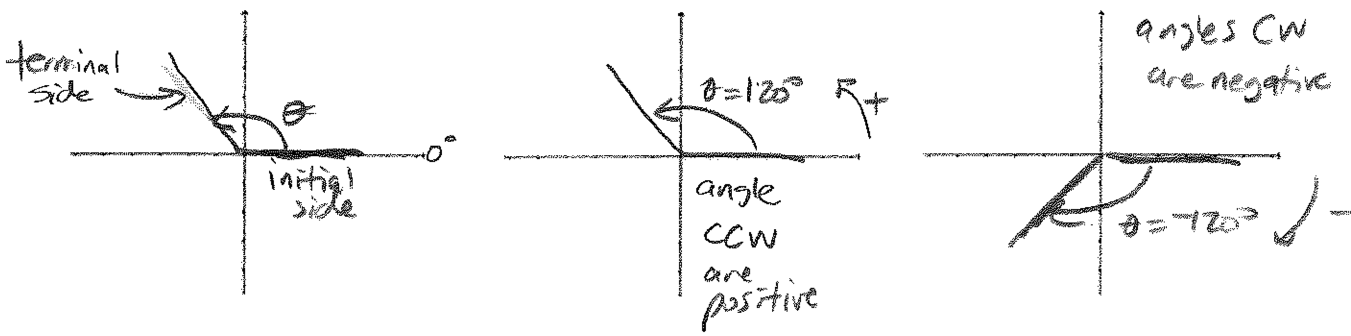
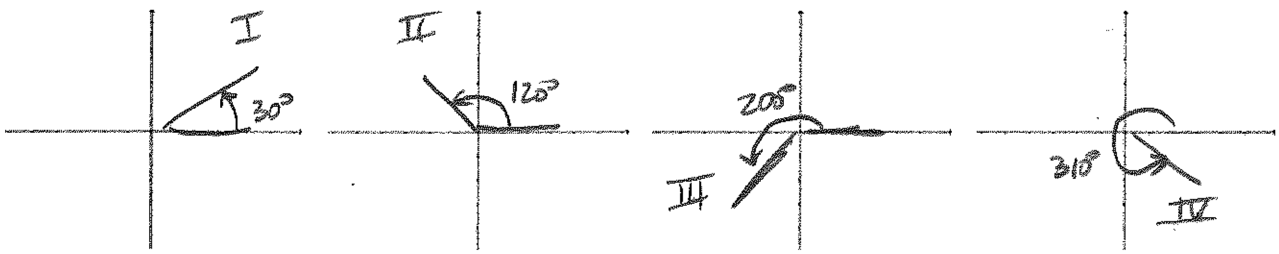
Precalculus – Lesson Notes: Chapter 4 Trigonometry Fundamentals

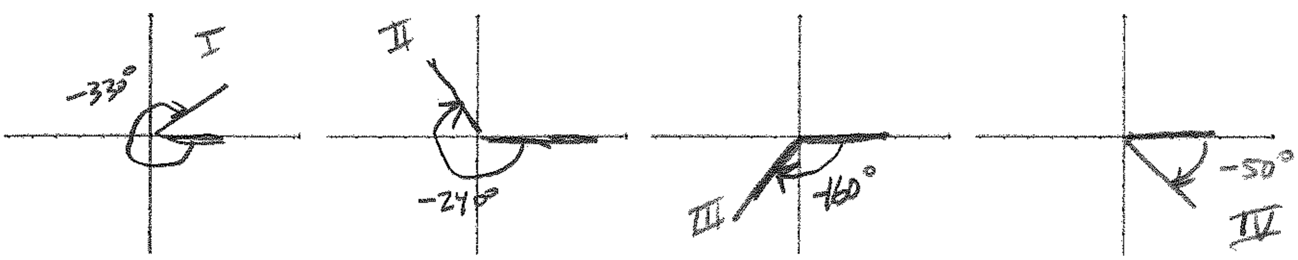
4.1 Radian and Degree measure angles

An angle in 'standard position': 0° to the right (along + x-axis)



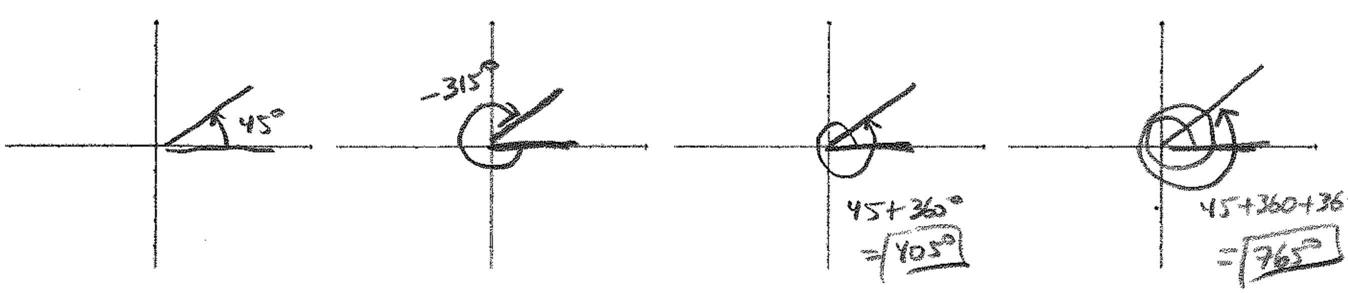
The terminal side can be in any quadrant:



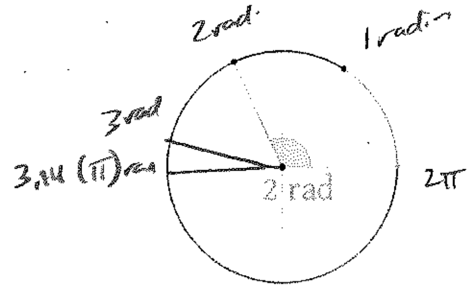


Coterminal angles = angles with the same terminal side.

All of the following are coterminal:



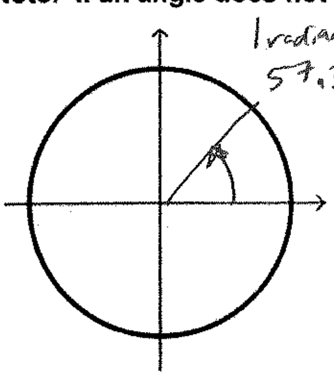
Radians:

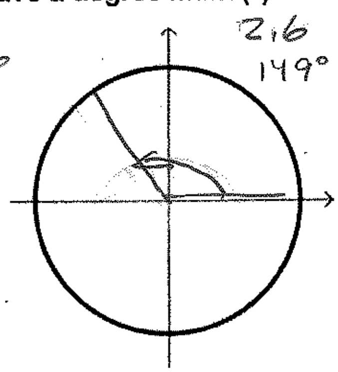


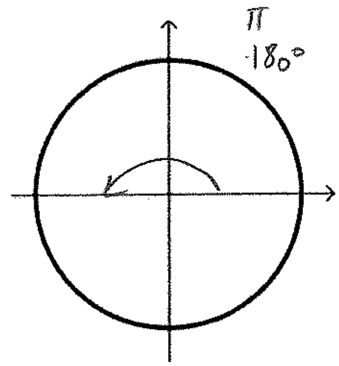
*An angle in radians is how many 'radiuses' around the circle from the x-axis

Any angle can be expressed in either degrees or radians:

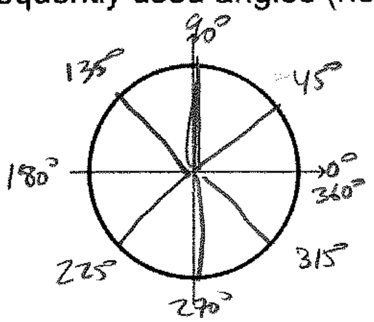
Note: If an angle does not have a degree mark (°) it is in radians.

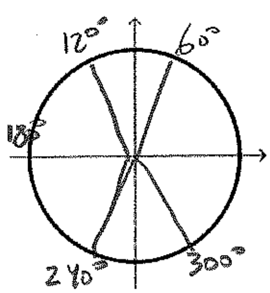


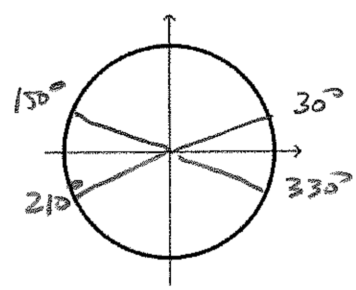


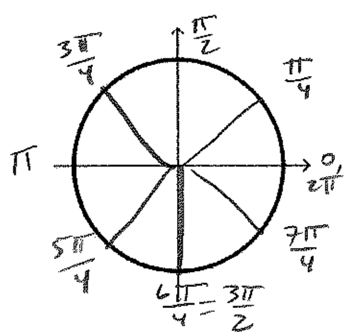


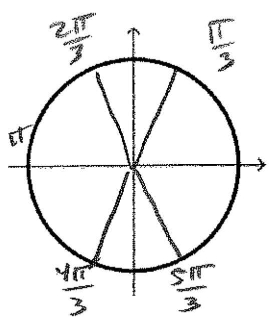
Frequently used angles (how to quickly locate radian measure angles)...

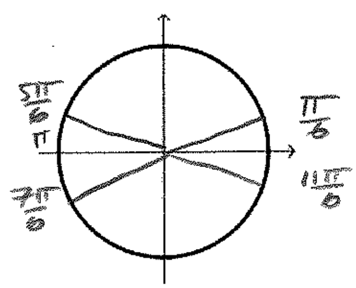












Converting any angle between radians and degrees:

Do unit conversion, like you would to convert feet to yards: $180° = \pi$ radians

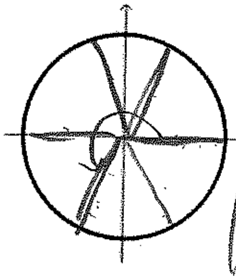
$3\text{ yds}\left(\frac{3\text{ ft}}{1\text{ yd}}\right) = 9\text{ ft}$

$67°\left(\frac{\pi}{180°}\right) = \frac{67}{180}\pi = \boxed{\frac{67}{180}\pi}$

$2.3\left(\frac{180°}{\pi}\right) = \boxed{131.78°}$

Finding coterminal angles in radians: (add or subtract 2π radians)

– you can use unit circle (if special angle).

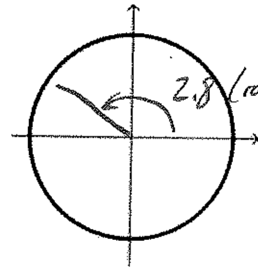


$\theta = \frac{4\pi}{3}$

coterminal:

$\boxed{\theta = \frac{-2\pi}{3} \quad \theta = \frac{10\pi}{3}}$

not a special angle? add/subtract 2π



$\theta = 2.8$

coterminal:

$2.8 + 2\pi = \boxed{9.083}$

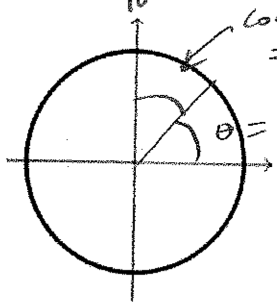
$2.8 - 2\pi = \boxed{-3.483}$

– or add/subtract 2π:

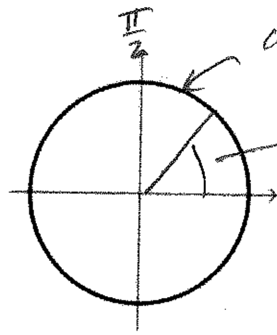
$\frac{4\pi}{3} + \frac{2\pi}{1} = \frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

$\frac{4\pi}{3} - \frac{2\pi}{1} = \frac{4\pi}{3} - \frac{6\pi}{3} = \frac{-2\pi}{3}$

Complementary angles: angles that add to 90°, angles that add to $\frac{\pi}{2}$ radian



complement $= 90° - 50° = \boxed{40°}$

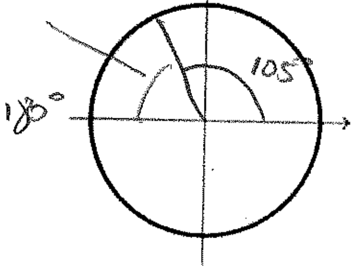


complement $= \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \boxed{\frac{\pi}{6}}$

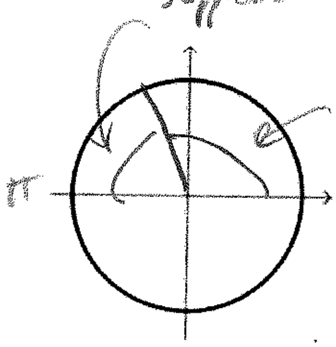
$\theta = \frac{\pi}{3}$

Supplementary angles: angles that add to 180°, angles that add to π radians

supplement $= 180° - 105° = \boxed{75°}$



supplement $= \pi - 2.1 = \boxed{1.0416}$



$\theta = 2.1$ (radians)

4.1 day 2: Degrees-Minutes-Seconds (DMS) and Arc length problems

How do we indicate a fraction of a degree?

Could use $41\frac{3}{4}°$, but usually we use Degrees-Minutes-Seconds (DMS):

1 degree = 60 minutes (60')
1 minute = 60 seconds (60")

Converting means multiplying or dividing by 60

Examples: $41\frac{3}{4}° = \boxed{41°45'}$

$\frac{3°}{4} \cdot \frac{60'}{1°} = 45'$

$127°15'\underline{45}" =$

$45" \cdot \frac{1'}{60"} = 0.75'$

$127°15.75'$

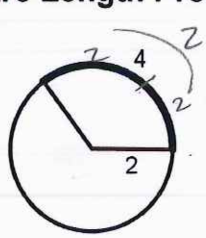
$15.75' \cdot \frac{1°}{60'} = 0.2625$

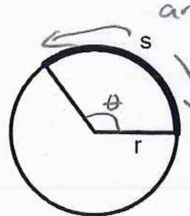
$\boxed{127.2625°}$

Calculator feature – degree conversions:

- Decimal degrees to DMS:
  - Enter decimal degrees.
  - 2nd – APPS (angle screen).
  - >DMS, enter twice.
- DMS to decimal degrees: example, convert $127°15'45"$
  - Enter 127, 2nd-APPS(angle), select degree mark, enter
  - Enter 15, 2nd-APPS(angle), select minutes mark, enter
  - Enter 45, Alpha-+key (" mark), enter.

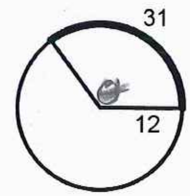
Arc Length Problems:





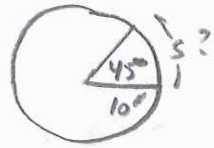
$\theta_{rad} = \frac{s}{r}$

so $\boxed{s = r\theta_{rad}}$
(arc length)



if arclength is 31 and radius is 12 what is angle?

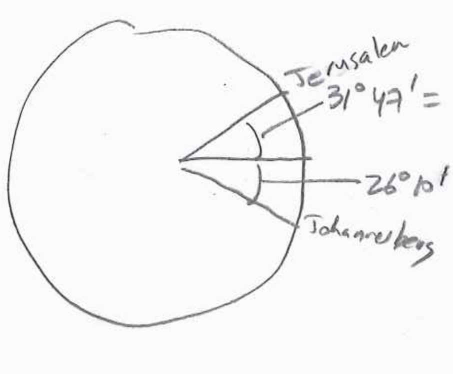
$\theta = \frac{s}{r} = \frac{31}{12} = (2.58\overline{3}\ \text{rad.})$

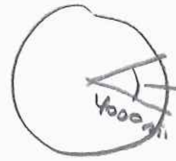


$s = r\theta_{rad}$

$s = 10\left(45\frac{\pi}{180}\right) = \boxed{7.85"}$

Example: Find the north-south distance between Jerusalem, Israel (with lattitude $31°47'N$) and Johannesberg, South Africa (with lattitude $26°10'S$). The radius of the earth is approximately 4000 miles.



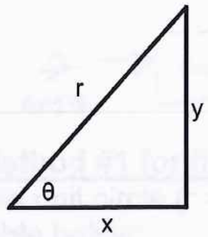


$s = 4000\left(57.95° \frac{\pi}{180°}\right)$

$= \boxed{4045.7 \text{ miles}}$

4.2 day 1: *Unit Circle Definition of Sine and Cosine*

Remember from geometry:



$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

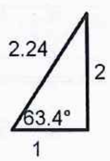
$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

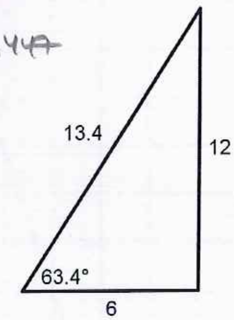
$x = r\cos\theta$

$y = r\sin\theta$

Sine, cosine, and tangent are ratios, so they don't really depend upon the size of the triangle, only on the angle:

$\cos 63.4° = \frac{1}{2.24} = 0.447$

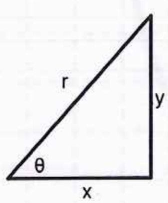




$\cos 63.4° = \frac{6}{13.4} = 0.447$

If we think of the angle as the input and the ratio as a number which is the output, sine and cosine can also be thought of, not just as ratios, but also as functions. Here is how the sine and cosine functions are defined:

1) Start with this triangle…

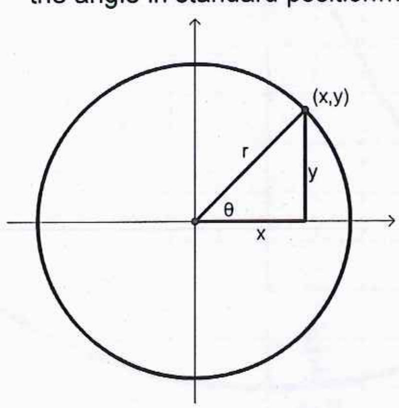


…for which:

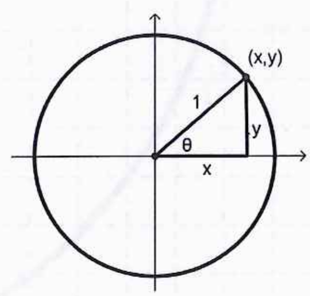
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

2) Put this triangle on a circle with the angle in standard position…



3) Reduce the radius to 1 unit…



…and on this 'unit circle'…

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

The input to the sine and cosine functions is an angle. If this angle is in standard position on a 'unit circle' (radius 1), we define:

$\cos\theta$ = the x-coordinate of the point on the unit circle at angle θ

$\sin\theta$ = the y-coordinate of the point on the unit circle at angle θ

The angle can be any angle…in degrees or radians can be positive or negative.

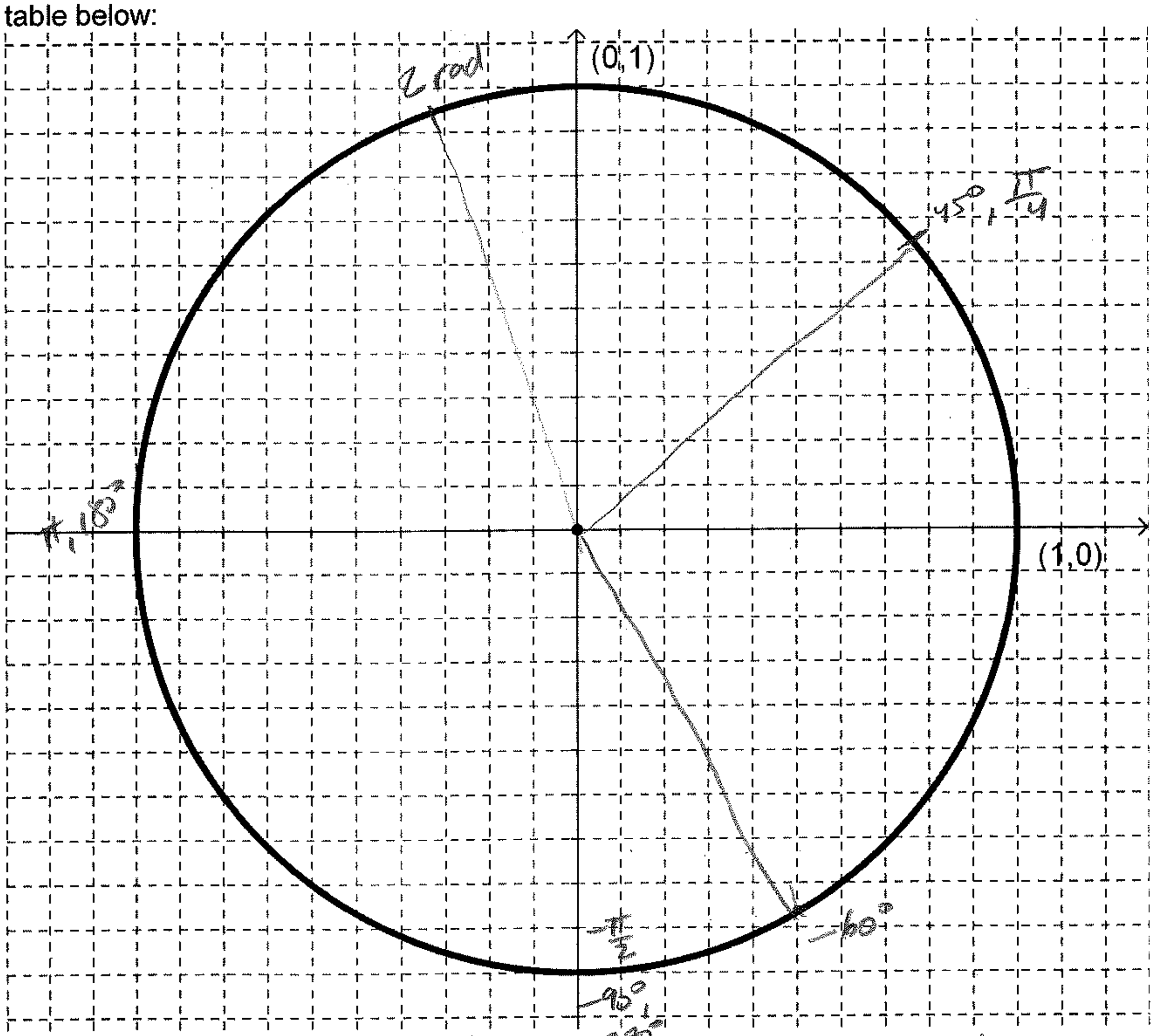
We define cosine and sine functions to be the x and y coordinates of a point on the unit circle with angle in standard position.

* different kind of function: no algebraic rule, must use graphical procedure.

θ (angle) → [cos] → number, x-coordinate

θ (angle) → [sin] → number, y-coordinate

Method #1 for finding sine and cosine of an angle #1 – Measuring x or y: Use the grid on this unit circle (r = 1) to estimate the value for cosine and sine of each angle to complete the table below:

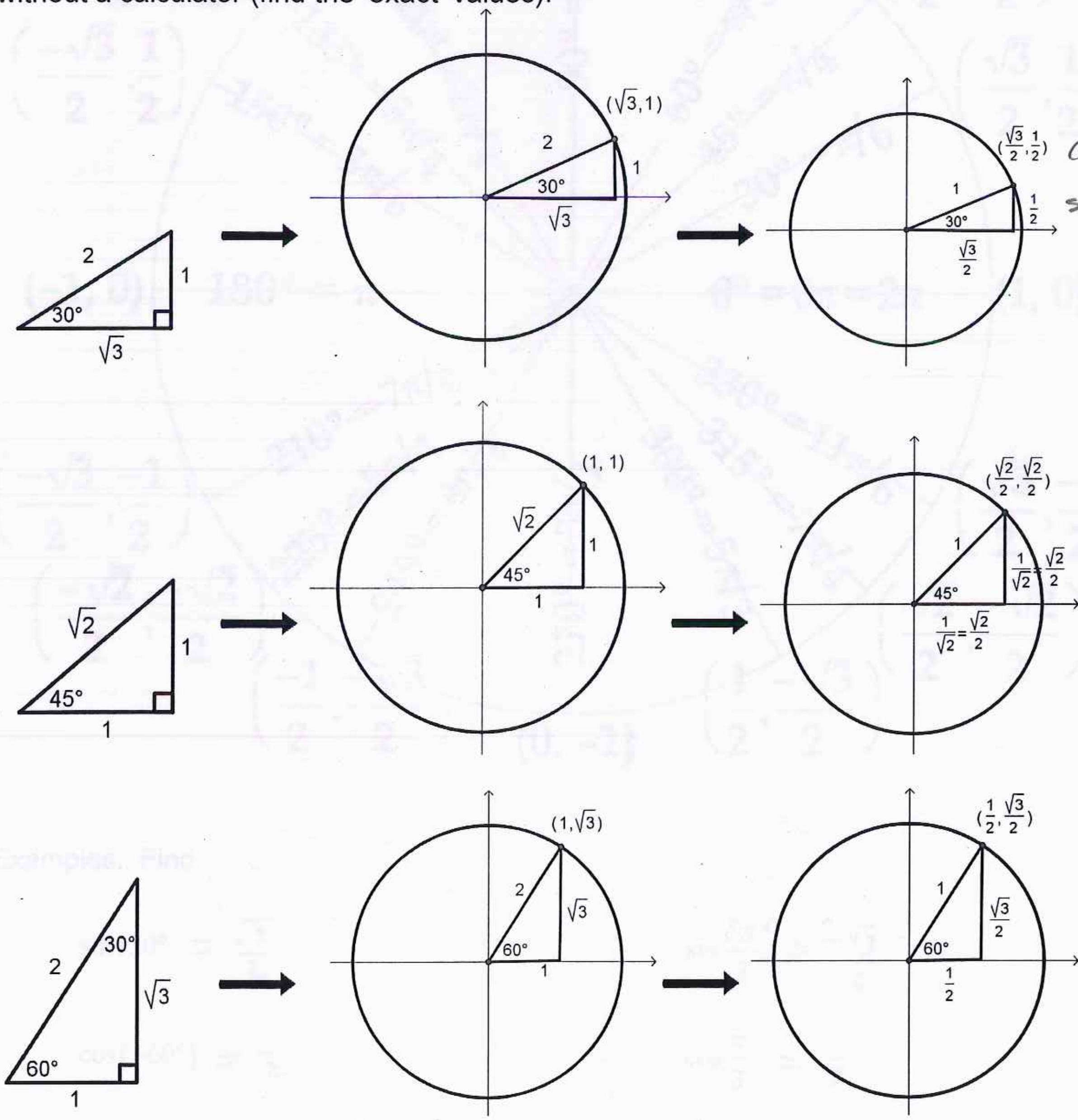


angle θ(deg)	$\cos\theta$ (x)	$\sin\theta$ (y)
45°	0.71	0.71
180°	−1	0
−60°	0.5	−0.86
630° = 270	0	−1
−90°	0	−1

angle θ(radians)	$\cos\theta$ (x)	$\sin\theta$ (y)
$\frac{\pi}{4}$	0.71	0.71
$-\frac{\pi}{2}$	0	−1
0	1	0
2	−0.31	0.84
-1.5	0.19	−0.98

Method #2 for finding sine and cosine of an angle – Calculator: You can also find cosine or sine of an angle with your calculator. Go back and verify a few of these values in the table with your calculator (be sure to set mode to 'degrees' or 'radians' as needed.)

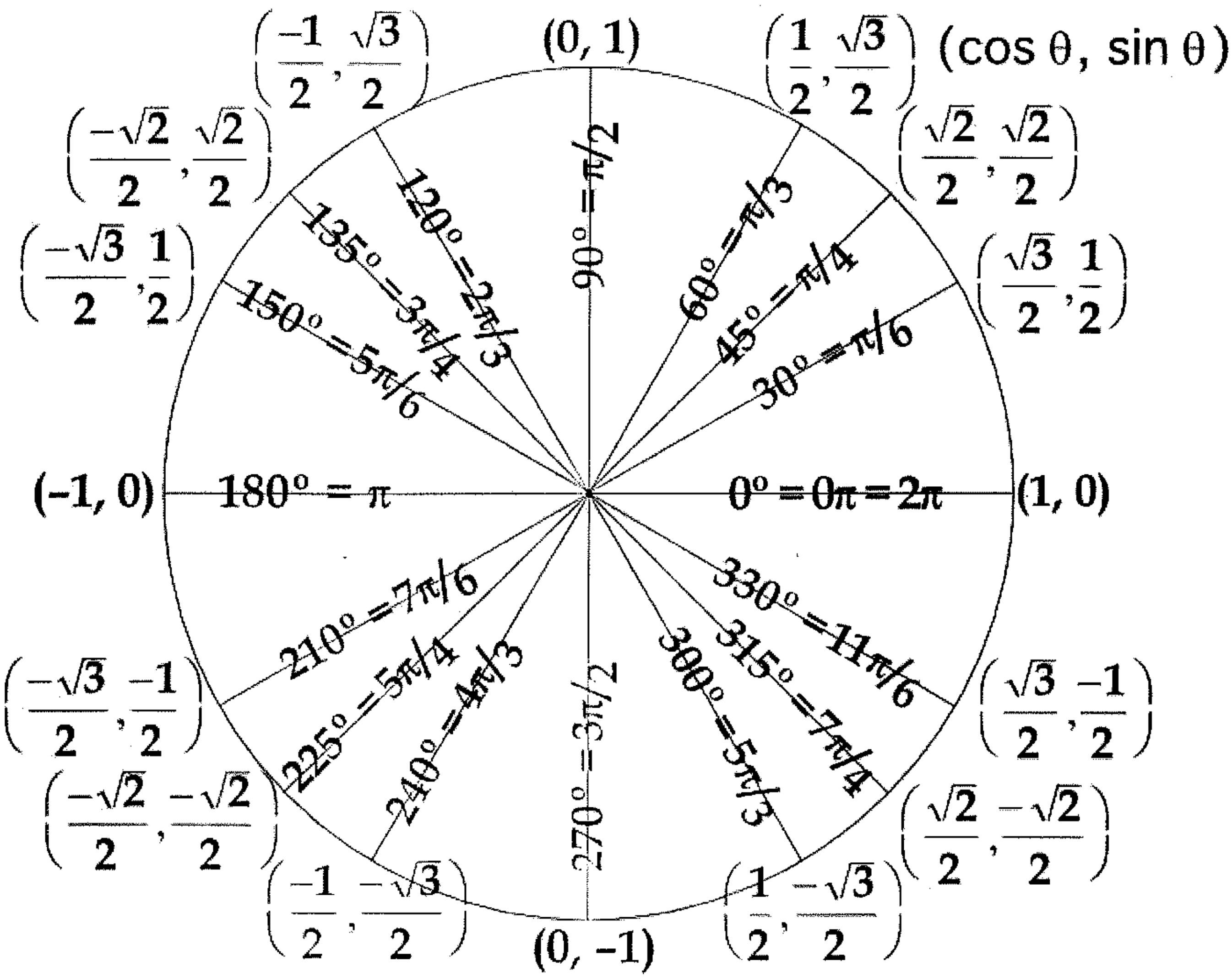
Method #3 for finding sine and cosine of an angle – Unit Circle chart:
For some 'special' values of θ (multiples of 30 and 45 degrees), we can find cosine or sine without a calculator (find the 'exact' values).



$\cos 30° = \frac{\sqrt{3}}{2}$ (0.866)

$\sin 30° = \frac{1}{2}$ (0.5)

The 'unit circle' chart is a list of all the special values. Multiples of 30 and 45 degrees are found by using symmetry of the 30, 45 and 60 degree values. You can only use the chart to find sine and cosine for multiples of 30 and 45 degrees (use a calculator to find any values for 'non-special' angles).



Examples...Find:

$\sin 120° = \frac{\sqrt{3}}{2}$ $\qquad$ $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos(-60°) = \frac{1}{2}$ $\qquad$ $\cos\frac{\pi}{3} = \frac{1}{2}$

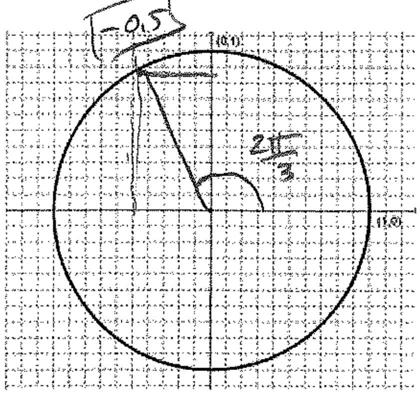
$\sin 240° = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sin\frac{\pi}{2} = 1$

4.2 day 2: *'Memorize' unit circle, other 4 trigonometric functions*

Review:

Values for sine and cosine or an angle can be found in 3 different ways: $\cos\left(\frac{2\pi}{3}\right)$

1) Measure x (cos) or y (sin)



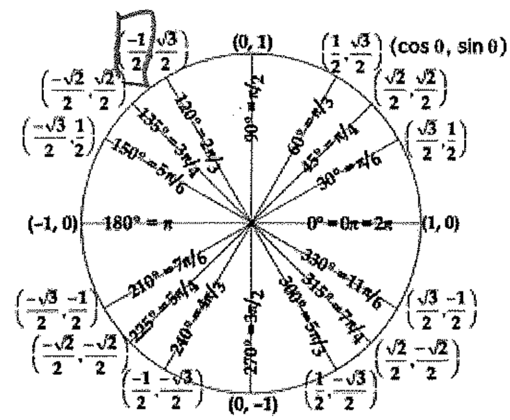
2) Calculator

(make sure you are in degrees or radians mode as appropriate)

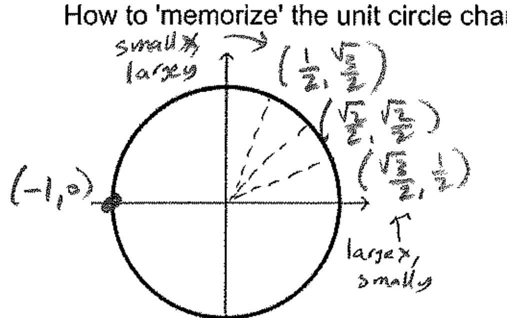
$\cos\left(\frac{2\pi}{3}\right)$ rad mode

$\boxed{-0.5}$

3) Unit circle chart for special angles



How to 'memorize' the unit circle chart:

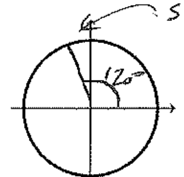


3 'special' angles in each quadrant

values for sin, cos: $\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}$

smallest — largest

$\cos(120°)$

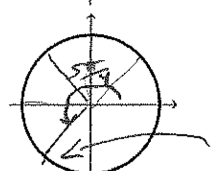


smallest x, largest y

x (−), y (+)

cos is x, so $\cos(120°) = \boxed{-\frac{1}{2}}$

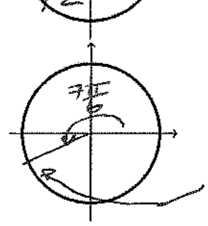
$\sin\left(\frac{5\pi}{4}\right)$



'middle' values for x & y, both (−)

$\sin\left(\frac{5\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

$\cos\left(\frac{7\pi}{6}\right)$



largest x (−), smallest y (−), cos is x

$\cos\left(\frac{7\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

Other trigonometry functions:

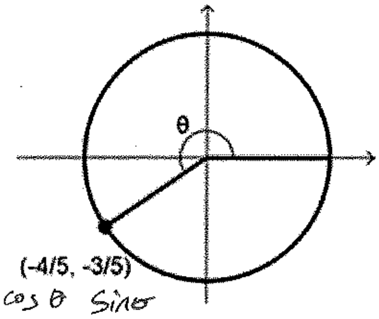
From sin(t) and cos(t) we can define 4 additional trigonometric functions, for a total of 6 trigonometric functions
(these are general functions...the independent variable could be anything, but we evaluate by treating the input variable as an angle):

sine: $\sin(t) = \frac{1}{\csc t}$ cosecant: $\csc(t) = \frac{1}{\sin t}$

cosine: $\cos(t) = \frac{1}{\sec t}$ secant: $\sec(t) = \frac{1}{\cos t}$

tangent: $\tan(t) = \frac{\sin t}{\cos t}$ cotangent: $\cot(t) = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$

Find all 6 trig functions:



$\cos\theta$ $\sin\theta$

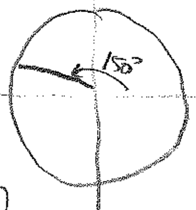
$\sin\theta = \boxed{\frac{-3}{5}}$ $\qquad \csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{-5}{3}}$

$\cos\theta = \boxed{\frac{-4}{5}}$ $\qquad \sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{-5}{4}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{3}{5}\right)5}{\left(-\frac{4}{5}\right)5} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$ $\qquad \cot\theta = \frac{1}{\tan\theta} = \boxed{\frac{4}{3}}$

$\theta = 150°$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$\sin\theta = \boxed{\frac{1}{2}} \rightarrow \csc\theta = \boxed{2}$

$\cos\theta = \boxed{\frac{-\sqrt{3}}{2}} \rightarrow \sec\theta = \frac{-2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/2}{-\sqrt{3}/2} = \frac{-1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$ $\qquad \cot\theta = \boxed{-\sqrt{3}}$

$\theta = 238°$ (not a 'special' angle, so use calculator)

$(-.5299, -.8480)$

$\sin\theta = \boxed{-.8480}$ $\qquad \csc\theta = \frac{1}{-.8480} = \boxed{-1.1792}$

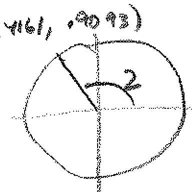
$\cos\theta = \boxed{-.5299}$ $\qquad \sec\theta = \frac{1}{-.5299} = \boxed{-1.8871}$

$\tan\theta = \frac{-.8480}{-.5299} = \boxed{1.600}$

$\cot\theta = \frac{1}{1.600} = \boxed{.6249}$

$\theta = 2$ (not 'special', calculator radian mode

$(-.4161, .9093)$



$\sin\theta = \boxed{.9093}$ $\qquad \csc\theta = \frac{1}{.9093} = \boxed{1.0997}$

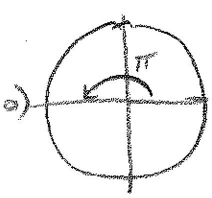
$\cos\theta = \boxed{-.4161}$ $\qquad \sec\theta = \boxed{-2.4032}$

$\tan\theta = \frac{.9093}{-.4161} = \boxed{-2.1853}$

$\cot\theta = \boxed{-.4576}$

$\theta = \pi$ (special angle)

$(-1, 0)$



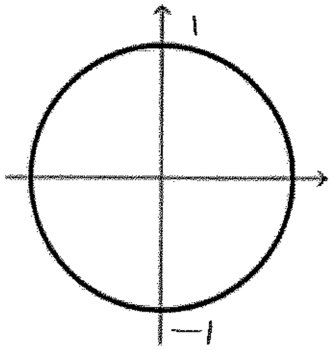
$\sin\theta = \boxed{0}$ $\qquad \csc\theta = \frac{1}{0}$ $\boxed{\text{undef}}$

$\cos\theta = \boxed{-1}$ $\qquad \sec\theta = \frac{1}{-1} = \boxed{-1}$

$\tan\theta = \frac{0}{-1} = \boxed{0}$ $\qquad \cot\theta = \frac{-1}{0} = \boxed{\text{undef}}$

4.2 day 3: Symmetries and Properties of the trigonometric functions

Domain and range of sine and cosine functions:



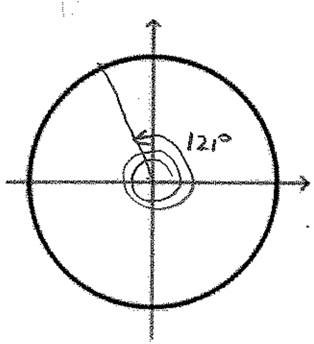
We can put any angle into the sine or cosine functions, so domain is:

$$D:(-\infty,\infty)$$

What is the range?

$R: [-1, 1]$

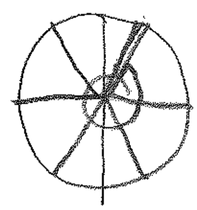
Periodicity of Sine and Cosine:



What is $\sin(841°)$? $841° - 360° = 481° - 360° = 121°$

$\sin(841°) = \sin(121°) = 0.8572$

What is $\cos\frac{7\pi}{3}$?



$\cos\left(\frac{7\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

The values for both sine and cosine repeat every 2π

A function that repeats this way is called a **periodic function:**

$$f(\theta+p)=f(\theta)$$
$$\sin(\theta+2\pi)=\sin(\theta)$$
$$\cos(\theta+2\pi)=\cos(\theta)$$

The **period** of the sine and cosine functions is 2π

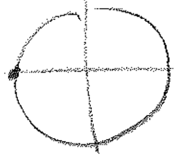
Homework examples..

#29. Evaluate the trigonometric function using its period as an aid.

$\sin 5\pi$ $5\pi - 2\pi = 3\pi - 2\pi = \pi$

$= \sin(\pi) = \boxed{0}$

$-3\pi, 5\pi, \pi$



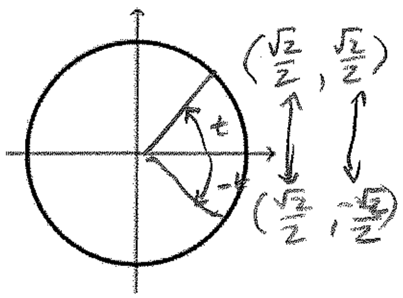
$\cos(-3\pi) = \cos(\pi) = \boxed{-1}$

$-3\pi + 2\pi = -\pi + 2\pi = \pi$

Even/Odd functions: remember, definition of even and odd functions:

Even function if $f(-x) = f(x)$

Odd function if $f(-x) = -f(x)$



$\cos(-t) = \cos(t)$ so <u>cosine is an even function</u>

$\sin(-t) = -\sin(t)$ so <u>sine is an odd function</u>

Other trig functions are found by using sin and cos:

$\tan(-t) = -\tan(t)$ (odd) $\qquad \cot(-t) = -\cot(t)$ (odd)

$\csc(-t) = -\csc(t)$ (odd) $\qquad \sec(-t) = \sec(t)$ (even)

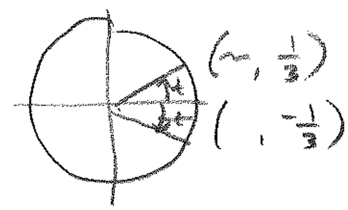
Homework examples..

#37. Use the value of the trigonometric function to evaluate the indicated function.

$\sin t = \frac{1}{3}$

a) $\sin(-t) = \boxed{-\frac{1}{3}}$

b) $\csc(-t) = \frac{1}{\sin(-t)} = \boxed{-3}$

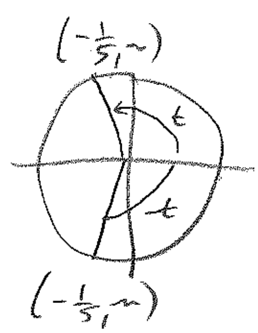


#39. Use the value of the trigonometric function to evaluate the indicated function.

$\cos(-t) = -\frac{1}{5}$

a) $\cos t = \boxed{-\frac{1}{5}}$

b) $\sec(-t) = \boxed{-5}$

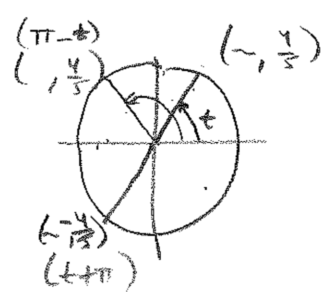


#41. Use the value of the trigonometric function to evaluate the indicated function.

$\sin t = \frac{4}{5}$

a) $\sin(\pi - t) = \boxed{\frac{4}{5}}$

b) $\sin(t + \pi) = \boxed{\frac{-4}{5}}$

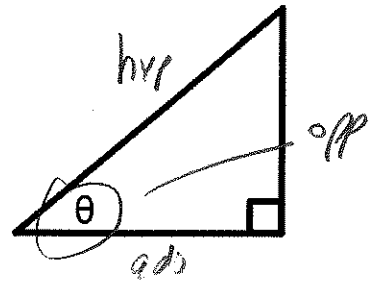


4.3 day 1: Right triangle trigonometry

Right triangle definitions of the 6 trig functions:

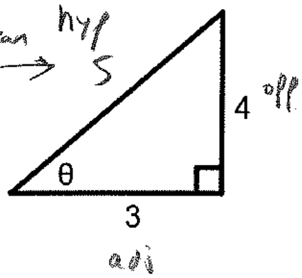
$\sin\theta = \dfrac{opp}{hyp}$ $\quad\cos\theta = \dfrac{adj}{hyp}$ $\quad\tan\theta = \dfrac{opp}{adj}$

$\csc\theta = \dfrac{hyp}{opp}$ $\quad\sec\theta = \dfrac{hyp}{adj}$ $\quad\cot\theta = \dfrac{adj}{opp}$



Examples...

(Pythagorean triple) →



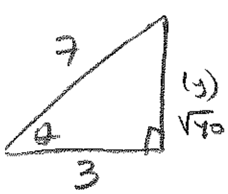
#1. Find all 6 trig functions:

$\sin\theta = \frac{4}{5}$ $\quad\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad\cot\theta = \frac{3}{4}$

#2. Sketch the triangle, if $\cos\theta = \dfrac{3}{7} = \dfrac{adj}{hyp}$



$3^2 + y^2 = 7^2$

$9 + y^2 = 49$

$y^2 = 40$

$y = \sqrt{40}$

Could find other trig functions:

$\sin\theta = \frac{\sqrt{40}}{7}$ $\quad\csc\theta = \frac{7}{\sqrt{40}}\frac{\sqrt{40}}{\sqrt{40}} = \frac{7\sqrt{40}}{40}$

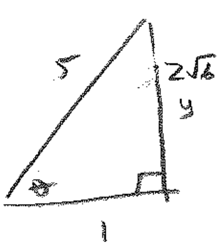
$\cos\theta = \frac{3}{7}$ $\quad\sec\theta = \frac{7}{3}$

$\tan\theta = \frac{\sqrt{40}}{3}$ $\quad\cot\theta = \frac{3}{\sqrt{40}}\frac{\sqrt{40}}{\sqrt{40}} = \frac{3\sqrt{40}}{40}$

#3. Given $\sec\theta = 5$ and $\tan\theta = 2\sqrt{6}$, find $\cos\theta, \cot\theta, \cot(90° - \theta), \sin\theta$

$\sec\theta = 5$

$\cos\theta = \frac{1}{5}$ $\quad \frac{adj}{hyp}$



$1^2 + y^2 = 5^2$

$1 + y^2 = 25$

$y^2 = 24$

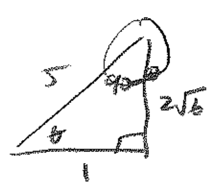
$y = \sqrt{24}$

$y = \sqrt{4}\sqrt{6}$

$y = 2\sqrt{6}$

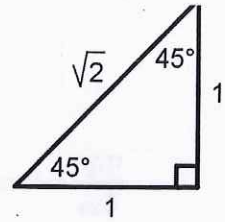
$\cos\theta = \frac{adj}{hyp} = \boxed{\frac{1}{5}}$ $\quad\sin\theta = \frac{opp}{hyp} = \boxed{\frac{2\sqrt{6}}{5}}$

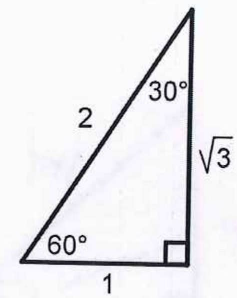
$\cot\theta = \frac{1}{2\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{2 \cdot 6} = \boxed{\frac{\sqrt{6}}{12}}$



$\cot(90° - \theta) = \frac{adj}{opp} = \frac{2\sqrt{6}}{1} = \boxed{2\sqrt{6}}$

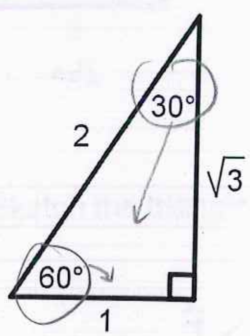
Special Right triangles (memorize these):





#4. Find $\cos 60° = \frac{adj}{hyp} = \boxed{\frac{1}{2}}$ (special angle value)

Cofunctions:



Find $\cos 60° = \frac{1}{2}$

Find $\sin 30° = \sin(90° - 60°) = \frac{1}{2}$

$\sin(90° - \theta) = \cos\theta$ $\quad$ $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$ $\quad$ $\cot(90° - \theta) = \tan\theta$

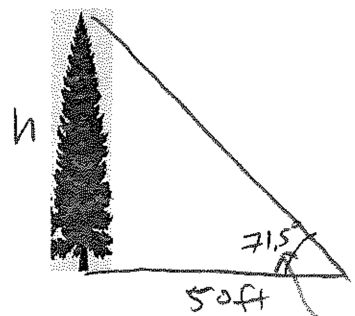
$\sec(90° - \theta) = \csc\theta$ $\quad$ $\csc(90° - \theta) = \sec\theta$

4.3 day 2: Right triangle application problems

Real-world application problems

Angle of Elevation:

Example: A surveyor is standing 50 feet from the base of a large tree. The surveyor measures the angle of elevation to the top of the tree as 71.5°. How tall is the tree?



angle of elevation

$\tan\theta = \frac{opp}{adj}$

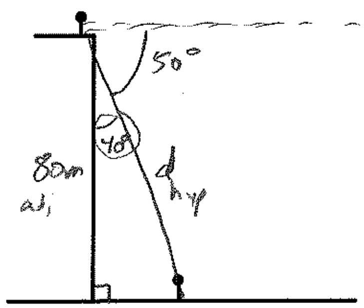
$\tan 71.5° = \frac{h}{50}$

$h = 50\tan 71.5°$

$\boxed{h = 149.4\text{ ft}}$

Angle of Depression:

Example: You are standing at the edge of the roof of an 80m tall building. Your friend is on the ground below. If you measure the angle of depression to your friend to be 50°, and you can be heard (if you shout) from 100m away, will your friend hear you if you shout?



$\cos\theta = \frac{adj}{hyp}$

$\cos 40° = \frac{80}{d}$

$\frac{\cos 45°}{1} = \frac{80}{d}$

$d\cos 40° = 80$

$d = \frac{80}{\cos 40°}$

$d = 104.4\text{ m}$

(no, too far to hear)

Notice:

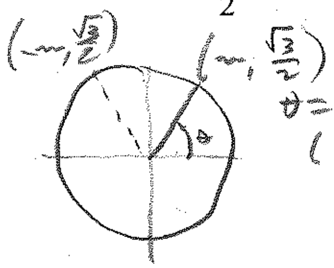
$\cos 40° = \frac{80}{d}$

became swap

$d = \frac{80}{\cos 45°}$

Given a trig function value, you can find the angle (with unit circle or with calculator)

Example: $\sin\theta = \frac{\sqrt{3}}{2}$, find θ in radians and degrees:



$\theta = 60^\circ$ or 120°
$\left(\frac{\pi}{3}\text{ or }\frac{2\pi}{3}\right)$

Calculator for any angle:

θ angle $\rightarrow$ $\boxed{\sin}$ $\rightarrow$ y-coord.

y-coord $\rightarrow$ $\boxed{\sin^{-1}}$ $\rightarrow$ θ angle

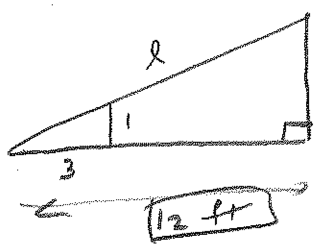
is $\sin\theta = \frac{\sqrt{3}}{2}$

$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$

(calculator can only give one angle)

Inverse trig function calculator problems:
You want to build a skateboard ramp that rises 1 foot for every 3 feet of horizontal length. If the ramp is to be 4 feet tall, find the lengths of the other sides, and the angle the ramp makes with the ground.



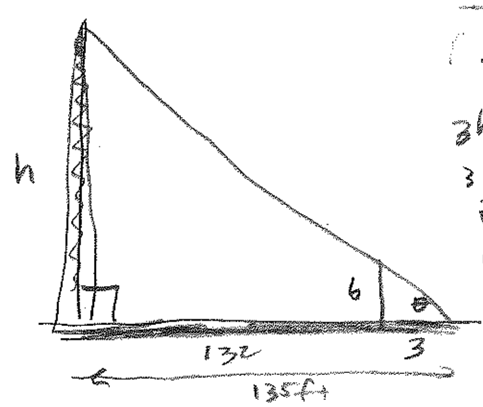
$l^2 = 4^2 + 12^2$
$l^2 = 16 + 144$
$l^2 = 160$
$l = \sqrt{160} = \boxed{12.65\text{ ft}}$

$\tan\theta = \frac{opp}{adj}$
$\tan\theta = \frac{1}{3}$
$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{1}{3}\right)$
$\theta = \tan^{-1}\left(\frac{1}{3}\right)$
$\boxed{\theta = 18.4^\circ}$

You can use 2nd sin (or cos or tan) to find the angle if you know sin (or cos or tan).

Homework problem #63:
A 6 ft person walks from the base of a broadcasting tower directly toward the tip of the shadow ~~case~~ caused by the tower. When the person is 132 ft from the tower and 3 ft from the tip of the tower's shadow, the person's shadow starts to appear beyond the tower's shadow.

(a) Draw a picture to represent the problem, including the unknown height of the tower.
(b) Write an equation involving the unknown.
(c) What is the height of the tower?



using ratios

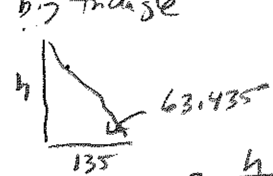
$\frac{6}{3} = \frac{h}{135}$
$3h = 6(135)$
$3h = 810$
$\boxed{h = 270\text{ ft}}$

using trig

small triangle

$\tan\theta = \frac{6}{3} = 2$
$\theta = \tan^{-1}(2)$
$(\theta = 63.435^\circ)$

big triangle



$\tan 63.435^\circ = \frac{h}{135}$
$h = 135\tan 63.435^\circ$
$\boxed{h = 270\text{ ft}}$

4.4: Reference angles, Finding trig functions or angles given points, constraints.

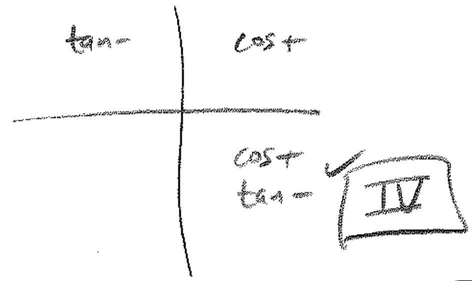
Cosine=x, Sine=y
and the other 4 trig functions are defined from sine and cosine.
Signs of the 6 trig functions in each quadrant:

Quadrant II	Quadrant I
sinθ + cscθ + cosθ − secθ − tanθ − cotθ − Students	sinθ + cscθ + cosθ + secθ + tanθ + cotθ + All
Quadrant III	**Quadrant IV**
sinθ − cscθ − cosθ − secθ − tanθ + cotθ + Take	sinθ − cscθ − cosθ + secθ + tanθ − cotθ − Calculus

State in which quadrant the angle lies:

$\sec\theta > 0$ *and* $\cot\theta < 0$

cos (+)   tan (−)



Procedure for sketching:
1) determine quadrant, and draw in a radius,
2) start at origin and draw x, then y, to make a triangle
3) fill in 2 values you are given
4) solve for 3rd value
5) use sketching rules for trig values:

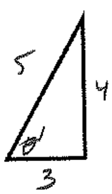
$\sin\theta = \frac{y}{r}$ (r is always positive)

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Find sin, cos, tan of an angle given that point (3, 4) is on the terminal side of the angle.

Method #1) Use a triangle

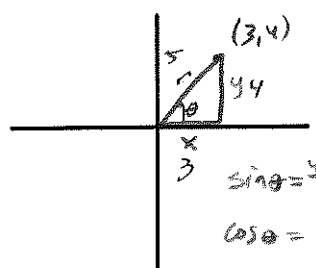


$\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$

$\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$

$\tan\theta = \frac{opp}{adj} = \frac{4}{3}$

Method #2) Use an x-y sketch



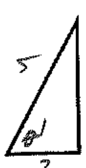
$\sin\theta = \frac{4}{5}$ $\left(\frac{y}{r}\right)$

$\cos\theta = \frac{3}{5}$ $\left(\frac{x}{r}\right)$

$\tan\theta = \frac{4}{3}$ $\left(\frac{y}{x}\right)$

Find sin, cos, tan of an angle given that point (-3, 4) is on the terminal side of the angle.

Method #1) Use a triangle



$\sin\theta' = \frac{4}{5}$

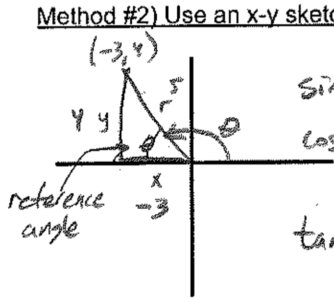
$\cos\theta' = \frac{3}{5}$

$\tan\theta' = \frac{4}{3}$

in II, cos & tan (−)

So $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{-3}{5}$, $\tan\theta = \frac{-4}{3}$

Method #2) Use an x-y sketch



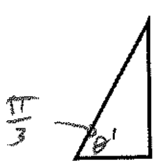
$\sin\theta = \frac{4}{5}$ $\left(\frac{y}{r}\right)$

$\cos\theta = \frac{-3}{5}$ $\left(\frac{x}{r}\right)$

$\tan\theta = \frac{4}{-3}$ $\left(\frac{y}{x}\right)$

Find sin, cos, tan of the angle $\theta = \frac{5\pi}{3}$

Method #1) Use a triangle



$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

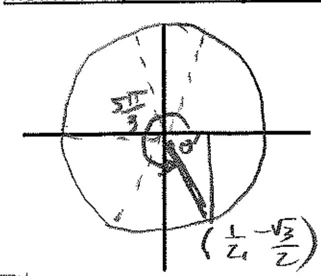
$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

in IV, sin, tan (−)

So $\sin\frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$, $\cos\frac{5\pi}{3} = \frac{1}{2}$, $\tan\frac{5\pi}{3} = -\sqrt{3}$

Method #2) Use an x-y sketch



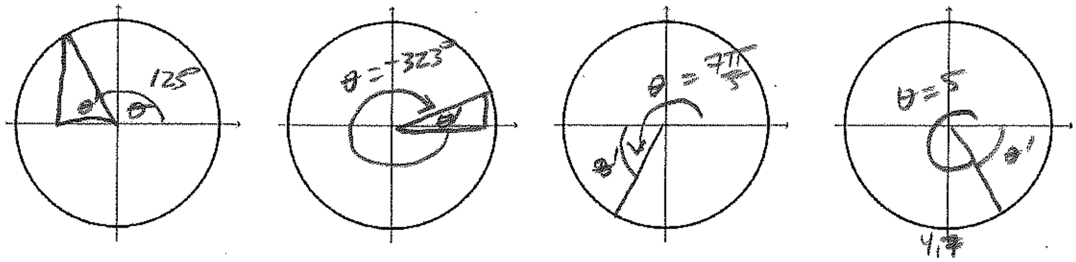
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

$\tan\theta = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

Reference angle = The acute angle θ' formed by the terminal side of θ and the horiz (x) axis. (always positive, always < 90°)



$\theta = 125°$

$\theta' = 180° - 125° = \boxed{55°}$

$\theta = -323°$

$\theta' = 360° - 323° = \boxed{37°}$

$\theta = \dfrac{7\pi}{5}$

$\theta' = \frac{7\pi}{5} - \pi = \frac{7\pi}{5} - \frac{5\pi}{5} = \boxed{\frac{2\pi}{5}}$

$\theta = 5$

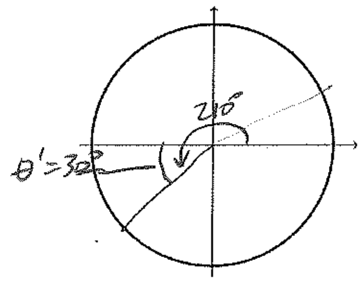
$\theta' = 2\pi - 5 = \boxed{1.283}$

Can use reference angle to evaluate a trig function:

Ex: Find $\sin 210°$

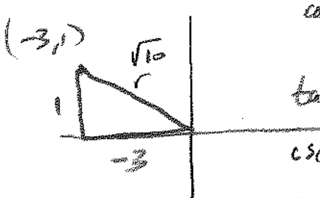
$\sin 30° = \frac{1}{2}$

so $\sin 210° = \boxed{-\frac{1}{2}}$



Given x, y coordinates of a point (any radius) find 6 trig functions of the angle.

(-3, 1)



$1^2 + 3^2 = r^2$

$1 + 9 = r^2$

$r^2 = 10$

$r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{-3\sqrt{10}}{10}}$

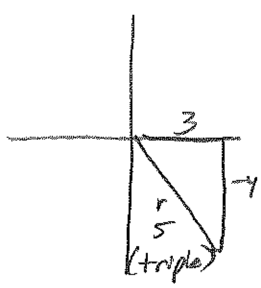
$\tan\theta = \frac{y}{x} = \boxed{\frac{1}{-3}}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{\sqrt{10}}$

$\sec\theta = \frac{1}{\cos\theta} = \boxed{-\frac{\sqrt{10}}{3}}$

$\cot\theta = \boxed{-3}$

(3, -4)



$\sin\theta = \frac{y}{r} = \boxed{\frac{-4}{5}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{3}{5}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{-4}{3}}$

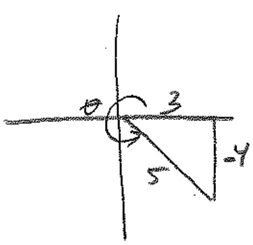
$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{-5}{4}}$

$\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{5}{3}}$

$\cot\theta = \boxed{-\frac{3}{4}}$

Find all 6 trig functions if $\tan\theta = -\dfrac{4}{3}$ $\frac{y}{x}$

constraint: angle lies in quadrant IV



$\sin\theta = \frac{y}{r} = \boxed{\frac{-4}{5}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{3}{5}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{-4}{3}}$

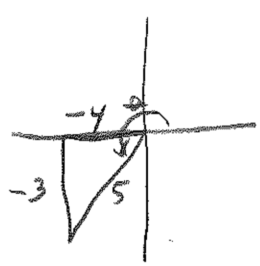
$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{-5}{4}}$

$\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{5}{3}}$

$\cot\theta = \frac{1}{\tan\theta} = \boxed{\frac{-3}{4}}$

Find all 6 trig functions if $\cos\theta = -\dfrac{4}{5}$ $\frac{x}{r}$

constraint: angle lies in quadrant III



$\sin\theta = \frac{y}{r} = \boxed{\frac{-3}{5}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-4}{5}}$

$\tan\theta = \frac{y}{x} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{-5}{3}}$

$\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{-5}{4}}$

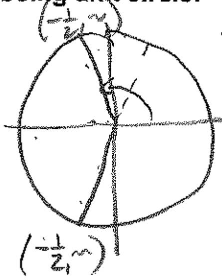
$\cot\theta = \frac{1}{\tan\theta} = \boxed{\frac{4}{3}}$

Solving for the angle, given a trig function value:

(day2)

Ex: Solve for θ: $\cos\theta = -\frac{1}{2}$ $(0 \le \theta \le 360°)$

Using unit circle:



$\theta = \boxed{120°}$

or

$\theta = \boxed{240°}$

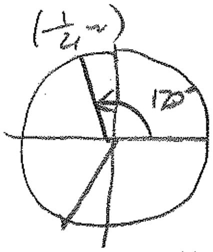
Using calculator:

$\cos\theta = -\frac{1}{2}$

$\cos^{-1}(\cos\theta) = \cos^{-1}(-\frac{1}{2})$

$\theta = \cos^{-1}(-\frac{1}{2})$

$\theta = \boxed{120°}$



(* cosine = x so other value is across x-axis)

by symmetry:

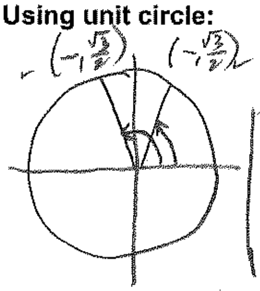
$\theta = 360° - 120° = \boxed{240°}$

Solve for θ: $\csc\theta = \frac{2\sqrt{3}}{3}$ $(0 \le \theta \le 2\pi)$

$\sin\theta = \frac{3}{2\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$\sin\theta = \frac{\sqrt{3}}{2}$

Using unit circle:



$\theta = \frac{\pi}{3}$

or

$\theta = \frac{2\pi}{3}$

Using calculator:

$\sin\theta = \frac{\sqrt{3}}{2}$

$\sin^{-1}(\sin\theta) = \sin^{-1}(\frac{\sqrt{3}}{2})$

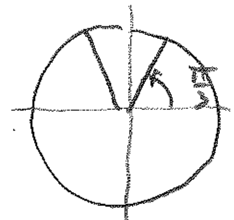
$\theta = \sin^{-1}(\frac{\sqrt{3}}{2})$

$\theta = 1.047197551$

divide by π

math → frac

$\boxed{\frac{1}{3}\pi}$



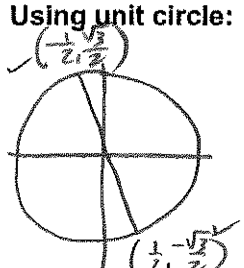
(* sine = y so other value is across y-axis)

by symmetry:

$\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

Solve for θ: $\cot\theta = -\sqrt{3}$ $(0 \le \theta \le 360°)$

Using unit circle:



$\tan\theta = -\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$\tan\theta = -\frac{\sqrt{3}}{1}$

trick: divide top & bottom by 2:

$\tan\theta = \frac{-\sqrt{3}/2}{1/2}$ ← y ← x

$\boxed{\theta = 120° \text{ or } 300°}$

Using calculator:

$\tan\theta = -\sqrt{3}$

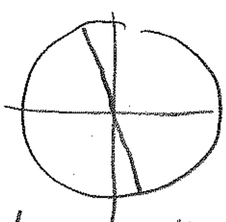
$\tan^{-1}(\tan\theta) = \tan^{-1}(-\sqrt{3})$

$\theta = \tan^{-1}(-\sqrt{3})$

$\theta = -60° = \boxed{300°}$

$\theta = 300° - 180°$

$= \boxed{120°}$



(* tangent, other value is across the circle)

Solve for θ: $\cos\theta = 0.9848$ $(0 \le \theta \le 360°)$

Using unit circle:

0.9848

not a special value

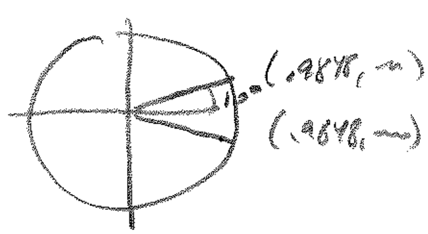
can't use unit circle

Using calculator:

$\cos\theta = .9848$

$\cos^{-1}(\cos\theta) = \cos^{-1}(.9848)$

$\theta = \boxed{10°}$



cosine = x, so other value is across x-axis

$\theta = 360° - 10°$

$= \boxed{350°}$