## *7.2 – Average Value of a Function*

With a set of discrete data values, we find the average value by adding the values and dividing by the number of values:

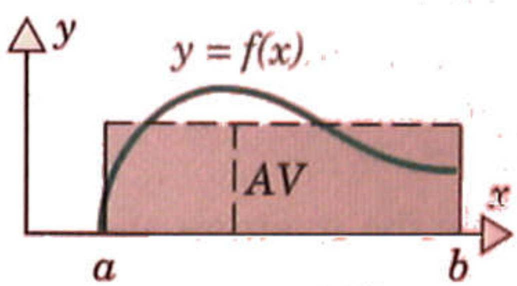
3, 6, 10, 13

$$AV = \frac{3+6+10+13}{4}$$
$$AV = 8$$

Another way to think of average value - Summing n average values yields the same sum as summing the original values:

$$3+6+10+13=32$$
$$8\ +8\ +8\ +8=32$$

We can define an Average Value (AV) for a function over a given interval similarly. 'Summing' the function over an interval gives the area under the curve. The AV for the function is the single y value that would give the same area:




$$AV(b-a) = \int_a^b f(x)\,dx$$

Length and width of a rectangle containing the same area

Bounded Area

$$AV = \frac{1}{b-a}\int_a^b f(x)\,dx$$

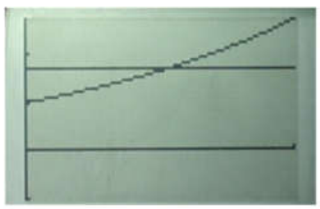
Find the average value of the given function over the given interval.

**2.** $f(x) = 2x^2, \quad \text{over } [2, 4]$

**Average Speed** A car starting from rest accelerates at the rate of 3 meters per second per second. Find its average speed over the first 8 seconds.

Find the average value, AV. Graph $y = f(x)$ and $y = \text{AV}$ on the same pair of axes within the $x$ range [a,b]

$f(x) = e^x \quad \text{over } [0, 1]$



*Example*: Assume that in a certain city the temperature (in °F) $t$ hours after 9 A.M. is represented by the function

$$T(t) = 50 + 14 \sin \frac{\pi t}{12}.$$

Find the average temperature in that city during the period from 9 A.M. to 9 P.M.

## *Added topic – Discrete Probability Functions*

A ***random variable*** is a quantity that is measured in connection with a random experiment. Unlike an algebraic variable (which is a placeholder for a specific value), a random variable represents a quantity which is *expected to vary from trial to trial in an experiment or observational setting.* If we are observing the time between arrivals of customers at a gas station, then the time between arrivals of customers would be a random variable because it is expected to vary randomly each time.

**If a fair coin is flipped 3 times, what is the probability of obtaining exactly 2 heads?**

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

| $number\ of\ heads = X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $probability = P$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |
| | $TTT$ | $TTH$<br>$THT$<br>$HTT$ | $HHT$<br>$HTH$<br>$THH$ | $HHH$ |

**Probability Model**
Assigning probabilities to each random variable value.

### Random Variable

A **random variable** is a function that assigns a numerical value to each outcome of a sample space $S$.

**Discrete Sample Space; Discrete Random Variable**
A sample space is **discrete** if it contains a finite number of outcomes or as many outcomes as there are whole numbers. A random variable is said to be **discrete** if it is defined over a discrete sample space.

The random experiment of flipping a fair coin 3 times and counting the number of heads that appear is represented by a discrete sample space that is finite.

$X = \{0, 1, 2, 3\}$

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| $TTT$ | $TTH$<br>$THT$<br>$HTT$ | $HHT$<br>$HTH$<br>$THH$ | $HHH$ |

The random experiment of flipping a fair coin until a head appears is represented by a discrete sample space that is infinite.

$X = \{1, 2, 3, 4, 5 \ldots\}$

| 1 | 2 | 3 | 4 | 5 .... |
|---|---|---|---|---|
| $H$ | $TH$ | $TTH$ | $TTTH$ | $TTTTH$ |

The sample space associated with the random experiment of measuring the height of each citizen of the U.S. is, of course, finite. This random variable could assume any real number so that the number of possible heights is infinite and the random variable is ***continuous***.

A **continuous random variable** has values that consist of an entire interval of real numbers. We say the sample space is **continuous**.

The relationship between the probability and the random variables is called a **probability function**.

**discrete probability function** - the random variable is discrete (16.2)
**continuous probability function** - the random variable is continuous (16.3)

Classify each given random variable as **discrete** or **continuous**.
If it is discrete, state whether the sample space is infinite or finite.

6) $X$ is the number of defective items in a lot of 10,000 items.

8) $X$ denotes the time elapsed in minutes between the arrival of airplanes at an airport.

## Discrete probability function models

### Binomial Probability Function

$$P(X=k)=\binom{n}{k}p^k q^{n-k}$$

*n = number of trials*
*k = number of successes*
*p = probability of success*
*q = probability of failure*

- Binomial model requires n independent trials (probability of one trial doesn't depend upon other trials)
- Binomial probability approximated by Normal distribution for large n or when p is close to 0.5.

***binompdf(n, p, k)***
Probability that exactly k trials succeed.

***binomcdf(n, p, k)***
Probability that 0-k trials succeed.

### Poisson Probability Function

$$P(X=x)=\frac{(np)^x e^{-np}}{x!}$$

*n = number of trials*
*p = probability of success*
*x = a value of the random variable*
*(np) = λ*

- Poisson model can be used when trials are not independent.
- Many scenarios have data which match Poisson probability productions:
  - Number of cars arriving at tollbooth per hour.
  - Bacteria distribution in a culture.
  - Number of defects in a manufactured products.

***poissonpdf(np, x)***
Probability that x occurs.

***poissoncdf(np, x)***
Probability that 0-x occur.

A machine produces parts to meet certain specifications, and the probability that a part is defective is .05. A sample of 50 parts is taken. What is the probability of having exactly 1 defective part in the sample?

Binomial Probability Function

$$P(X=k)=\binom{n}{k}p^k q^{n-k}$$

Poisson Probability Function

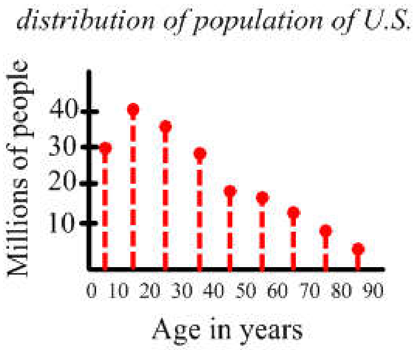
$$P(X=x)=\frac{(np)^x e^{-np}}{x!}$$

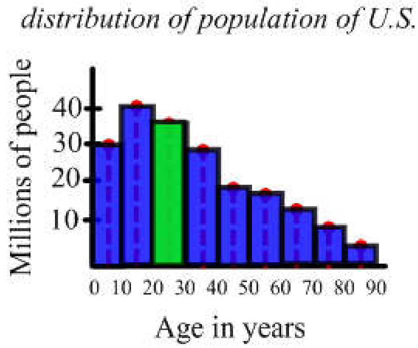
What is the probability of having more than 5 defective parts in the sample?

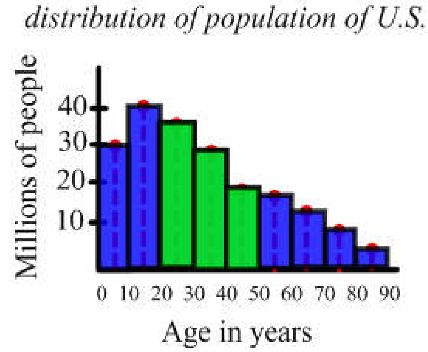
14. **Insurance Policy** An insurance company insures 5000 people against the loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumption that on the average 8 people in 100,000 will have car accidents each year that result in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year?

## *7.3 – Probability Density Functions*

The random experiments we have encountered so far have resulted in probability models with sample spaces that are either finite or discrete. However, we often have to deal with random experiments that are neither finite nor discrete.

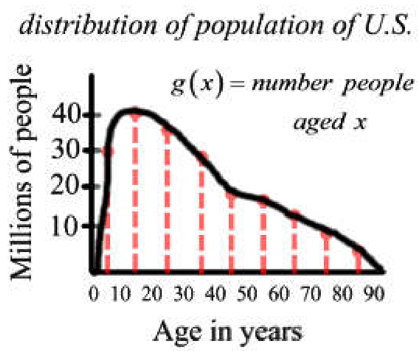


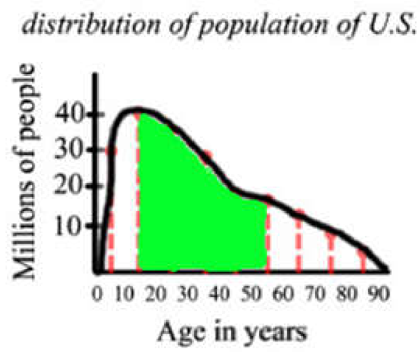



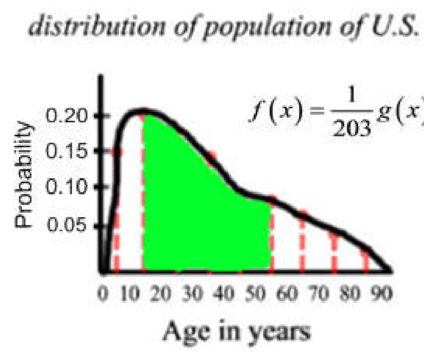




$$P(20 to 30) = \frac{35}{203} = 0.17$$

$$P(20 to 50) = \frac{35+30+20}{203} = 0.42$$




$$P(20 to 50) = \frac{\int_{20}^{50} g(x)}{\int_{0}^{90} g(x)} = 0.42$$

$$P(20 to 50) = \int_{20}^{50} f(x) = 0.42$$

### $f(x)$ is a **probability density function**

Properties of a Probability Density Function

I. $\int_a^b f(x)\,dx = 1$ where the interval $[a, b]$, contains **all** values that the random variable $X$ can assume

II. $f(x) \geq 0$

The probability that the outcome of an experiment results in a value of a random variable $X$ between $c$ and $d$ is

$$P(c \leq X \leq d) = \int_c^d f(x)\,dx$$

$f(x)$ can be any function whose area under the curve is zero. Here are two commonly encountered PDFs:

**Uniform PDF**

$f(x) = a\ constant$

**Exponential PDF**

$f(x) = \lambda e^{-\lambda x}$

If $f(x) \geq 0$ is not a probability density function, we can find a constant $k$ such that $\int_a^b k f(x) = 1$

(Determine the constant $k$ that will make the function a probability density function over the interval indicated.)

14. $f(x) = 10x - x^2$ , over [0, 8]

Verify that the function is a probability density function over the indicated interval.

4. $f(x) = \frac{6}{27}\left(3x - x^2\right)$ , over [0 ,3]

28. The demand for an inventory item has a " pdf " given by

$$f(x) = 0.2e^{-0.2x}$$

where $f(x)$ is the probability that $x$ items will be in demand over a 1-week period.

What is the probability that fewer than 5 items will be in demand?

Fewer than 100?

More than 10?

## Expected Value of a Continuous Random Variable

From 8.3, if $X$ is a discrete random variable then

$$E = m_1 p_1 + m_2 p_2 + m_3 p_3 + \ldots + m_n p_n$$

Example: 100 tickets sold in a raffle. 1 winning ticket pays $50, 2 runner-up tickets pay $5 each and the other tickets pay nothing. If you play this raffle many times, what will be your average (expected) winnings?

**Expected Value of a Continuous Random Variable**

If $X$ is a continuous random variable with the probability density function $f(x)$, $a \le x \le b$, the **expected value of $X$** is

$$E(X) = \int_a^b x\, f(x)\, dx$$

Compute the expected value for the probability density function.

18. $f(x) = \frac{1}{5}$ , over [0, 5]