Honors Brief Calculus – Lesson Notes: Trig Limits and Derivatives; L'Hospital's Rule

Trig Limits

Remember:

$$\lim_{x\to 0} c = c$$

$$\lim_{x\to c}\left[f(x)\cdot g(x)\right] = \lim_{x\to c} f(x)\cdot\lim_{x\to c} g(x)$$

$$\lim_{x\to c}\left[f(x)\right]^n = \left[\lim_{x\to c} f(x)\right]^n$$

Graph to find the following limits.

$$\lim_{x\to 0}\frac{\sin x}{x} =$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} =$$

Use the previous limits to find the following.

1. $\lim_{x\to 0}\dfrac{\sin\frac{1}{2}x}{x} =$

5. $\lim_{x\to 0}\dfrac{\tan x}{x} =$

6. $\lim_{x\to 0}\dfrac{\sin 3x}{\sin 2x} =$

10. $\lim_{x\to 0}\dfrac{1-\cos x}{x^2} =$

15. $\lim_{\theta\to 0}\dfrac{\sin^2\theta}{\theta} =$

Trig Derivatives

$$\frac{d}{dx}\sin x=\lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

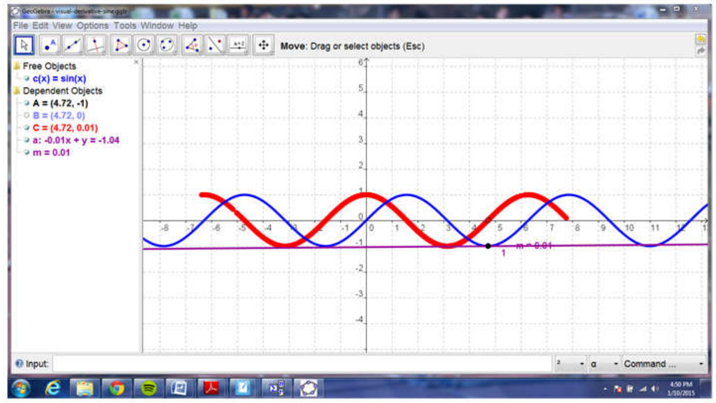
$$\frac{d}{dx}\sin x=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h}$$

$$\frac{d}{dx}\sin x=\lim_{h\to 0}[-\sin x]\left[\frac{1-\cos h}{h}\right]+\lim_{h\to 0}[\cos x]\left[\frac{\sin h}{h}\right]$$

$$\frac{d}{dx}\sin x=[-\sin x]\lim_{h\to 0}\left[\frac{1-\cos h}{h}\right]+[\cos x]\lim_{h\to 0}\left[\frac{\sin h}{h}\right]$$

$$\frac{d}{dx}\sin x=[-\sin x][0]+[\cos x][1]$$

$$\boxed{\frac{d}{dx}\sin x=\cos x}$$



Geogebra demonstration: Visual Derivative Sine

$$\frac{d}{dx}\cos x=\lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$$\frac{d}{dx}\cos x=\lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos(x)}{h}$$

$$\frac{d}{dx}\cos x=\lim_{h\to 0}[-\cos x]\left[\frac{1-\cos h}{h}\right]-\lim_{h\to 0}[\sin x]\left[\frac{\sin h}{h}\right]$$

$$\frac{d}{dx}\cos x=[-\cos x]\lim_{h\to 0}\left[\frac{1-\cos h}{h}\right]-[\sin x]\lim_{h\to 0}\left[\frac{\sin h}{h}\right]$$

$$\frac{d}{dx}\cos x=[-\cos x][0]-[\sin x][1]$$

$$\boxed{\frac{d}{dx}\cos x=-\sin x}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan(x+h)-\tan(x)}{h}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\frac{\tan x+\tan h}{1-\tan x\tan h}-\tan x}{h}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\left(\frac{\tan x+\tan h}{1-\tan x\tan h}-\tan x\right)(1-\tan x\tan h)}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan x+\tan h-\tan x-\tan^2 x\tan h}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan x+\tan h-\tan x-\tan^2 x\tan h}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan h-\tan^2 x\tan h}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan h(1-\tan^2 x)}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan h\sec^2 x}{h(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan h}{h}\frac{\sec^2 x}{(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\sin h}{h}\frac{1}{\cos h}\frac{\sec^2 x}{(1-\tan x\tan h)}$$

$$\frac{d}{dx}\tan x=[1]\left[\frac{1}{1}\right]\frac{\sec^2 x}{(1-\tan x[0])}$$

$$\boxed{\frac{d}{dx}\tan x=\sec^2 x}$$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\csc x = -\csc x \cot x$ (memorize)

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

17. Find the equation of the tangent line to

$y = 2\cos x - \cos 2x$ at $x = \frac{\pi}{3}$

Find the derivative, *dy/dx*

18. $y = \sec 3x$

20. $y = \cos^6 x$

24. $y = \left(\ln\left(\cos e^{3x}\right)\right)^4$

L'Hospital's Rule

$\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$ The 'divide everything by largest degree x from denominator' trick doesn't work here.

Cases like this are called 'Indeterminate forms':

$\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$

For indeterminate forms, we can use **L'Hospital's Rule**

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Pronounced "L-oh-pe-tal" (French)

In other words, find the derivative of the numerator and the derivative of the denominator and try again.

Find the limit using L'Hospital's Rule.

$\lim\limits_{x \to \infty} \dfrac{2x^2 - 2}{x+1}$

25. $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$

27. $\lim\limits_{\theta \to 0} \dfrac{\tan \theta}{\theta}$

28. $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sin x}$

29. $\lim_{x \to 3} \frac{x-3}{3x^2 - 13x + 12}$

30. $\lim_{t \to 0} \frac{te^t}{1-e^t}$

34. $\lim_{x \to 0} \frac{x - \ln(x+1)}{1 - \cos 2x}$

31. $\lim_{x \to 0^+} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$

35. $\lim_{x \to 0} \frac{2 - x^2 - 2\cos x}{x^4}$