H Finite Mathematics – Lesson Notes: Unit 8 Precalculus Review (multiple chapters)

Function Properties

Facts about functions:

1. *f(x)* is the image of *x*, or the value of *f* at *x* when the rule *f* is applied to an *x* in the domain.
2. To each *x* in the domain of *f*, there is one and only one image *f(x)* in the range.
3. *f* is the symbol we use to denote the function
   a. *x* - element of the domain - independent
   b. *f(x)* - element of the range - dependent
4. Vertical line test

Finding Intercepts

1. To find the *x*-int., let $y = 0$. (Cover-up)
2. To find the *y*-int., let $x = 0$. (Cover-up)

Find the following values for each function:
$f(x) = -3x^2 + 2x - 4$

$f(0) = -3(0)^2+2(0)-4 = \boxed{-4}$

$f(1) = -3(1)^2+2(1)-4 = \boxed{-5}$

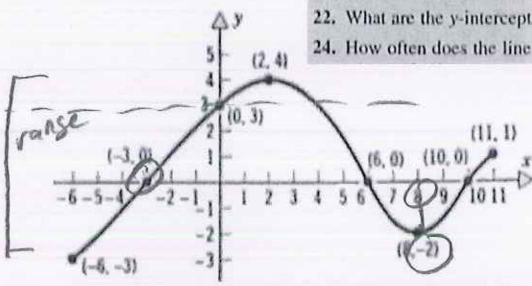
$f(-1) = -3(-1)^2+2(-1)-4 = \boxed{-9}$

$f(2) = -3(2)^2+2(2)-4 = \boxed{-12}$

$f(8) = \boxed{-2}$
$f(-3) = \boxed{0}$

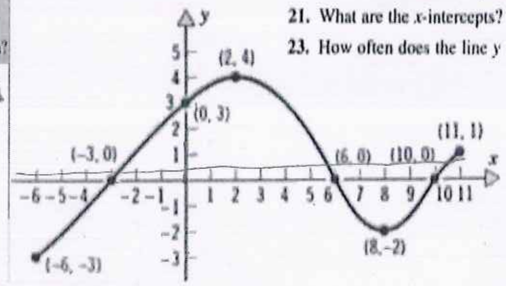
14. Find $f(8)$ and $f(-3)$.
16. Is $f(-6)$ positive or negative?
18. For what numbers x is $f(x) > 0$? $(-3,6) \cup (10,11]$
20. What is the range of f? $[-3,4]$
22. What are the y-intercepts? $(0,3)$
24. How often does the line $y = 3$ intersect the graph?

$\boxed{\text{twice}}$



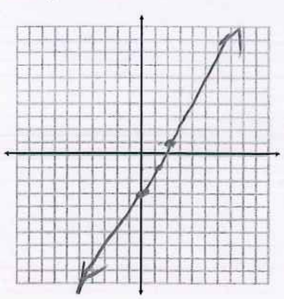
13. Find $f(0)$ and $f(2)$. $f(0) = \boxed{3}$ $f(2) = \boxed{4}$
~~15. Is $f(11)$ positive or negative?~~
17. For what numbers x is $f(x) = 0$? $y=0$ at $\boxed{x=-3,\ x=6,\ x=10}$
~~19. What is the domain of f?~~
21. What are the x-intercepts? $(-3,0)\,(6,0)\,(10,0)$
23. How often does the line $y = \frac{1}{2}$ intersect the graph?

$\boxed{3 \text{ times}}$



Graph each function. Find any intercepts.

26. $y = 2x - 3$

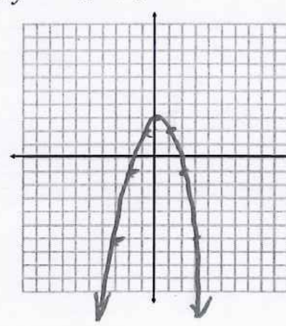


y int (x=0)
$y = 2(0) - 3$
$y = -3$
$\boxed{(0,-3)}$

x int (y=0)
$0 = 2x - 3$
$2x = 3$
$x = \frac{3}{2}$
$\boxed{(\frac{3}{2},0)}$

always in form of a point

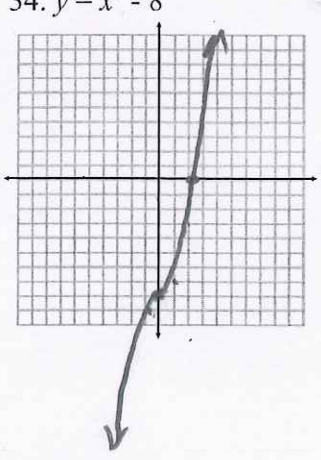
30. $y = -x^2 + 3$



y int (x=0)
$y = -(0)^2 + 3$
$y = 3$
$\boxed{(0,3)}$

x int (y=0)
$0 = -x^2 + 3$
$x^2 = 3$
$x = \pm\sqrt{3}$
$\boxed{(\sqrt{3},0)\ (-\sqrt{3},0)}$

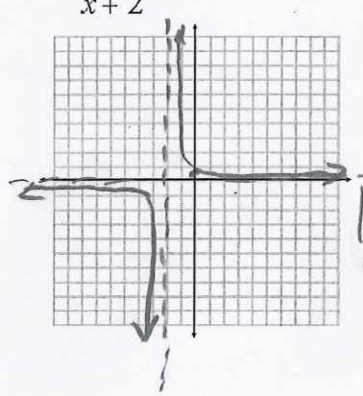
34. $y = x^3 - 8$



y int (x=0)
$y = (0)^3 - 8$
$y = -8$
$\boxed{(0,-8)}$

x int (y=0)
$0 = x^3 - 8$
$x^3 = 8$
$x = \sqrt[3]{8}$
$x = 2$
$\boxed{(2,0)}$

38. $y = \dfrac{1}{x+2}$



Vertical Asymptote where denom = 0 $x = -2$

y int (x=0)
$y = \frac{1}{(0)+2}$
$y = \frac{1}{2}$
$\boxed{(0,\frac{1}{2})}$

x int (y=0)
$0 = \frac{1}{x+2}$
$\boxed{\text{no x int}}$

when $x \to \infty$ horizontal asymptote
$y = \frac{1}{\text{large \#}} \to 0$
$y = 0$

45. $f(x) = \dfrac{2x^2}{x^4+1}$

a. Is the point (-1,1) on the graph of f ?

b. If $x = 2$, what is $f(x)$?

c. If $f(x) = 1$, what is x ?

d. What is the domain of f ?

a) $1 \stackrel{?}{=} \dfrac{2(-1)^2}{(-1)^4+1}$ b) $f(2) = \dfrac{2(2)^2}{(2)^4+1} = \boxed{\dfrac{8}{17}}$

$1 \stackrel{?}{=} \dfrac{2}{1+1}$ c) $1 = \dfrac{2x^2}{x^4+1}$ d) $\boxed{(-\infty,\infty)}$

$1 \stackrel{?}{=} \dfrac{2}{2}$ $x^4+1 = 2x^2$

$x^4 - 2x^2 + 1 = 0$

$1 = 1$ $(x^2-1)(x^2-1) = 0$

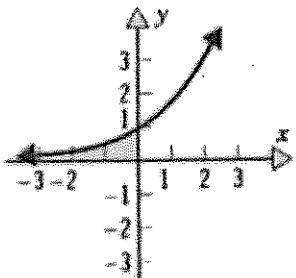
$\boxed{\text{yes}}$ $x^2 - 1 = 0$

$x^2 = 1$

$\boxed{x = \pm 1}$

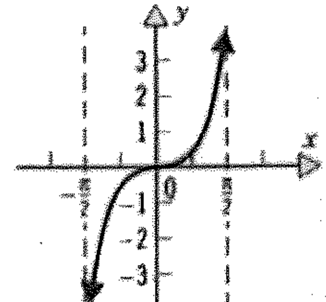
If the graph is a graph of a function, find its domain and range and intercepts

48.



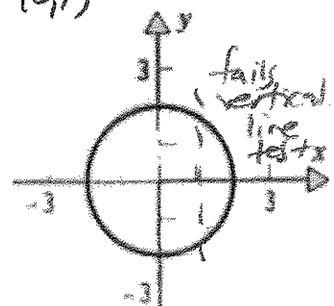
D: $(-\infty,\infty)$
R: $(0,\infty)$
$(0,1)$

50.



D: $(-\frac{\pi}{2}, \frac{\pi}{2})$
R: $(-\infty,\infty)$
$(0,0)$

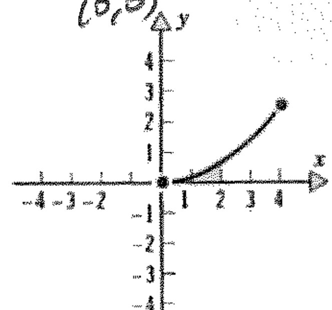
52.



fails vertical line test

not a function

54.



D: $[0,4]$
R: $[0,3]$
$(0,0)$

71. Find the domain. $f(x) = \sqrt{(x^2-9)}$

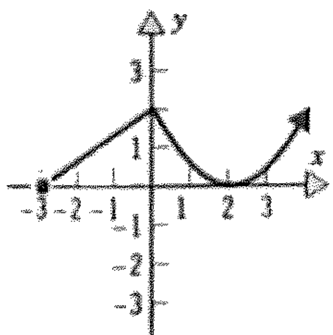
$x^2 - 9 \geq 0$

$x^2 \geq 9$ outside case

$x \geq 3$ or $x \leq -3$

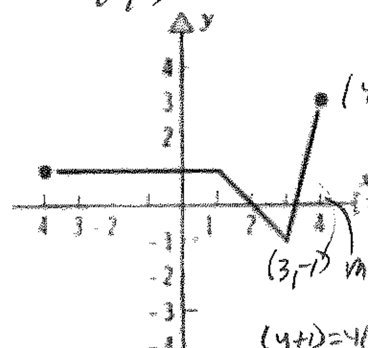
$\boxed{(-\infty,-3] \cup [3,\infty)}$

56.



D: $[-3,\infty)$
R: $[0,\infty)$
$(0,2)$ $(-3,0)$ $(2,0)$

58.



$(4,3)$

$(3,-1)$ $m = \dfrac{3-(-1)}{4-3} = 4$

$(y+1) = 4(x-3)$

x int $y = 0$

$(0+1) = 4(x-3)$

$1 = 4x - 12$

$4x = 13$ $x = \frac{13}{4}$

D: $[4,4]$
R: $[-1,3]$
$(0,1)$ $(2,0)$ $(\frac{13}{4},0)$

77. If $f(x) = \dfrac{(3x+8)}{(2x-A)}$ and $f(0) = 2$, what is the value of A?

$\dfrac{3(0)+8}{2(0)-A} = 2$

$\dfrac{8}{-A} = 2$

$8 = -2A$

$A = \dfrac{8}{-2} = \boxed{-4}$

91. **Demand Equation** The price p and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{5}x + 100 \qquad 0 \leq x \leq 500$$

Express the revenue R as a function of x.

$R = px$

$R(x) = (-\frac{1}{5}x + 100)x$

$\boxed{R(x) = -\frac{1}{5}x^2 + 100x}$

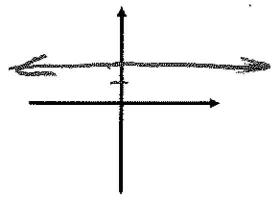
Function Families, Difference Quotient

Polynomials

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_0$$

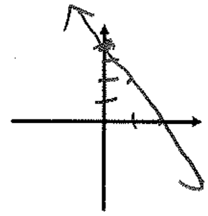
Highest exponent = degree

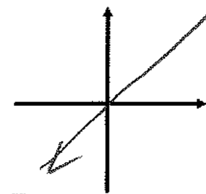
degree 0: $f(x) = 2$ "constant function"



degree 1: $f(x) = -2x + 4$ "linear function"

$f(x) = x$ "identity function"



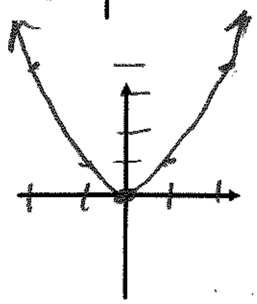


(identity)

degree 2: $f(x) = 2x^2 - 5x + 7$ "quadratic function"

basic shape: $f(x) = x^2$

x	x^2
0	0
1	1
-1	1
2	4
-2	4



can use vertex and intercepts to sketch more complicated cases:

$f(x) = -x^2 + 4x - 3$

$y = -x^2 + 4x - 3$

$y + 3 = -x^2 + 4x$

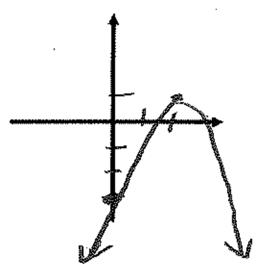
$y + 3 = (-1)(x^2 - 4x)$

$y + 3 - \underline{4} = (-1)(x^2 - 4x + \underline{4})$

$(y - 1) = (-1)(x - 2)^2$

$(y-1) = -(x-2)^2$ "vertex form"

vertex at $\boxed{(2,1)}$

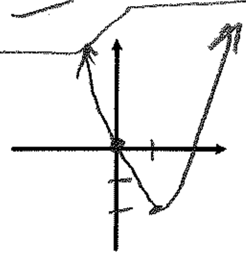


y int $(x=0)$

$y = -(0)^2 + 4(0) - 3$

$y = -3$ $\boxed{(0,-3)}$

$f(x) = 2x^2 - 4x$



$y = 2x^2 - 4x$

$y = 2(x^2 - 2x)$

$y + \underline{2} = 2(x^2 - 2x + \underline{1})$

$(y+2) = 2(x-1)^2$

vertex at $\boxed{(1,-2)}$

y int $(x=0)$

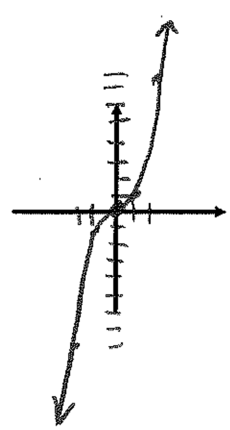
$y = 2(0)^2 - 4(0) = 0$

$\boxed{(0,0)}$

degree 3: "cubic function"

basic shape: $f(x) = x^3$

x	x^3
0	0
1	1
-1	-1
2	8
-2	-8



"S shape"

For degree 3 or higher, complicated cases, could sketch using zeros and left/right hand behavior...

even degree polynomials: left and right side move together (positive leading coefficient = 'up', negative leading coefficient = 'down').

odd degree polynomials: left and right side move opposite (positive leading coefficient = positive 'slope', negative leading coefficient = negative 'slope')

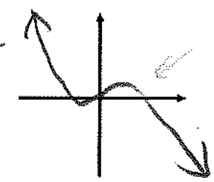
zeros / multiplicity: odd multiplicities 'through', even multiplicities 'bounce'

...but usually just graph in a calculator.

neg odd

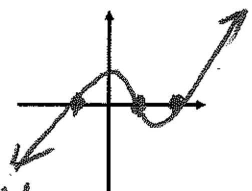
$f(x) = -x^3 - 2x^2 - x + 2$

LH ↖



RH ↘

degree 3: $f(x) = x^3 - 2x^2 - x + 2$ "cubic function"



Sometimes you get lucky and can factor to find zeros:

$(x^3-2x^2)+(-x+2)=0$

$x^2(x-2)-(x-2)=0$

$(x-2)(x^2-1)=0$

$(x-2)(x+1)(x-1)=0$

zeros at $x=2$, $x=-1$, $x=1$

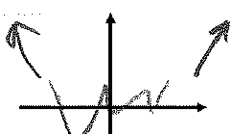
x^3 odd ...

positive so left ↙ right ↗

pos even

degree 4: $f(x) = 3x^4 - 5x^3 - 2x^2 - x + 2$

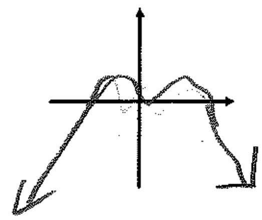
"quartic function"



Something more complex here

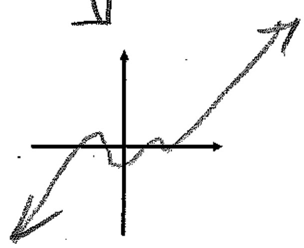
neg even

$f(x) = -3x^4 - x^3 - 2x^2 - x + 2$



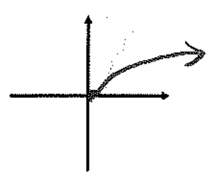
pos odd

degree 5: $f(x) = x^5 + 2x^4 + 8x^3 - 2x^2 - 3x + 6$ "quintic function"

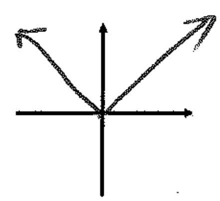


Other basic function shapes

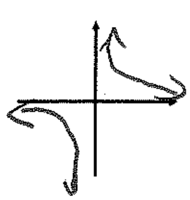
$f(x) = \sqrt{x}$ "square-root function"



$f(x) = |x|$ "absolute-value function"

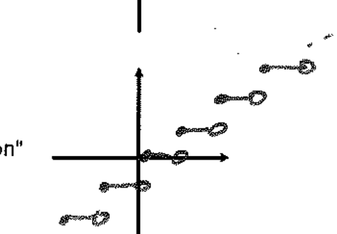


$f(x) = \frac{1}{x}$ "reciprocal function"



$f(x) = [\![x]\!]$ "greatest-integer function"

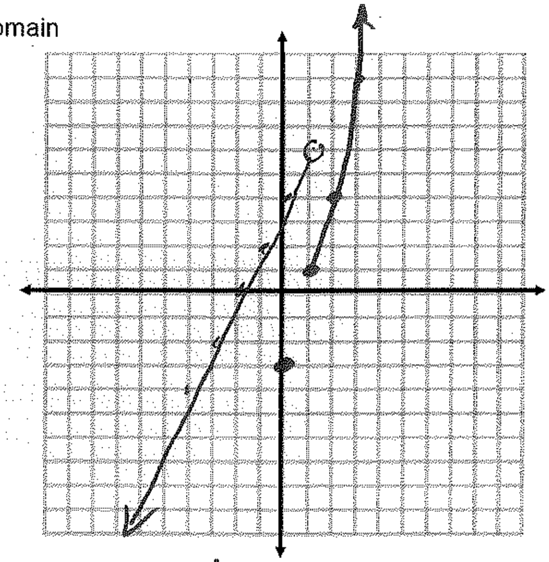
"round down to nearest integer"



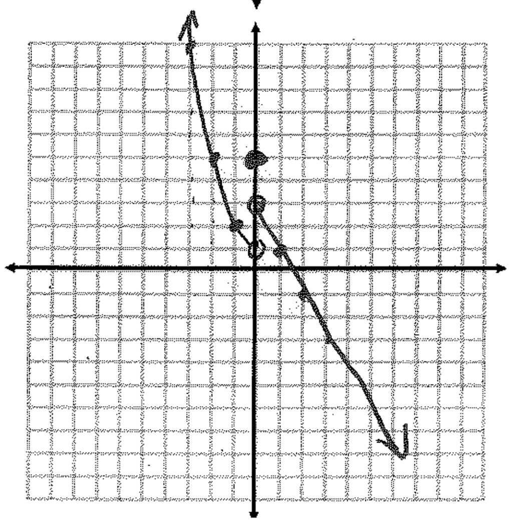
Piece-wise defined functions

Different rules for different parts of the domain

$$f(x)=\begin{cases} 2x+4, & x<1 \\ -3, & x=1 \\ x^2, & x>1 \end{cases}$$



$$f(x)=\begin{cases} x^2+1, & x<0 \\ 4, & x=0 \\ -2x+3, & x>0 \end{cases}$$



In Problems 1–8 match each graph to the function whose graph most resembles the one given.

(a) Constant function	*(b) Linear function*	*(c) Square function*
(d) Cube function	*(e) Square root function*	*(f) Reciprocal function*
(g) Absolute value function	*(h) Greatest integer function*	

1. 

Square

2. 

Constant

3. 

Squareroot

4. 

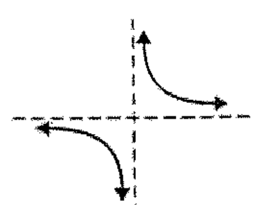
absolute value.

5. 

linear

6. 

cubic
(cube)

7. 

reciprocal

8. 

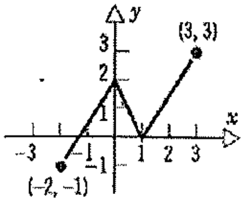
greatest integer

The graph of the following function is given: Find:

a) Domain and Range.

b) Intervals of increasing, decreasing, or constant.

c) The intercepts, if any.



D: $[-2, 3]$

R: $[-1, 3]$

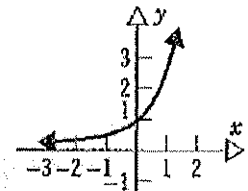
incr: $(-2, 0) \cup (1, 3)$

decr: $(0, 1)$

yint $(0, 2)$

xint $(-\frac{1}{3}, 0)$

$(1, 0)$



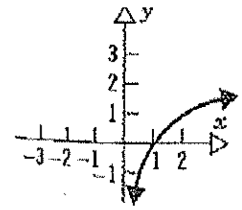
D: $(-\infty, \infty)$

R: $(0, \infty)$

incr: $(-\infty, \infty)$

yint $(0, 1)$

xint none



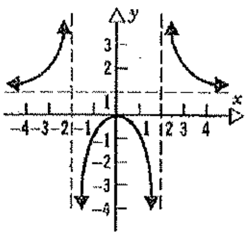
D: $(0, \infty)$

R: $(-\infty, \infty)$

incr: $(0, \infty)$

yint (none)

xint $(1, 0)$



D: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

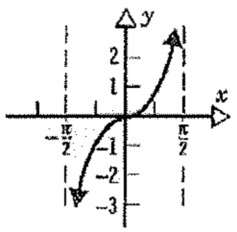
R: $[-\infty, 0) \cup (1, \infty)$

incr: $(-\infty, -2) \cup (-2, 0)$

decr: $(0, 2) \cup (2, \infty)$

yint $(0, 0)$

xint $(0, 0)$



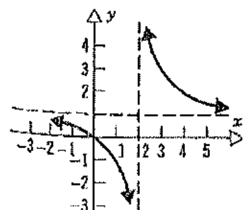
D: $(-\frac{\pi}{2}, \frac{\pi}{2})$

R: $(-\infty, \infty)$

incr $(-\frac{\pi}{2}, \frac{\pi}{2})$

yint $(0, 0)$

xint $(0, 0)$



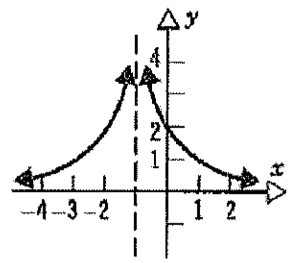
D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 1) \cup (1, \infty)$

decr $(-\infty, 2) \cup (2, \infty)$

yint $(0, 0)$

xint $(0, 0)$



D: $(-\infty, -1) \cup (-1, \infty)$

R: $(0, \infty)$

incr $(-\infty, -1)$

decr $(-1, \infty)$

yint $(0, 2)$

xint none

Given: $f(x) = \dfrac{x}{x^2+1}$ Find:

a. $f(-x)$

$\dfrac{(-x)}{(-x)^2+1} = \boxed{\dfrac{-x}{x^2+1}}$

b. ~~$f(x)$~~ $-f(x)$

$\boxed{\dfrac{-x}{x^2+1}}$

c. $f(2x)$

$\dfrac{(2x)}{(2x)^2+1} = \boxed{\dfrac{2x}{4x^2+1}}$

d. $f(x - 3)$

$\dfrac{(x-3)}{(x-3)^2+1} = \dfrac{x-3}{x^2-6x+9+1}$

$= \boxed{\dfrac{x-3}{x^2-6x+10}}$

e. $f(1/x)$

$\dfrac{(\frac{1}{x})}{(\frac{1}{x})^2+1} = \dfrac{(\frac{1}{x})x^2}{(\frac{1}{x^2}+1)x^2}$

$= \boxed{\dfrac{x}{1+x^2}}$

f. $\dfrac{1}{f(x)}$

$\boxed{\dfrac{x^2+1}{x}}$

g. $f(x^2)$

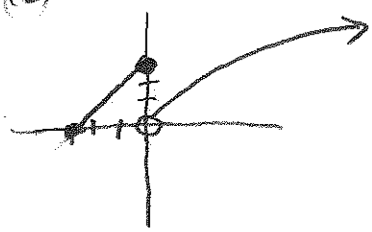
$\dfrac{(x^2)}{(x^2)^2+1} = \boxed{\dfrac{x^2}{x^4+1}}$

66. (a) Find the domain of the function
(b) Locate any intercepts
(c) Graph the function
(d) Based on the graph, find the range

$$f(x)=\begin{cases}3+x & if \quad -3\le x<0\\ 3 & if \quad x=0\\ \sqrt{x} & if \quad x>0\end{cases}$$

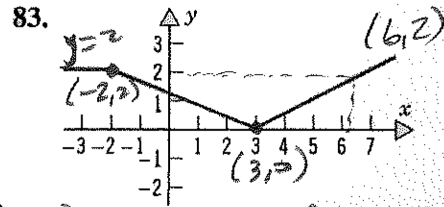
(a) D. $[-3,\infty)$ (b) $(0,3)$ $(-3,0)$

(c)



(d) R: $[0,\infty)$

83. Find a piecewise-defined function whose graph is shown in the given figure. Note that each graph is made up of line segments.



$m=\frac{2-0}{-2-3}=\frac{2}{-5}$

$(y-0)=-\frac{2}{5}(x-3)$

$y=-\frac{2}{5}x+\frac{6}{5}$

$m=\frac{2-0}{6-3}=\frac{2}{3}$

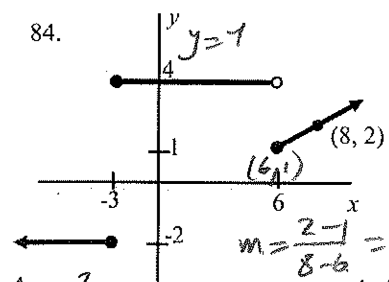
$(y-0)=\frac{2}{3}(x-3)$

$y=\frac{2}{3}x-\frac{6}{3}$

$y=\frac{2}{3}-2$

$$\boxed{f(x)=\begin{cases}2, & x<-2\\ -\frac{2}{5}x+\frac{6}{5}, & -2\le x\le 3\\ \frac{2}{3}x-2, & x>3\end{cases}}$$

84.



$m=\frac{2-1}{8-6}=\frac{1}{2}$

$(y-1)=\frac{1}{2}(x-6)$

$y=\frac{1}{2}x-3+1$

$y=\frac{1}{2}x-2$

$$f(x)=\begin{cases}-2, & x<-3\\ 4, & -3\le x<6\\ \frac{1}{2}x-2, & x\ge 6\end{cases}$$

79. Determine whether the given function opens upward or downward. Find the vertex, the y-intercept, and the x-intercepts, if any. Graph the function.

$$y=f(x)=x^2-7x+12$$

$y-12+\frac{49}{4}=x^2-7x+\frac{49}{4}$

$y-\frac{48}{4}+\frac{49}{4}=(x-\frac{7}{2})^2$

$(y+\frac{1}{4})=(x-\frac{7}{2})^2$

vertex at $(\frac{7}{2},-\frac{1}{4})$

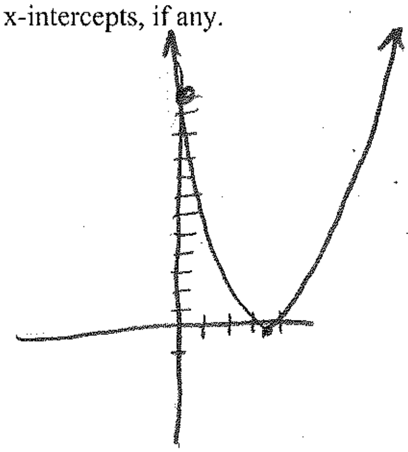
y int ($x=0$)

$y=(0)^2-7(0)+12$

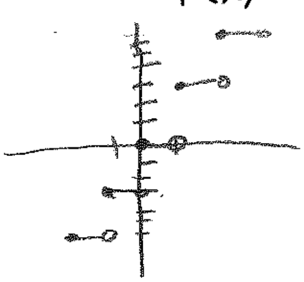
$y=12$

$(0,12)$

$(\frac{7}{2}=3\frac{1}{2})$



70. Graph $f(x)=3[[x]]$



Rational Functions

Fraction ('ratio') of polynomials:

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_n x^n + a_{n-1}x^{n-1}+\dots+a_0}{b_m x^n + b_{m-1}x^{n-1}+\dots+b_0}$$

May have one horizontal asymptote and/or one or more vertical asymptotes.

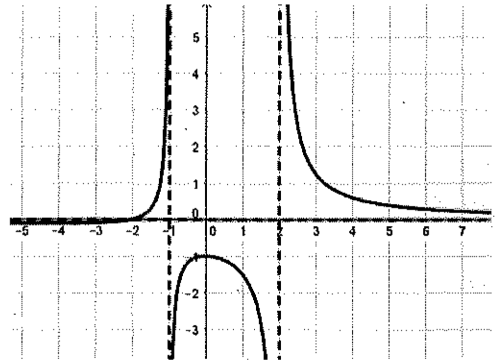
Horizontal asymptote:
Occurs if the function approaches a specific y-value as x gets large in the positive or negative direction.

Vertical asymptotes:
Occur at any uncanceled zeros in the denominator.

(Finding x, and y-intercepts also helpful for sketching.)

Find asymptotes and intercepts

$$f(x)=\frac{x+2}{x^2-x-2}$$



V.A. denom ⇒

$x^2-x-2 \Rightarrow$

$(x-2)(x+1) \Rightarrow$

$\boxed{x=2 \quad x=-1}$

H.A. when $x \to$ large

$f(x) \approx \frac{x}{x^2-x} = \frac{x(1)}{x(x-1)} = \frac{1}{x-1} = \frac{1}{\text{large}} = 0$ $\boxed{y=0}$

Find asymptotes and intercepts

$$f(x)=\frac{x-6}{x^2-x-6}$$

V.A $x^2-x-6=0$

$(x+2)(x-3)=0$

$\boxed{x=-2 \quad x=3}$

H.A. $x \to$ large

$f(x) \approx \frac{x}{x^2-x}$

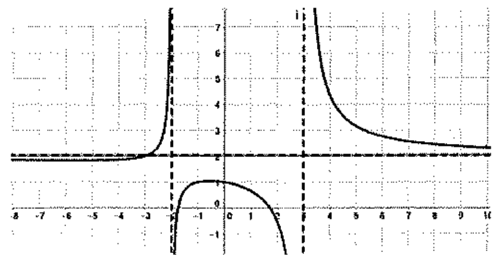
$= \frac{x(1)}{x(x-1)}$

$= \frac{1}{x-1} = \frac{1}{\text{large}}$

$\boxed{y=0}$

Find asymptotes

$$f(x)=\frac{2x^2-6}{x^2-x-6}$$



V.A $x^2-x-6=0$

$(x+2)(x-3)=0$

$\boxed{x=-2 \quad x=3}$

H.A. $x \to$ large

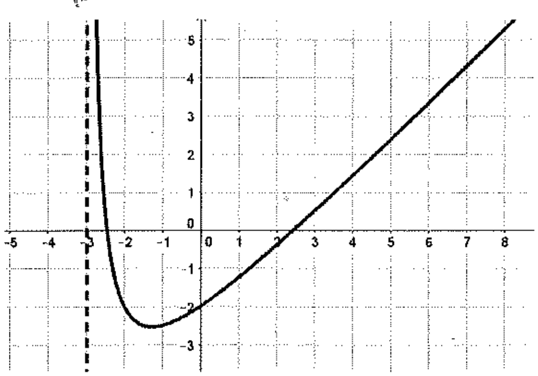
$f(x) \approx \frac{2x^2}{x^2-x} = \frac{x(2x)}{x(x-1)}$

$= \frac{2x}{x-1} \approx \frac{2\cancel{x}}{\cancel{x}} = 2$

$\boxed{y=2}$

Find asymptotes

$$f(x)=\frac{x^2-6}{x+3}$$



V.A.

$x+3=0$

$\boxed{x=-3}$

H.A. $x \to$ large

$f(x) \approx \frac{x^2}{x}$

$= \frac{x}{1}$

$=$ large

$x \to \infty$

then $y \to \infty$

$\boxed{\text{no H.A.}}$

Difference Quotient

A specific multiple function structure used frequently in calculus

$$\frac{[f(x+h)]-[f(x)]}{h} \quad (h \neq 0)$$

Find the difference quotient for $f(x) = 2x^2 - 4x$

$$\frac{[2(x+h)^2 - 4(x+h)] - [2(x)^2 - 4(x)]}{h}$$

$$\frac{2(x^2+2xh+h^2) - 4x - 4h - 2x^2 + 4x}{h}$$

$$\frac{2x^2 + 4xh + 2h^2 - 4x - 4h - 2x^2 + 4x}{h}$$

$$\frac{4xh + 2h^2 - 4h}{h}$$

$$\frac{h(4x+2h-4)}{h(1)} = \boxed{4x+2h-4}$$

Find the difference quotient for $f(x) = 3x + 4$

$$\frac{[3(x+h)+4] - [3(x)+4]}{h}$$

$$\frac{3x+3h+4-3x-4}{h}$$

$$\frac{3h}{h}$$

$$\boxed{3}$$

The Exponential Function

Exponential function
- a is a positive real number
- $a \neq 1$
- domain = set of all real numbers

$$f(x) = a^x$$

The base e

$$e = \left(1 + \frac{1}{n}\right)^n$$

as n increases without bound

$$e \approx \left(1 + \frac{1}{n}\right)^n \approx 2.718$$

Laws of exponents

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^m a^n = a^{m+n}$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(ab)^m = a^m b^m$$

but:
$(a+b)^m \neq a^m + b^m$
$(a+b)^3 = (a+b)(a+b)(a+b)$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{m/n} = \left(a^{\frac{1}{n}}\right)^m$$

Summary of info on: $f(x) = a^x$ $\quad a > 1$

$f(x) = 2^x$

domain $(-\infty, \infty)$

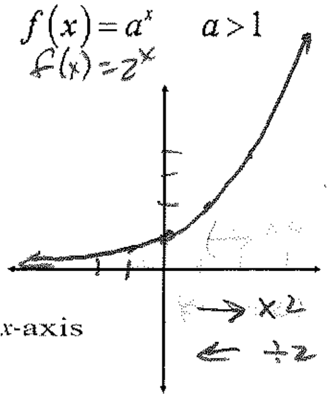
range $(0, \infty)$

x-int. none

y-int. $(0,1)$

horizontal asymptote: x-axis

increasing function



Summary of info on: $f(x) = a^x$ $\quad 0 < a < 1$

$f(x) = \left(\frac{1}{2}\right)^x$

domain $(-\infty, \infty)$

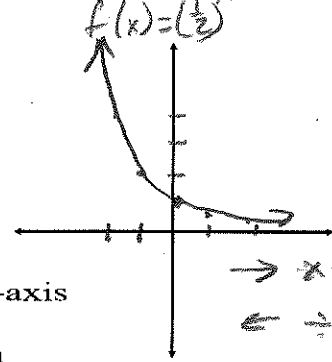
range $(0, \infty)$

x-int. none

y-int. $(0,1)$

hor. asymptote: x-axis

decreasing function



Exponential equations

If $a^u = a^v$, then $u = v$
(1:1 property)

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous interest

$$A = Pe^{rt}$$

P = principal (initial amt)
r = annual interest rate (decimal)
t = years invested
n = number of times compounded per year

Evaluate each expression.

#14 $\left(27^{3/2}\right)\left(\frac{1}{3}\right)^{3/2}$

$\left(\frac{27}{3}\right)^{3/2}$

$(9)^{3/2} = \left(9^{1/2}\right)^3 = \left(\sqrt{9}\right)^3 = 3^3 = \boxed{27}$

15. $\left[\left(8^{-1}\right)\left(8^{1/3}\right)\right]^3$

$\left[\frac{8^{1/3}}{8^1}\right]^3$

$\left(\frac{\sqrt[3]{8}}{8}\right)^3 = \left(\frac{2}{8}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \boxed{\frac{1}{64}}$

Solve each equation.

#47 $\left(\frac{1}{8}\right)^x = 16$

$\left(\frac{1}{2^3}\right)^x = 2^4$

$\left(2^{-3}\right)^x = 2^4$

$2^{-3x} = 2^4$

$-3x = 4$

$\boxed{x = \frac{-4}{3}}$

#51 $e^{2x+1} = e^3$

$2x + 1 = 3$

$2x = 2$

$\boxed{x = 1}$

$A = Pe^{rt}$

$2P = Pe^{r(10)}$

$2 = e^{10r}$

$e^{10r} = 2 \longrightarrow \log_e(e^{10r}) = \log_e(2)$

need inverse to "undo" e^x

$10r = \ln(2)$

$r = \frac{\ln(2)}{10} = \boxed{.0693}$

t	A
0	P
10	2P

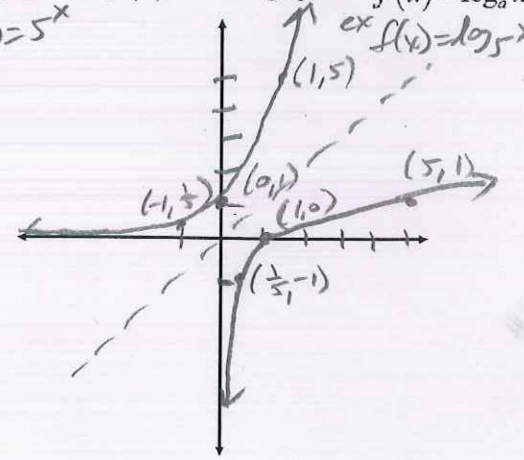
The Logarithmic Function

If you invest money in an account and your money doubles in 10 years, what was the account's interest rate? (assume continuous compounding)

Logarithm / exponential are inverses

graph of $f(x) = a^x$ to graph of $f(x) = \log_a x$

$f(x) = 5^x$ ex $f(x) = \log_5 x$



Logarithmic function: $f(x) = \log_a x \quad a > 0, \quad a \neq 1$

domain: $(0, \infty)$ No log of a negative

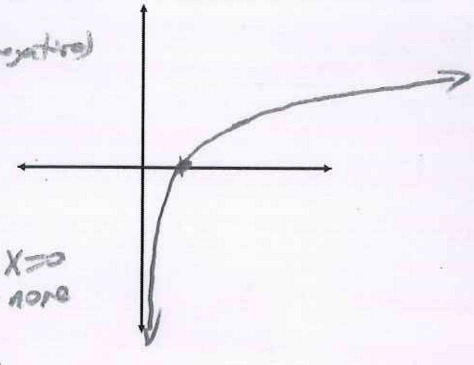
range: $(-\infty, \infty)$

x-int.: $(1, 0)$

y-int.: none

hor/vert asym.: V.A. $x = 0$ H.A none

inc./dec.: increasing everywhere



Log form / exponential form

$y = \log_a x$ means $a^y = x$

(log form) (exponential form)

REMEMBER: "LOGS ARE EXPONENTS"

Write each logarithm expression using exponential notation.

#4. $\log_{1/2}\left(\frac{1}{16}\right) = 4$ "Circle of logs"

$\boxed{\left(\frac{1}{2}\right)^4 = \frac{1}{16}}$ exp. form

$\log_2 8 = 3$ "Circle of logs" $2^3 = 8$

Properties of logs:

If m and n are positive real numbers and r is any real number, then

$\log_a(mn) = \log_a(m) + \log_a(n)$

$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$

$\log_a m^r = r\log_a(m)$

More about logs

$\log_a 1 = 0$

$\log_a a = 1$

$\log_a a^x = x$

$a^{\log_a x} = x$

logs & exp. are inverses

If

$\log_a u = \log_a v$

then

$u = v$

(1:1 property)

Evaluate each expression.

11. $\log_2 32 = x$

$2^x = 32$

$x = \boxed{5}$

15. $\log_2 24 - \log_2 12 = \log_2\left(\frac{24}{12}\right) = \log_2(2) = \boxed{1}$

Use $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, $\log_{10} 5 = 0.6990$

To compute each quantity.

18. $\log_{10} 250$

$\log_{10}(5 \cdot 5 \cdot 5 \cdot 2)$

$\log_{10} 5 + \log_{10} 5 + \log_{10} 5 + \log_{10} 2$

$.6990 + .6990 + .6990 + .3010$

$\boxed{2.398}$

22. $\log_{10} 30 = \log_{10}(5 \cdot 3 \cdot 2)$

$= \log_{10} 5 + \log_{10} 3 + \log_{10} 2$

$= .6990 + .4771 + .3010$

$= \boxed{1.4771}$

Use the properties of logs to write each expression as a single log.

26. $\ln x^6 - \ln x^3$

$\ln\left(\frac{x^6}{x^3}\right)$

$\boxed{\ln(x^3)}$

28. $\ln 2 - \ln x + \ln 4$

$\ln\left(\frac{(2)(4)}{x}\right)$

$\boxed{\ln\left(\frac{8}{x}\right)}$

Find x or y.

34. $\log_2 64 = y$

$2^y = 64$

$\boxed{y = 6}$

36. $\log_{16} x = \frac{1}{2}$

$16^{1/2} = x$

$\sqrt{16} = x$

$\boxed{4 = x}$

Use the properties of logs to solve for x.

42. $\ln x - \ln(x-1) = \ln 2$

$\ln\left(\frac{x}{x-1}\right) = \ln(2)$

$\frac{x}{x-1} = 2$

$x = 2(x-1)$

$x = 2x - 2$

$\boxed{2 = x}$

check:

$\ln(2) - \ln(2-1) \stackrel{?}{=} \ln(2)$

$.6931 = .6931$ ✓

47. $6 + (3)2^{(x+1)} = 8$

$-6 \qquad -6$

$3(2)^{x+1} = 2$

$(2)^{x+1} = \frac{2}{3}$

$\log_2(2)^{x+1} = \log_2\left(\frac{2}{3}\right)$

$x + 1 = \log_2\left(\frac{2}{3}\right)$

$x = \log_2\left(\frac{2}{3}\right) - 1$

$x = \frac{\log_{10}\left(\frac{2}{3}\right)}{\log_{10}(2)} - 1 = \boxed{-1.585}$

53. If $3^x = e^{cx}$, find c.

$\ln(3^x) = \ln(e^{cx})$

$x\ln(3) = cx$

if $x \neq 0$

$\boxed{c = \ln(3)}$

6. Half-life

- exponential decay
- time required for **half** the amount present to decay

Example: The quantity of a given radioactive substance which remains after t years is given by:

$$Q = 100e^{-0.24t}$$

(a) What is the initial quantity?
(b) What is the half-life?

(a) initial: $t = 0$

$Q = 100e^{-.24(0)}$

$Q = 100e^0$

$Q = 100(1)$

$\boxed{Q = 100}$

(b) half life is t when quantity is half

t	Q
0	100
t_{HL}	50

$Q = 100e^{-.24t}$

$50 = 100e^{-.24t}$

$\frac{1}{2} = e^{-.24t}$

$\ln\left(\frac{1}{2}\right) = \ln(e^{-.24t})$

$-.24t = \ln\left(\frac{1}{2}\right)$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-.24} = \boxed{2.89 \text{ years}}$

Curve fitting and Regression

Why do we spend time studying the graphs of different families of functions? One reason is so that we can determine an equation which models real-world data.

Could fit curve to 2 points:

$y = mx + b$ $\quad (4.2, 31.7)$ $\quad (1.1, 24.4)$

$m = \frac{31.7 - 24.4}{4.2 - 1.1}$

$m = 2.355$

$y = 2.355x + b$

$(24.4) = 2.355(1.1) + b$

$b = 21.8$

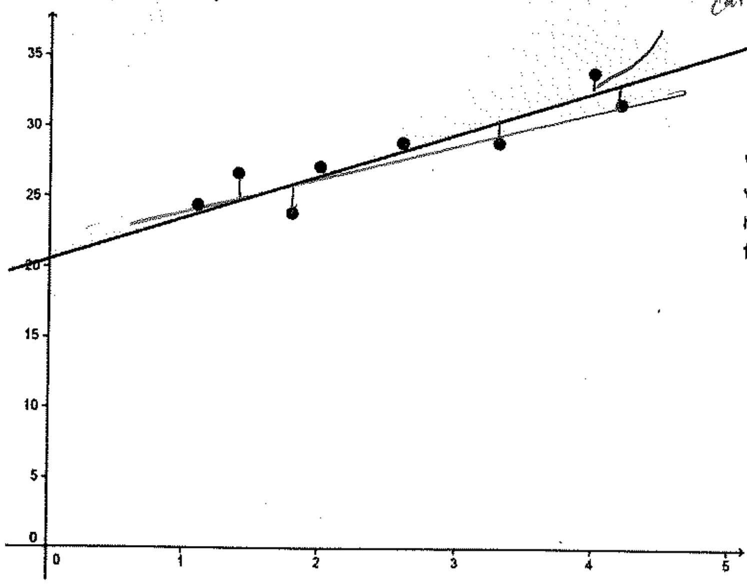
$y = 2.355x + 21.8$

Let's try to find an equation of a function which closely matches these data:

(x, y)
(1.1, 24.4)
(1.5, 26.7)
(1.8, 23.9)
(2.0, 27.2)
(2.6, 28.9)
(3.3, 28.9)
(4.0, 33.9)
(4.2, 31.7)

'x': Cricket chirps per second
'y': Air Temperature (C)

error is called 'residual'



y=mx+b

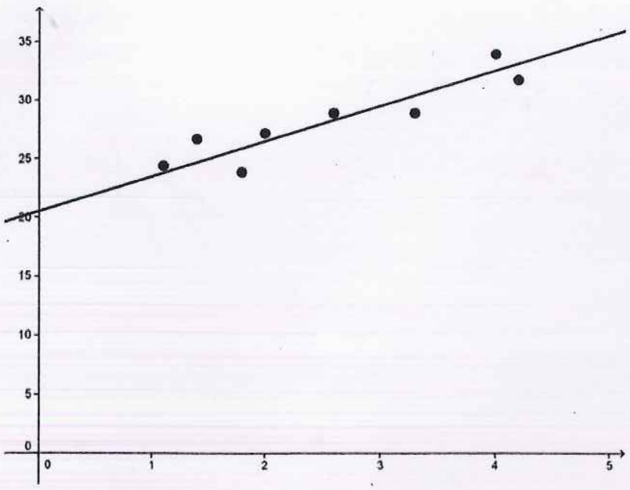
We need to find values for constants m and b so line best fits the data.

(x, y)
(1.1, 24.4)
(1.5, 26.7)
(1.8, 23.9)
(2.0, 27.2)
(2.6, 28.9)
(3.3, 28.9)
(4.0, 33.9)
(4.2, 31.7)

<u>Linear Regression using Ti83/84</u>

1) Clear any y= entries
2) Clear your lists (Stat, Edit, up arrow to top of column clear, enter)
3) Enter 'x' values in L1 and 'y' values in L2
4) Run Linear Regression (Stat, Calc, LinReg)
   LinReg L1, L2, Y1
5) Record equation produced
6) Turn on statistics plot (2nd Y=, just turn on 1st plot)
7) Zoom 9

(To turn off data point plotting, in Y=, up arrow to Plot1 and 'enter' to un-high-light).



y=2.7x+21.3

Now use model to predict temperature if observe 3 chirps per second:

$y = 2.7(3) + 21.3$

$= \boxed{29.4\,^\circ C}$

Your calculator can do regression analysis with a variety of functions:

- Linear - LinReg: $y = ax + b$
- Quadratic - QuadReg: $y = ax^2 + bx + c$
- Cubic - CubicReg: $y = ax^3 + bx^2 + cx + d$
- Quartic - QuartReg: $y = ax^4 + bx^3 + cx^2 + dx + e$
- Natural Logarithmic - LnReg: $y = a + b\ln(x)$
- Exponential - ExpReg: $y = a\bullet(b)^x$
- Power - PwrReg: $y = a\bullet(x)^b$
- Sine - SinReg: $y = a\sin(bx + c) + d$
-
- Logistic - Logistic: $y = \dfrac{c}{1 + a(e)^{-bx}}$