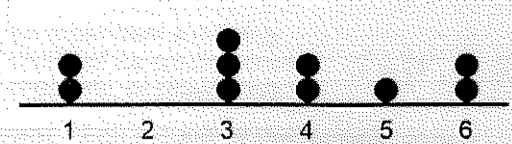
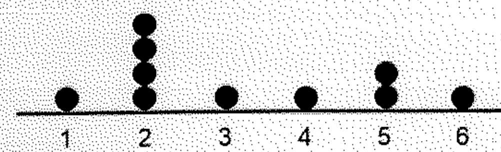
Honors Finite Mathematics – Lesson Notes: Unit 6 Probability (multiple chapters)

7.4 – Probability: Sample spaces, assignment of probabilities

If we rolled a 6-sided die 10 times, here is one possible result:



...and here is another:



If you had no previous experience with dice and someone asked you, "what is the probability of rolling a 2?" You would give difference answers depending upon the result of your 'experiment'. A probability determined in this way is called an **Experimental Probability.**

But if we rolled the die 1000 times (and it was a fair die) we would expect each side to come up equally often, and we could compute the **Theoretical Probability**. We might express this by showing all possible outcomes and the probability for each (called a *probability model)*:

X	1	2	3	4	5	6
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Anything can happen if you measure an outcome - experimental probability

If you consider a large number of cases, the measured probability will approach the theoretical probability (law of large numbers)

Rolling a die one time produces a probability model with *equally likely outcomes*. But not all probability models have equally likely outcomes. If we rolled two dice and found the sum, there are more ways to get a sum of 7 than a sum of 2 or 12:

X	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$
	.028	.056	.083	.111	.139	.167	.139	.111	.083	.056	.028

Probability terms:

Outcome: One possible result that can occur in an experiment.

Examples of outcomes:

A single roll of a die: 5

Tossing a coin twice: *TH*

An experiment surveying choice of ice cream (vanilla, chocolate, or strawberry): *C* (chocolate)

Sample space: Set of all outcomes that can occur as the result of an experiment.

Examples of sample spaces:

A single roll of a die: $S = \{1, 2, 3, 4, 5, 6\}$

Tossing a coin twice: $S = \{HH, HT, TH, TT\}$

An experiment surveying choice of ice cream (vanilla, chocolate, or strawberry): $S = \{V, C, S\}$

Event: Any subset of the sample space.

Examples of events:

Rolling an even number: $E = \{2, 4, 6\}$

Obtaining at least one tail: $E = \{HT, TH, TT\}$

Simple Event: An event that consists of only one outcome.

Choosing vanilla: $e = \{V\}$

Probability of an Outcome: A numerical value between 0 and 1 representing the likelihood of the outcome occurring. Denoted $P(e)$.

a) $P(e) \geq 0$ for each outcome e in S.
b) The sum of all the probabilities of the outcomes in $S = 1$.

Probability of an Event: A numerical value between 0 and 1 representing the likelihood of the event occurring. Denoted $P(E)$.

a) $P(\varnothing) = 0$
b) What is the probability of E if it is a simple event? $P(E) = P(e)$
c) What is the probability of the **union** of r simple events?

$$P(E) = P(e_1) + P(e_2) + P(e_3) + \ldots + P(e_r)$$

Constructing a probability (stochastic) model:

Roll a fair 6-sided die once. What is the probability of rolling an even number?

List the sample space: S = $\{1, 2, 3, 4, 5, 6\}$

Assign probabilities: $\frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}$

List the outcomes in the event: E = $\{2, 4, 6\}$

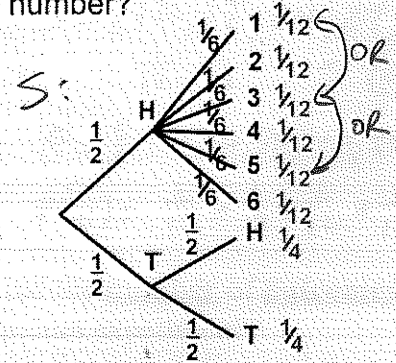
Compute the probability of the event: $P(E) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{2}}$

'or' = add (usually)

Constructing a probability (stochastic) model:

A fair coin is tossed. If it comes up heads, then a fair die is rolled. If it comes up tails, then the coin is tossed once more. What is the probability of tossing heads, then rolling an odd number?

S:



$$E = \{H1, H3, H5\}$$

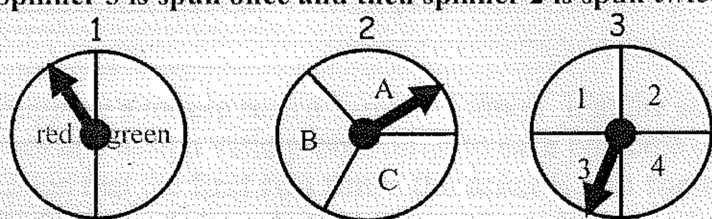
$$P(E) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$P(E) = \frac{1}{4}$$

('and'=multiply, 'or'=add) (usually)

#14 Use the pictured spinners to **list** the outcomes of a sample space associated with the experiment.

Spinner 3 is spun once and then spinner 2 is spun twice.



S = {1AA, 1AB, 1AC,
1BA, 1BB, 1BC,
1CA, 1CB, 1CC,
2AA, 2AB, 2AC,
2BA, 2BB, 2BC,
2CA, 2CB, 2CC,
3AA, 3AB, 3AC,
3BA, 3BB, 3BC,
3CA, 3CB, 3CC,
4AA, 4AB, 4AC,
4BA, 4BB, 4BC,
4CA, 4CB, 4CC}

Find the **number** of **outcomes** of a sample space associated with the random experiment.

#18 Tossing a coin 5 times

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2^5 = \boxed{32}$

#19 Tossing 3 dice.

$6 \cdot 6 \cdot 6$

$6^3 = \boxed{216}$

#34 The random experiment consinsts of tossing 2 fair dice. Construct a probability model for this experiment and find the probability of each given event.

B = {(1,5), (2,4), (3,3), (4,2), (5,1)}.

S:

	1 2 3 4 5 6
1	(1,1) — (1,6)
2	⋮
3	⋮ (36 outcomes)
4	⋮
5	
6	(6,1) (6,6)

$P(\text{each outcome}) = \frac{1}{36}$

so $P(B) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{5}{36}}$

#39 The random experiment consists of tossing a fair die and then a fair coin. Construct a probability model for this experiment. Then find the probability of the given event.

The coin comes up heads.

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$\frac{1}{12}\ \frac{1}{12}$ - - (all $\frac{1}{12}$) - - -

$E = \{1H, 2H, 3H, 4H, 5H, 6H\}$

$P(E) = \frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12} = \boxed{\frac{1}{2}}$

Find the **number** of **outcomes** of a sample space associated with the random experiment.

#20 Tossing 2 dice and then a coin.

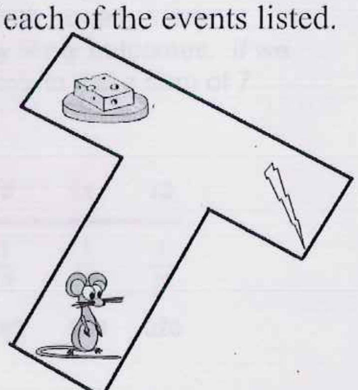
$\underset{die}{6} \cdot \underset{die}{6} \cdot \underset{coin}{2} = \boxed{72 \text{ outcomes}}$

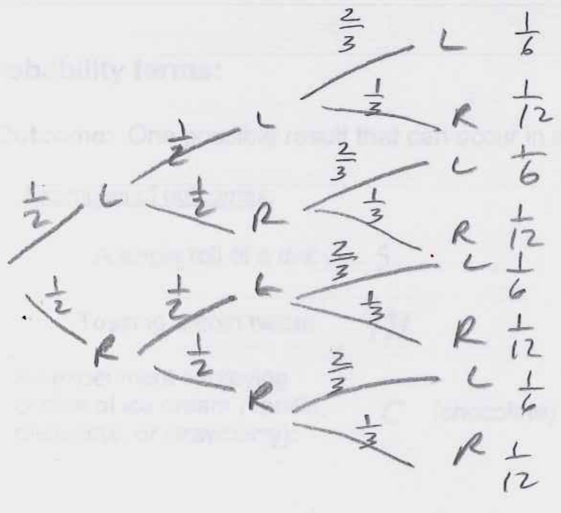
#22 Selecting 3 cards from a regular deck of 52 cards (assume order is not important).

${}_{52}C_3 = \boxed{22{,}100 \text{ outcomes}}$

#59 Suppose a mouse runs a T-maze 3 times. List the set of all possible outcomes and assign valid probabilities to each outcome under the assumption that the **first two times the maze is run the mouse chooses equally between left and right**, but on the **third run**, the mouse is **twice as likely choose cheese**. Find the probability of each of the events listed.

a) Run to the right 2 consecutive times
b) Never run to the right.
c) Run to the left on the first trial.
d) Run to the right of the second trial.





(a) $E = \{\underset{\frac{1}{12}}{LRR}, \underset{\frac{1}{6}}{RRL}, \underset{\frac{1}{2}}{RRR}\}$ $P(E) = \frac{4}{12} = \boxed{\frac{1}{3}}$

(b) $E = \{\underset{\frac{1}{6}}{LLL}\}$ $P(E) = \boxed{\frac{1}{6}}$

(c) $E = \{\underset{\frac{1}{6}}{LLL}, \underset{\frac{1}{12}}{LLR}, \underset{\frac{1}{6}}{LRL}, \underset{\frac{1}{12}}{LRR}\}$ $P(E) = \frac{6}{12} = \boxed{\frac{1}{2}}$

(d) $E = \{\underset{\frac{1}{6}}{LRL}, \underset{\frac{1}{12}}{LRR}, \underset{\frac{1}{6}}{RRL}, \underset{\frac{1}{12}}{RRR}\}$ $P(E) = \frac{6}{12} = \boxed{\frac{1}{2}}$

7.5 – Properties of the Probability of an Event

Roll a fair 6-sided die once. $S = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

What are the probabilities of the following events?

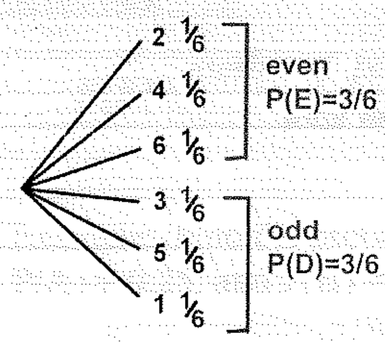
<u>Result is odd</u>	<u>Result is even</u>	<u>Result is divisible by 3</u>
$E = \{1,\ 3,\ 5\}$	$E = \{2,\ 4,\ 6\}$	$E = \{3,\ 6\}$
$\frac{3}{6}$	$\frac{3}{6}$	$\frac{2}{6}$

('or' = add)
(sometimes)

<u>Result is even or odd</u>	<u>Result is even or divisible by 3</u>
$E = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$	$E = \{2,\ 3,\ 4,\ 6\}$
$\frac{3}{6} + \frac{3}{6} = \frac{6}{6}$	$\frac{3}{6} + \frac{2}{6} \neq \frac{4}{6}$
$P(\text{even}) + P(\text{odd}) = P(\text{even or odd})$	$P(\text{even}) + P(\div 3) \neq P(\text{even or} \div 3)$

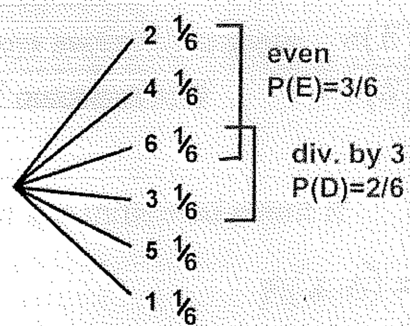
Mutually Exclusive

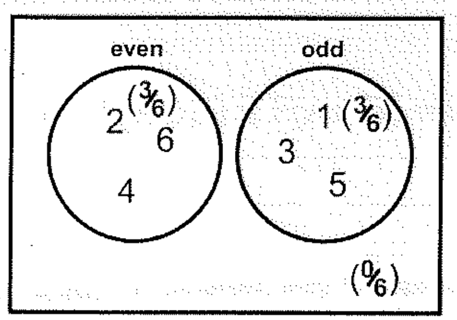
<u>Result is even or odd</u>

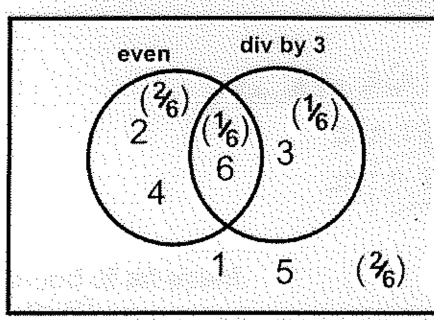


Not Mutually Exclusive

<u>Result is even or divisible by 3</u>







$$P(E \cup D) = P(E) + P(D)$$

$$P(E \cup D) = P(E) + P(D) - \underbrace{P(E \cap D)}_{}$$

subtracting the overlap so it isn't counted twice

Properties of the Probability of an Event

Additive Rule: $P(E \cup D) = P(E) + P(D) - P(E \cap D)$

Mutually Exclusive: Two or more events of a sample space are mutually exclusive if and only if they have no outcomes in common.

For mutually exclusive events: $P(E \cup D) = P(E) + P(D) - \cancel{P(E \cap D)}$

$P(E \cup D) = P(E) + P(D)$

For every event *E* in *S*: $0 \le P(E) \le 1$

Null event, sum of all events: $P(\varnothing) = 0, \quad P(S) = 1$

Complement: $P(\overline{E}) = 1 - P(E)$

Equally likely outcomes: When the same probability is assigned to each outcome of a sample space. (Frequently true, but not always).

Probability of event E in sample space with equally likely outcomes:

$$P(E) = \frac{\text{number of possible ways event } E \text{ can occur}}{\text{total number of outcomes in } S} = \frac{c(E)}{c(S)}$$

Example: Find the probability of the indicated event if P(A) = 0.25, and P(B) = 0.40

#3 P(A∪B) if A, B, are mutually exclusive.

$P(A \cup B) = P(A) + P(B)$
$= 0.25 + 0.40$
$= \boxed{0.65}$

#5 P(A∪B) if P(A∩B) = 0.15

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.25 + 0.40 - 0.15$
$= \boxed{0.5}$

odds: T to F $\quad \frac{T}{\ }$
Probability: $\frac{T}{T+F}$

Odds: A different way to represent likelihood of an event.

Odds for an event E: $\quad odds\ for\ E = \frac{P(E)}{P(\overline{E})}$

ex: If the probability of rain is 0.3, what are the odds for rain?

$\frac{0.3}{0.7} \quad \frac{3}{7} \quad \boxed{3 \text{ to } 7}$

Odds against an event E: $\quad odds\ against\ E = \frac{P(\overline{E})}{P(E)}$

$\frac{0.7}{0.3} \quad \frac{7}{3} \quad \boxed{7 \text{ to } 3}$

ex: If the probability of rain is 0.3, what are the odds against rain?

Converting odds to probability:

If the odds **for** E are a to b, then $P(E) = \dfrac{a}{a+b}$

If the odds **against** E are a to b, then $P(E) = \dfrac{b}{a+b}$

ex: What is the probability of rain if the odds are 3 to 7 for rain? $\frac{3}{3+7}$ $\frac{3}{10}$ $\boxed{0.3}$

#13 A card is drawn at random from a regular deck of 52 cards. Calculate the probability that a card with a number less than 6 is drawn. (count the ace as 1)

$E = \{\underbrace{A, 2, 3, 4, 5}_{\text{clubs}}, \underbrace{A, 2, 3, 4, 5}_{\text{spades}}$ --- 4 suits 20 outcomes

$P(E) = \frac{20}{52} = \boxed{\frac{5}{13}}$

#29 The Chicago Bears football team has a probability of winning of 0.65 and of tying of 0.05. What is their probability of losing?

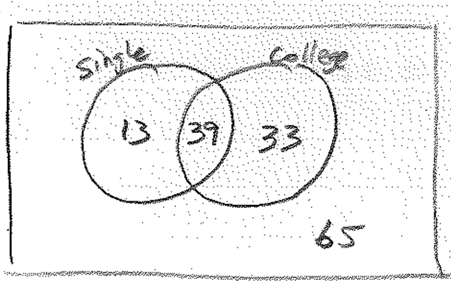
$S = \{\text{win, lose, tie}\}$

0.65 P(L) 0.05 must sum to 1

$P(L) = 1 - 0.65 - 0.05$
$= \boxed{0.3}$

#39 From a sales force of 150 people, 1 person will be chosen to attend a special sales meeting. If 52 are single, 72 are college graduates, and of the 52 who are single 0.75 are college graduates, what is the probability that a salesman selected at random will be neither single nor a college graduate?

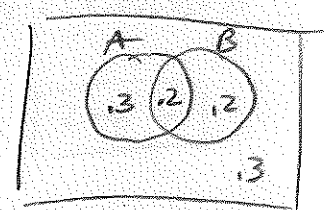
$0.75 \cdot 52 = 39$



$P(\overline{\text{single} \cap \text{college}}) = \frac{65}{150} = \boxed{\frac{13}{30}} = .433$

#33 Let A and B be events of a sample space S and let P(A) = 0.5, P(B) = 0.4, and P(A∩B) = 0.2 Find the probabilities of each of the following events:

a) A or B
b) A but not B
c) B but not A
d) Neither A nor B



(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= .5 + .4 - .2 = \boxed{.7}$

(b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
$= .5 - .2 = \boxed{.3}$

(c) $P(B \cap \bar{A}) = P(B) - P(A \cap B)$
$= .4 - .2 = \boxed{.2}$

(d) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - .7 = \boxed{.3}$

#41 Determine the probability of E for the given odds.
3 to 1 for E.

$$\frac{3}{3+1}$$

$$P(E) = \boxed{\frac{3}{4}}$$

#47 Determine the odds for and against each event for the given probability.
P(E) = 0.6

$$\frac{0.6}{0.4} \quad \frac{6}{4} \quad \frac{3}{2}$$

$\boxed{\begin{array}{l}\text{3 to 2 for} \\ \text{2 to 3 against}\end{array}}$

#51 If two fair dice are thrown, what are the odds of obtaining:
a 7?
an 11?
a 7 or 11?

36 outcomes $\quad \underline{6} \cdot \underline{6}$
$c(S) = 36$

$7 = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$
$c(7) = 6$
$P(7) = \frac{6}{36} = \frac{1}{6} = \frac{1}{1+5}$ $\boxed{\text{1 to 5}}$

$11 = \{(5,6)(6,5)\}$
$c(11) = 2$
$P(11) = \frac{2}{36} = \frac{1}{18} = \frac{1}{1+17}$ $\boxed{\text{1 to 17}}$

$7 \text{ or } 11 = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)(5,6)(6,5)\}$
$c(7 \text{ or } 11) = 8$
$P(7 \text{ or } 11) = \frac{8}{36} = \frac{2}{9} = \frac{2}{2+7}$ $\boxed{\text{2 to 7}}$

7.5 day 2 – Probability: Using counting techniques

Probabilities are often formed by counting number of outcomes in an event and dividing by number of outcomes in the sample space (especially for equally likely outcome situations).

We can sometimes use unit 5 counting techniques to determine the number of outcomes.

2. What is the probability that a seven-digit phone number contains the number 7?

all 10 digits available, 10^7 possible numbers.
everything except 7 available, 9^7 "

$P(\text{no } 7) = \frac{9^7}{10^7}$ $\quad P(\text{contain } 7) = 1 - P(\text{no } 7)$

$= 1 - \frac{9^7}{10^7} = \boxed{0.5217}$

4. Four letters, with repetition allowed, are selected from the alphabet. What is the probability that none of them is a vowel (a, e, i, o, u)?

$$\frac{21 \cdot 21 \cdot 21 \cdot 21}{26 \cdot 26 \cdot 26 \cdot 26} = \frac{21^4}{26^4} = \frac{194481}{456976} = \boxed{0.4256}$$

6. A fair coin is tossed 6 times.

total outcomes: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

a. Find the probability that exactly 1 tail appears.

THHHHH how many ways to
HTHHHH choose where T will be?
⋮
${}_6C_1$

$P(\text{ex } 1T) = \frac{{}_6C_1}{2^6} = \frac{6}{64} = \boxed{.09375}$

b. Find the probability that no more than 1 tail appears.

1 tail or 0 tails

$$\frac{{}_6C_1}{2^6} + \frac{{}_6C_0}{2^6} = \frac{6}{64} + \frac{1}{64} = \frac{7}{64} = \boxed{.1094}$$

groups

12. What is the probability that, in a group of 6 people, at least 2 were born in the same month (disregard day and year)?

total outcomes $= 12^6$ $\quad P(\text{at least } 2) = 1 - P(\text{none match})$

ways to choose all different months:

$12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = {}_{12}P_6$

$P(\text{at least } 2) = 1 - \frac{{}_{12}P_6}{12^6}$

$= 1 - \frac{665280}{2985984} \approx \boxed{.777}$

24. 5 cards are dealt at random from a regular deck of 52 playing cards. Find the probability that:

total outcomes $= C(52,5)$

a. All are hearts. $\frac{C(13,5)}{C(52,5)} = \frac{1287}{2598960} \approx 0.000495$

b. Exactly 4 are spades.

$\frac{C(13,4) \cdot C(39,1)}{C(52,5)}$ ← remaining when spades removed $= \frac{715 \cdot 39}{2598960} = \boxed{\frac{27885}{2598960} \approx .0107}$

c. Exactly 2 are clubs.

$\frac{C(13,2) \cdot C(39,3)}{C(52,5)} = \frac{78 \cdot 9139}{2598960} = \boxed{\frac{712842}{2598960} \approx .2743}$

20. If the seven letters in the word DEFAULT are rearranged, what is the probability the word will end in E and begin with T?

ways to pick letter 1: 1 (T)

ways to pick 5 middle letters: $P(5,5)$ or $5!$

ways to pick last: 1 (E)

$5! = 120$

$P = \frac{120}{7!} = \frac{120}{5040} = \boxed{\frac{1}{42} \approx .0238}$

17. If the five letters in the word VOWEL are rearranged, what is the probability the L will precede the E?

either E…L or L…E

Could swap, so $\boxed{\frac{1}{2}}$

25. In a game of bridge find the probability that a hand of 13 cards consists of 5 spades, 4 hearts, 3 diamonds, and 1 club.

$$P = \frac{C(13,5) \cdot C(13,4) \cdot C(13,3) \cdot C(13,1)}{C(52,13)}$$

$$= \frac{1287 \cdot 715 \cdot 286 \cdot 13}{6.35 \cdot 10^{11}} \approx \boxed{.00539}$$

11. What is the probability that, in a group of 3 people, at least 2 were born in the same month?

$P(\text{at least 2 same}) = 1 - P(\text{all diff})$

$$= 1 - \frac{P(12,3)}{12^3}$$

$$= 1 - \frac{1320}{1728} = \frac{408}{1728} = \boxed{\frac{17}{72} \approx .236}$$

26.Find the probability of obtaining each of the followingpoker hands:
a. Royal flush (10, J, Q, K, A all of the same suit)
b. Straight flush (5 cards in sequence in a single suit, but not a royal flush)
c. Four of a kind (4 cards of the same face value)
d. Full house (one pair and one triple of the same face values)
e. Flush (5 nonconsecutive cards each of the same suit)
f. Straight (5 consecutive cards, not all the same suit)

total 5 card hands $= C(52,5) = 2598960$

(a) 4 royal flushes (1 each suit) $P = \frac{4}{2598960} = \boxed{\frac{1}{649740} \approx 0.00000154}$

(b) possible straight flush in each suit:
A–5, 2–6, 3–7, 4–8, 5–9, 6–10, 7–J, 8–Q, 9–K
9 per suit, so 36 possible
$P = \boxed{\frac{36}{2598960} \approx 0.0000139}$

(c) 4 of a kind: 13 possible, each w/ $(52-4)$ 48 "5th cards"
$P = \boxed{\frac{13 \cdot 48}{C(52,5)} \approx 0.00024}$

(d) Full house

ways to choose card for 3 of a kind	ways to choose card for pair	ways to get 3 of kind $C(4,3)$	ways to get 2 of kind $C(4,2)$
13	12	4	6

$13 \cdot 12 \cdot 4 \cdot 6 = \boxed{\frac{3744}{2598960} \approx 0.00144}$

(e) Flush:

ways to choose suit	ways to get 5 cards in suit $C(13,5)$
4	1287

but don't include royal flushes or straight flushes (a, b)

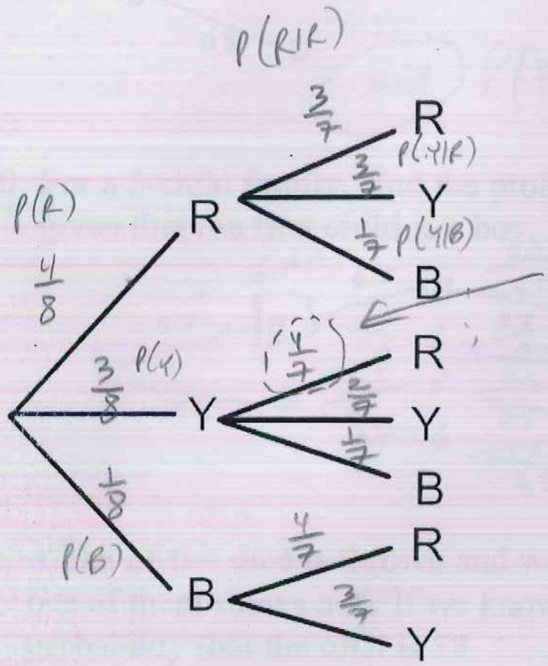
$4 \cdot 1287 = 5148 - 36 - 4$
$P = \boxed{\frac{5108}{C(52,5)} = 0.00197}$

f) Straight:
10 possible straights: A–5, 2–6, 3–7, 4–8, 5–9, 6–10, 7–J, 8–Q, 9–K, 10–A
4 suits for each of the 5 cards: 4^5
so: $10 \cdot 4^5 = 10240$ possible straights but don't include royal or straight flush cases
$- 36$
$- 4$
$= 10200$
so $P = \boxed{\frac{10200}{C(52,5)} \approx 0.00392}$

8.1 day 1 – Conditional probability

A jar contains 4 red balls, 3 yellow balls, and 1 blue ball. (8 total balls)

If a ball is removed, and then a 2nd ball is removed without replacing the 1st ball, what is the probability of removing a yellow ball, then a red ball?



conditional probability

$P(R|Y)$

"the probability of R given Y"

multiply ('and')

Product rule

$$P(\text{yellow then red}) = P(\text{yellow})\cdot P(\text{red given yellow})$$

$$P(R\cap Y) = P(Y)\cdot P(R\,|\,Y)$$

AND = multiply BUT one is a conditional probability

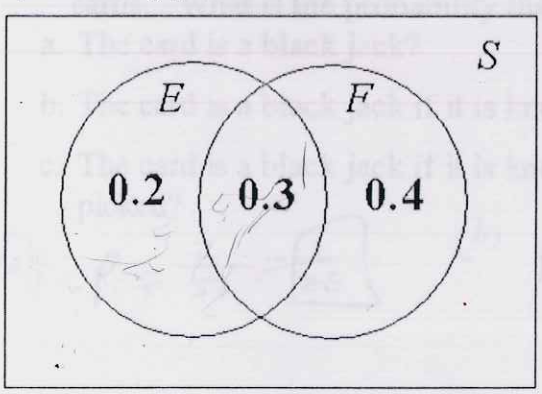
Product Rule

$$P(E\cap F) = P(F)\cdot P(E\,|\,F)$$

Conditional Probability

$$P(E\,|\,F) = \frac{P(E\cap F)}{P(F)}$$

Tree and Venn diagrams are useful tools to solve conditional probability problems



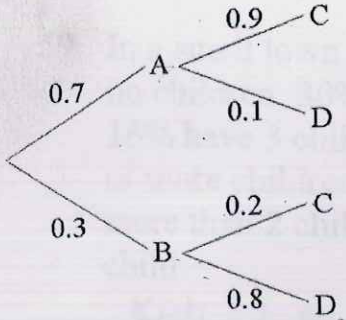
$P(E) = 0.2 + 0.3 = 0.5 \quad [= P(E\cap \bar{F}) + P(E\cap F)]$

$P(E\cap F) = 0.3$

$P(F|E) = \dfrac{0.3}{0.5} = \boxed{\dfrac{3}{5}}$

$P(F|E) = \dfrac{P(E\cap F)}{P(E)}$

E is the condition (portion of set "out of") so (bottom of fraction)



AD or BD

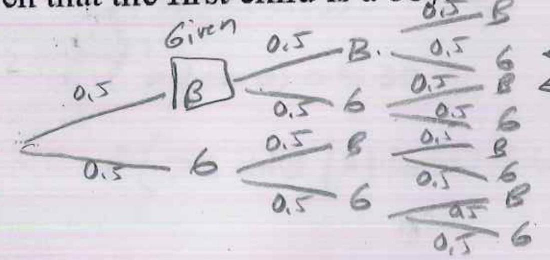
18. $P(D) = (0.7)(0.1) + (0.3)(0.8)$

$.07 + .24 = \boxed{.31}$

22. $P(D|B) = \boxed{0.8}$

add $P(B) = \boxed{0.3}$

30. For a 3-child family, find the probability of exactly 1 girl, given that the first child is a boy.



$(0.5)(0.5) + (0.5)(0.5)$

$0.25 + 0.25$

$\boxed{0.50}$

32. A pair of fair dice is thrown and we are told that at least one of them shows a 2. If we know this, what is the probability that the total is 7?

$P(7 | \text{at least a } 2)$

conditional $S = \{(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(1,2)(3,2)(4,2)(5,2)(6,2)\}$ (11)

$E = \{(2,5)(5,2)\}$ (2)

$P = \boxed{\frac{2}{11}}$

38. A card is drawn at random from a regular deck of 52 cards. What is the probability that
a. The card is a black jack?
b. The card is a black jack if it is known a jack was picked?
c. The card is a black jack if it is know a black card was picked?

(a) $P = \frac{2}{52} = \boxed{\frac{1}{26}}$ (b) $P = \frac{2}{4} = \boxed{\frac{1}{2}}$ (c) $P = \frac{2}{26} = \boxed{\frac{1}{13}}$

59. In a small town it is known that 20% of the families have no children, 30% have 1 child, 20% have 2 children, 16% have 3 children, 8% have 4 children, and 6% have 5 or more children. Find the probability that a family has more than 2 children if it is known that it has at least 1 child.

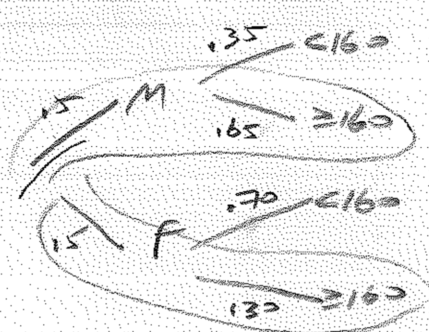
Kids	0	1	2	3	4	5+
P	.2	.3	.2	.16	.08	.06

$P(\text{at least 1 child}) = 0.80$

$P(>2\text{ child}) = 0.30$

$P(>2\text{ child} \mid \text{at least 1 child}) = \frac{.30}{.80} = \boxed{.375}$

57. In a sample survey it is found that 35% of the men and 70% of the women weigh less than 160 pounds. Assume that 50% of the sample are men. If a person is selected at random and this person weighs less than 160 pounds, what is the probability that this person is a woman?



$$P(W \mid <160) = \frac{P(W \cap <160)}{P(<160)}$$

$$= \frac{(0.5)(0.7)}{(0.5)(0.7) + (0.5)(0.35)}$$

$$= \frac{.35}{.525} = .\overline{666} = \boxed{\frac{2}{3}}$$

36. A box contains 2 red, 4 green, 1 black, and 8 yellow marbles. If 2 marbles are selected without replacement, what is the probability that 1 is red and 1 is green?

<u>using equations</u>

$E = \{RG, GR\}$

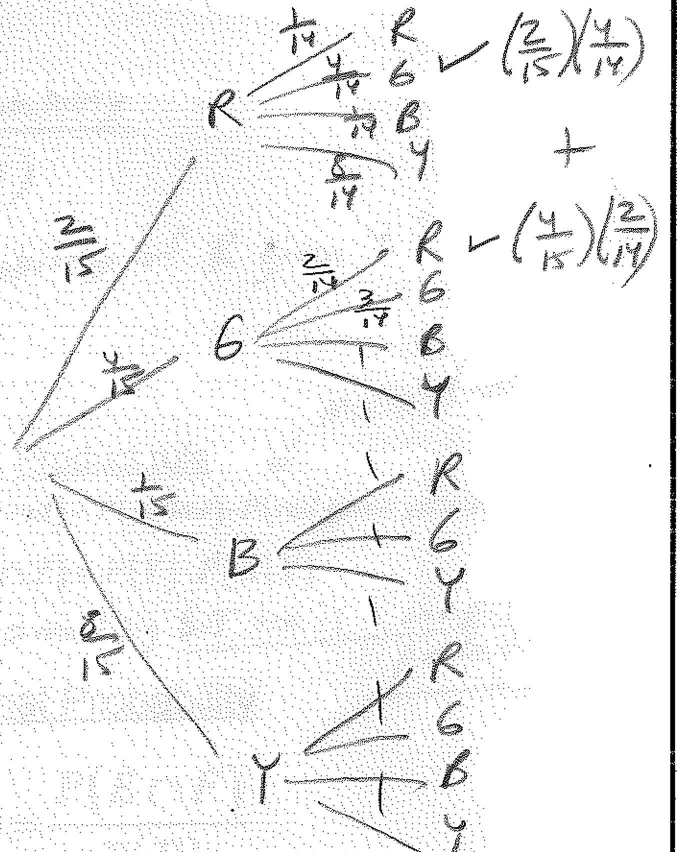
$\left(\frac{2}{15}\right)\left(\frac{4}{14}\right)+\left(\frac{4}{15}\right)\left(\frac{2}{14}\right)$

$\frac{8}{210} + \frac{8}{210}$

$\frac{16}{210}$

$\boxed{\frac{8}{105} \approx .076}$

<u>using tree</u>



8.2 – *Independent Events*

We polled 100 students and asked them to choose their favorite of 3 activities: reading a book, playing video games, or watching Netflix.

	girls	*boys*	
read a book	**18**	**4**	**22**
video games	**12**	**20**	**32**
watch Netflix	**30**	**16**	**46**
	60	**40**	**100**

We want to investigate: *"Does favorite activity depend upon gender?"*

Consider what you would say if I asked you the question, "What is the probability that a student likes to play video games?"

There are 3 probabilities you could find to try to answer this question.

You could find the probability of all students who like video games:

$$P(games) = \frac{32}{100}$$
$$P(games) = .32$$

You could find the probability of a girl liking video games:

$$P(games) = \frac{32}{100} \qquad P(games \mid girl) = \frac{12}{60}$$
$$P(games) = .32 \qquad P(games \mid girl) = .20$$

Or you could find the probability of a boy liking video games:

$$P(games) = \frac{32}{100} \qquad P(games \mid girl) = \frac{12}{60} \qquad P(games \mid boy) = \frac{20}{40}$$
$$P(games) = .32 \qquad P(games \mid girl) = .20 \qquad P(games \mid boy) = .50$$

So your answer to the question would have to be, "It *depends*. Are we talking about boys, girls, or all students?"

This means 'activity' **depends upon** 'gender'.

Or you could say 'activity' and 'gender' are **not independent**.

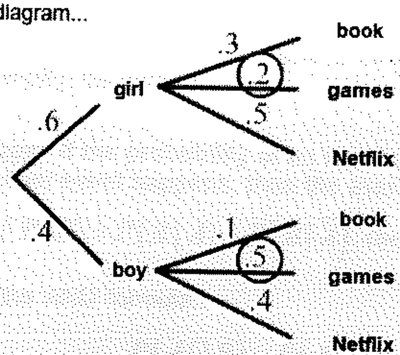
How can we determine if two events are independent?

One way is to check **any two** of the three probabilities of one of the events, like we just did.

$$P(games) = \frac{32}{100} \qquad P(games \mid girl) = \frac{12}{60}$$
$$P(games) = .32 \qquad P(games \mid girl) = .20$$

If you find that they don't match, then the events are not independent.

On a tree diagram...



...if you find that probabilities for the same 2nd choice don't match, then the events are not independent.

Here's what it would look like if 'gender' and 'activity' ***were* independent**:

The counts would have to be different from what they were in the not independent case.

	girls	*boys*	
read a book	12	8	20
video games	30	20	50
watch Netflix	18	12	30
	60	40	100

At first glance, this table doesn't look very different from the previous set of data.

Notice that the counts between the boys and girls and still *not the same*.

Yet, this data does show that 'gender' and 'activity' are independent.

We can't tell by comparing the ***counts*** between categories...

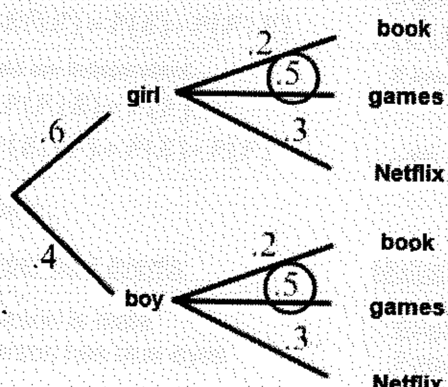
...we must compare the ***percentages.***

It's the ***percentages*** that would now be the same:

$P(games) = \frac{50}{100}$ $\quad P(games \mid girl) = \frac{30}{60}$ $\quad P(games \mid boy) = \frac{20}{40}$

$P(games) = .50$ $\quad P(games \mid girl) = .50$ $\quad P(games \mid boy) = .50$

On a tree diagram, corresponding conditional probabilities would be identical:



The special case 'disjoint events' modified the OR formula (it removed the overlap term).
Similarly, independent events are a special case for the AND formula and will modify (simplify) the formula:

<u>Special Case</u>: If the probability of an event does not change regardless of whether or not another event happens, then the events are <u>independent events.</u>

$$\textit{If } P(B)=P(B\mid A)=P\left(B\mid \overline{A}\right), \textit{then A and B are independent.}$$

...and the 'AND' formula...

$$P(A\cap B)=P(A)\bullet P(B\mid A)$$

...simplifies to:

$$P(A\cap B)=P(A)\bullet P(B)$$

We can also define a 'test for independent events':

<u>Test for independent events</u>:

Two events are independent if $P(B)=P(B\mid A)=P\left(B\mid \overline{A}\right)$

In other words: check any 2 of the 3 ways to find probability of B. If they don't match, then B is depending upon A and the events are not independent.

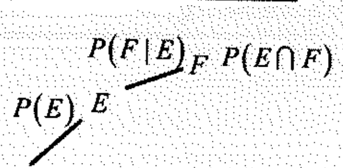
<u>Note</u>: Some books also use the simplified version of the AND formula as a 'test for independence'...

$$\textit{If } P(A\cap B)=P(A)\bullet P(B), \textit{then A and B are independent.}$$

...but this is more a consequence of independence, not the reason the events are independent.

Note: independent events and mutually-exclusive events are different.
Each is a 'special case' of the OR or AND rules:

<u>Independent events</u>



AND = Multiply

In general...

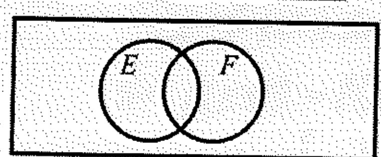
$$P(E\cap F)=P(E)\bullet P(F\mid E)$$

If events are independent...

$$P(F\mid E)=P(F)$$

...so $P(E\cap F)=P(E)\bullet P(F)$

<u>Mutually-exclusive events</u>



OR = Add

In general...

$$P(E\cup F)=P(E)+P(F)-P(E\cap F)$$

If events are mutually-exclusive...

$$P(E\cap F)=0$$

...so $P(E\cup F)=P(E)+P(F)$

Examples...

<u>Independent events</u>

E=person is male
F=person has blue eyes

E=person has heart disease
F=person is female

<u>Mutually-exclusive events</u>

E=person is male
F=person is female

E=shape is a circle
F=shape is a triangle

Examples:

If $P(E)=0.3$, $P(F)=0.2$, and $P(E\cup F)=0.4$, what is $P(E|F)$? Are E and F independent?

Addition Rule: $P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$0.4 = 0.3+0.2-P(E\cap F)$

$P(E\cap F)=0.1$

conditional probability: $P(E\cap F)=P(F)\cdot P(E|F)$

so $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0.1}{0.2}=0.5$

independent? yes if $P(E|F)=P(E)$

$0.5\neq 0.3$

$\boxed{\text{not independent}}$

#13

A fair die is rolled. Let E be the event "1, 2, or 3 is rolled" and let F be the event "3, 4, or 5 is rolled. Are E and F independent?

$E=\{1,2,3\}$ $\quad F=\{3,4,5\}$ $\quad E\cap F$

$P(E)=\frac{3}{6}=\frac{1}{2}$ $\quad P(F)=\frac{3}{6}=\frac{1}{2}$ $\quad \{1,2,3\}\cap\{3,4,5\}$

$\{3\}$

$P(E\cap F)=\frac{1}{6}$

general AND formula: $P(E\cap F)=P(E)\cdot P(F|E)$

if E & F indep, $P(F|E)=P(F)$

so $P(E\cap F)=P(E)\cdot P(F)$

$\frac{1}{6}\overset{?}{=}\frac{1}{2}\cdot\frac{1}{2}$

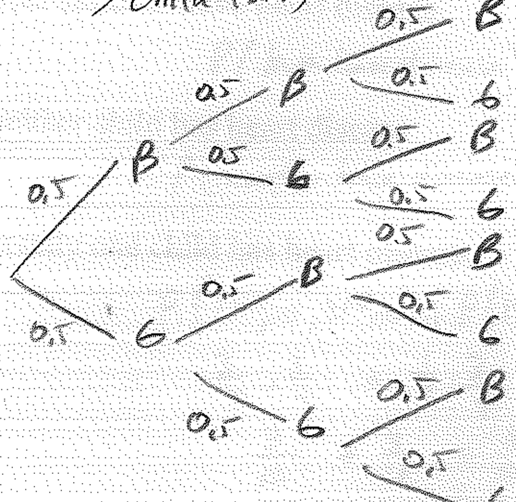
$\frac{1}{6}\neq\frac{1}{4}$

$\boxed{\text{not independent}}$

#15

For a 3-child family let E be the event "the family has at most 1 boy" and let F be the event "the family has children of each sex". Are E and F independent events?

3 child family



$P(BBB) = (0.5)^3$

$P(BBG) = (0.5)^3$

$P(GGG) = (0.5)^3$

$E = \{BGG, GBG, GGB, GGG\}$

$P(E) = (0.5)^3 + (0.5)^3 + (0.5)^3 + (0.5)^3$

$P(E) = \boxed{0.5}$ $\frac{1}{2}$

$F = \{BBG, BGB, BGG, GBB, GBG, GGB\}$

$P(F) = (0.5)^3 \cdot 6$

$P(F) = \boxed{0.75}$ $\frac{6}{8}$ $\frac{3}{4}$

$E \cap F = \{BGG, GBG, GGB\}$

$P(E \cap F) = (0.5)^3 \cdot 3$

$P(E \cap F) = \boxed{0.375}$ $\frac{3}{8}$

independent if $P(E \cap F) = P(E) \cdot P(F)$

$0.375 \stackrel{?}{=} (0.5)(0.75)$

$0.375 = 0.375$

$\boxed{\text{yes, E and F are independent}}$

23. Cardiovascular Disease Records show that a child of parents with heart disease has a probability of 3/4 of inheriting the disease. Assuming independence, what is the probability that, for a couple with heart disease and that has two children:

$P(\text{not disease}) = 1 - \frac{3}{4} = \frac{1}{4}$

a. Both children have heart disease

$P(\text{both have disease}) = \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{9}{16}}$

b. Neither child has heart disease

$P(\text{neither have disease}) = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

c. Exactly one child has heart disease.

$P(1^{st}\text{ yes}, 2^{nd}\text{ no}) + P(1^{st}\text{ no}, 2^{nd}\text{ yes})$

$\frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4}$

$\frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \boxed{\frac{3}{8}}$

33. Election A candidate for office believes that 4/5 of the registered voters in her district will vote for her in the next election. If two registered voters are independently selected at random, what is the probability that

$P(\text{not vote}) = \frac{1}{5}$

a. Both of them will vote for her in the next election

$\frac{4}{5} \cdot \frac{4}{5} = \boxed{\frac{16}{25}}$

b. Neither will vote for her in the next election

$\frac{1}{5} \cdot \frac{1}{5} = \boxed{\frac{1}{25}}$

c. Exactly one of them will vote for her in the next election?

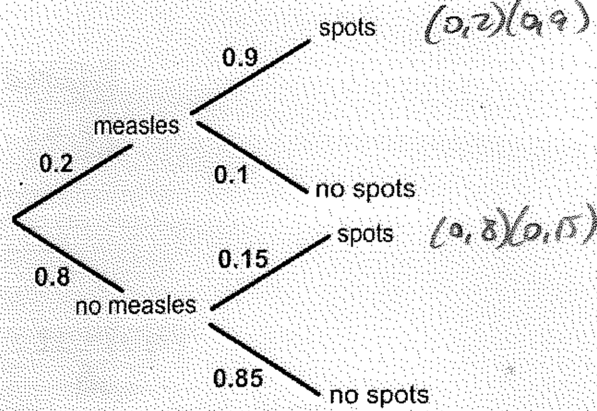
$P(1\text{ yes}, 1\text{ no}) * \#\text{ways to choose who will vote yes}$

$\frac{4}{5} \cdot \frac{1}{5} \binom{2}{1}$

$\frac{4}{25} \cdot 2 = \boxed{\frac{8}{25}}$

8.3 – Bayes' Formula

In a particular region, the probability that a person has measles is 20%. If a person has measles they have a 90% probability of having visible spots. A person who does not have measles has a 15% probability of visible spots. **If a patient displays spots, what is the probability that they have measles?**



$P(M \mid S) =$

$$\frac{\textit{\# of these ways that come from measles}}{\textit{\# ways to have spots}}$$

$$\frac{(0,2)(0,9)}{(0,2)(0,9)+(0,8)(0,15)}$$

$$=\boxed{0,6}$$

$$P(M \mid S) = \frac{P(M) \cdot P(S \mid M)}{P(S)}$$

Bayes' Formula

$$P(A \mid E) = \frac{P(A) \cdot P(E \mid A)}{P(E)}$$

Bayes' Formula

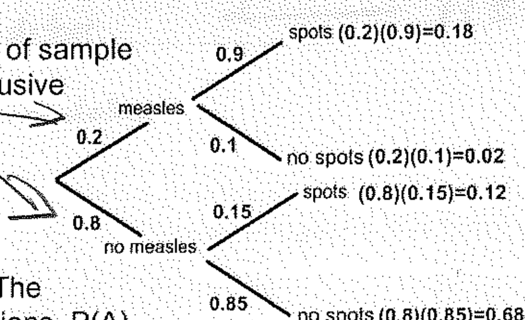
Let S be a sample space partitioned into n events, $A_1, \ldots, A_n$. Let E be any event of S for which $P(E) > 0$. The probability of the event $A_j (j = 1, 2, \ldots, n)$, given the event E, is

$$P(A_j \mid E) = \frac{P(A_j) \cdot P(E \mid A_j)}{P(E)} \tag{8}$$

$$= \frac{P(A_j) \cdot P(E \mid A_j)}{P(A_1) \cdot P(E \mid A_1) + P(A_2) \cdot P(E \mid A_2) + \cdots + P(A_n) \cdot P(E \mid A_n)}$$

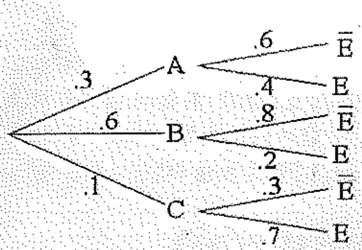
Terminology

Partitions: The division of sample space into mutually-exclusive subsets.



a priori probabilities: The probabilities of the partitions, P(A).

a posteriori probabilities: The probabilities of the partitions given additional event information P(A|E).



1. $P(E|A)$ $\boxed{0.4}$

2. $P(\overline{E}|A)$ $\boxed{0.6}$

6. $P(\overline{E}|C)$ $\boxed{0.3}$

7. $P(E)$

$P(A)\cdot P(E|A)$ $(0.3)(0.4)$
$+P(B)\cdot P(E|B)$ $(0.6)(0.2)$
$+P(C)\cdot P(E|C)$ $(0.1)(0.7)$

$\boxed{0.31}$

10. $P(B|\overline{E})$

Baye's:

$$= \frac{P(B)\cdot P(\overline{E}|B)}{P(\overline{E})}$$

$$= \frac{(0.6)(0.8)}{(0.3)(0.6)+(0.6)(0.8)+(0.1)(0.3)}$$

$$= \boxed{0.6957}$$

11. $P(C|E)$

Baye's:

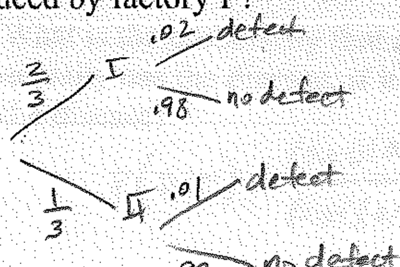
$$= \frac{P(C)\cdot P(E|C)}{P(E)}$$

$$= \frac{(0.1)(0.7)}{(0.3)(0.4)+(0.6)(0.2)+(0.1)(0.7)}$$

$$= \boxed{0.2258}$$

pg. 377 #28

Cars are being produced by two factories, but factory I produces twice as many cars as factory II in a given time. Factory I is known to produce 2% defectives and factory II produces 1% defectives. A car is examined and found to be defective. What are the a **priori** and a **posteriori** probabilities that the car was produced by factory I ?



a priori: $P(I) = \frac{2}{3} = \boxed{0.667}$

before knowing about defect 66.7% chance car from I

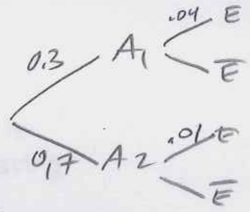
aposteriori: $P(I|D) = \frac{P(I)\,P(D|I)}{P(D)}$

$$= \frac{\left(\frac{2}{3}\right)\cdot(0.02)}{\left(\frac{2}{3}\right)(0.02)+\left(\frac{1}{3}\right)(0.01)}$$

$$= \boxed{0.800}$$

after knowing about defect 80% chance car from I pg. 376 #16

Events A_1 and A_2 form a partition of a sample space S with $P(A_1) = 0.3$ and $P(A_2) = 0.7$. If E is an event in S with $P(E|A_1) = 0.04$ and $P(E|A_2) = 0.01$, compute $P(E)$.



$P(E) = (0.3)(0.4) + (0.7)(.01) = \boxed{0.19}$

#20 Use the info in #16 to find $P(A_1|E)$ and $P(A_2|E)$

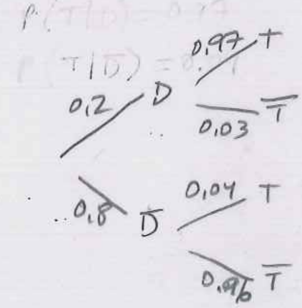
$$P(A_1|E) = \frac{P(A_1)\cdot P(E|A_1)}{P(E)} = \frac{(0.3)(0.04)}{0.19} \doteq \boxed{0.063}$$

$$P(A_2|E) = \frac{P(A_2)\cdot P(E|A_2)}{P(E)} = \frac{(0.7)(0.01)}{0.19} = \boxed{0.0368}$$

41. A scientist designed a medical test for a certain disease. Among **100** patients who have the disease, the test will show the presence of the disease in **97 cases out of 100**, and will fail to show the presence of the disease in the remaining **3 cases out of 100**. Among those who do not have the disease, the test will erroneously show the presence of the disease in **4 cases out of 100**, and will show that there is no disease in the remaining **96 cases out of 100**.

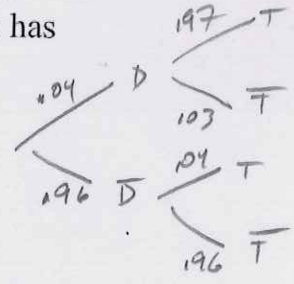
D = disease
T = test positive

a. What is the probability that a patient who tested positive on this test actually has the disease, if it is estimated that 20% of the population has the disease?



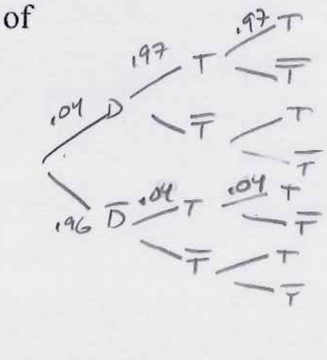
$$P(D|+) = \frac{P(D)\cdot P(T|D)}{P(T)} = \frac{(0.2)(0.97)}{(0.2)(0.97) + (0.8)(0.04)} = \boxed{0.858}$$

b. What is the probability that a patient who tested positive on this test actually has the disease, if it is estimated that 4% of the population has the disease?



$$P(D|T) = \frac{(0.04)(0.97)}{(0.04)(0.97) + (0.96)(0.04)} = \boxed{0.5026}$$

c. What is the probability that a patient who took the test twice and tested positive both times actually has the disease, if it is estimated that 4% of the population has the disease?



$$P(D|TT) = \frac{P(D)\cdot P(TT|D)}{P(TT)} = \frac{(.04)(0.97)^2}{(.04)(.97)^2 + (0.96)(.04)^2} = \boxed{0.9608}$$

7.6 – Expected Value

Expected Value = a typical or average number which represents data.

Examples:

- Average household has 3.17 people.
- Typical number of days of rain per year in Phoenix is 7.4 days of rain.

Note: expected value does not have to be an actual possible value in the data set.

Definition: $\mathbf{E} = m_1 p_1 + m_2 p_2 + \ldots$

m = assigned payoff value for event
p = probability of event

example:
1000 raffle tickets
prizes: 1 worth \$300 2 worth \$100
100 worth \$1 rest worth \$0

Find average (expected) value of each ticket.

$$E = 300\left(\frac{1}{1000}\right) + 100\left(\frac{2}{1000}\right) + 1\left(\frac{100}{1000}\right) + 0\left(\frac{897}{1000}\right)$$

$$\boxed{E = \$0.60}$$

If each ticket cost \$1, the "raffle"

If each sold for \$0.60

If a situation like a raffle has priced a ticket to be equal to the expected value, then the situation is called **fair**.

Fair: price to participate = expected value of return.

Favorable: price to participate < expected value of return.

Unfavorable: price to participate > expected value of return.

Expected Value for Bernoulli Trials

In a Bernoulli process with n trials, because the probability of success is the same for all trials, the expected number of successes is:

$$\mathbf{E} = np$$

Where p is the probability of success on any single trial

1) 2 teams played each other 14 times. Team A won 9 games, and team B won 5 games. They will play again next week. Bob offers to bet $6 on team A while you bet $4 on team B. The winner gets the $10. Is the bet fair to you in view of the past records of the two teams? Explain.

$P(A\text{win}) = \frac{9}{14}$ Bob: $\$4(\frac{9}{14}) - \$6(\frac{5}{14}) = \$0.43$

$P(B\text{win}) = \frac{5}{14}$ You: $-\$4(\frac{9}{14}) + \$6(\frac{5}{14}) = -\$0.43$

No, Bob's expected value is positive. Favorable to Bob, unfavorable to you.

2) You pay $1 to toss 2 coins. If you toss 2 heads, you get $2 (including your $1); if you toss only 1 head, you get back your $1; and if you toss no heads, you lose your $1. Is this a fair game to play?

$P(2H) = \frac{1}{4}$, $E = -1 + 2(\frac{1}{4}) + 1(\frac{2}{4}) + 0(\frac{1}{4})$

$P(1H) = \frac{2}{4}$ $E = 0$ yes, fair game

$P(0H) = \frac{1}{4}$

3) David gets $10 if he throws a double on a single throw of a pair of dice. How much should he pay for a throw?

1,1 $P(\text{double}) = \frac{6}{36}$ $E = 10(\frac{6}{36}) = \$1.67$

2,2

3,3

4,4

5,5

6,6

4) A coin is weighted so that $P(H) = \frac{1}{4}$ and $P(T) = \frac{3}{4}$. Find the expected number of tosses of the coin required in order to obtain either a head or 4 tails.

$E = 1(\frac{1}{4}) + 2(\frac{3}{4})(\frac{1}{4}) + 3(\frac{3}{4})^2(\frac{1}{4}) + 4(\frac{3}{4})^3(\frac{1}{4}) + 4(\frac{3}{4})^4$

P(ways to succeed: {H, TH, TTH, TTTH, TTTT}

Prob: $\frac{1}{4}$, $\frac{3}{4}\frac{1}{4}$, $(\frac{3}{4})^2\frac{1}{4}$, $(\frac{3}{4})^3(\frac{1}{4})$, $(\frac{3}{4})^4$

num of tosses: 1 2 3 4 4

$E = 2.73$

5) Assume that the odds for a certain race horse to win are 7 to 5. If a better wins $5 when the horse wins, how much should he bet to make the game fair?

X (winnings)	\$0	\$5
P	$\frac{5}{12}$	$\frac{7}{12}$

$P(\text{win}) = \frac{7}{12}$

$P(\text{loss}) = \frac{5}{12}$ $EV = (\$0)\frac{5}{12} + (\$5)(\frac{7}{12}) = \$2.92$

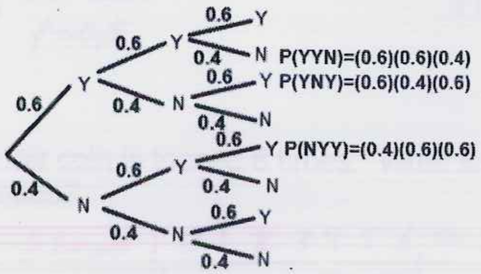
6) Find the number of times the face 5 is expected to occur in a sequence of 2000 throws of a fair die.

$E = pn$

$E = \frac{1}{6}(2000) = 333.3$ times

8.6 – Bernoulli Trials, Binomial probability model

Students at a particular university voted 60% of the time in the most recent election. If 3 students are picked at random, what is the probability that exactly 2 of them voted in that election?



probability of yes — probability of no

$$P(exactly\ 2\ Y) = 3(0.6)^2(0.4)^1$$

$${}_3C_2 = 3$$

#ways to choose which 2 of the 3 students were 'yes'

For us to use this 'pattern' these things must be true:

- Only 2 outcomes ('yes'/'no', 'success'/'fail')
- Probabilities have to be constant for each 'trial'
- Probabilities of trials must be independent
- Must have a fixed number of trials (here the 3 students)

In general:

$$P(exactly\ k\ Yes\ out\ of\ n\ students) = {}_nC_k\left[P(Y)\right]^k\left[P(N)\right]^{n-k}$$

The first 3 items on this list define what is called a ***Bernoulli Trial***:

- Only 2 outcomes ('yes'/'no', 'success'/'fail')
- Probabilities have to be constant for each trial
- The trials must be independent of each other

The last item defines this further what is called a ***Binomial Setting***:

- Must have a fixed number of trials (here, the 3 students)

The reason this is called a ***Binomial Setting*** is because the format for the pattern looks like a term from a Binomial expansion:

$$\textbf{P(exactly 2 voted)} = {}_3C_2\,(0.6)^2(0.4)^1$$

$$(a+b)^3 = {}_3C_0(a)^3(b)^0 + {}_3C_1(a)^2(b)^1 + {}_3C_2(a)^1(b)^2 + {}_3C_3(a)^0(b)^3$$

Binomial Probability Model / Calculator Function

$$binompdf(n,p,k) = {}_nC_k\,p^k q^{n-k}$$

$n = \#\ of\ trials$

$k = \#\ of\ successes$

$p = probability\ of\ success$

$q = probability\ of\ failure\ (q = 1-p)$

Note: the $binomcdf(n,p,k)$ function is different. It computes a sum of probabilities from 0 to k.

Students at a particular university voted 60% of the time in the most recent election. If 3 students are picked at random, what is the probability that exactly 2 of them voted in that election?

Does it meet the criteria for a Binomial model?

- Only 2 outcomes? ✓ (Y=voted, N=didn't vote)
- Probabilities constant? ✓ (p=.60)
- Trials independent? ✓ (likely, if students selected randomly)
- Fixed number of trials? ✓ (3 students asked)

#voted	0	1	2	3

$P(ex2) = binompdf(3, 0.6, 2) = \boxed{0.432}$

or by hand:

$P(ex2) = {}_3C_2(.6)^2(.4)^1 = 0.432$

A fair coin is tossed 8 times. What is the probability of obtaining **exactly** 6 heads?

X = #Heads | 0 1 2 3 4 5 6 7 8

yes = Heads
p = 0,5

$P(X=6) = \text{binompdf}(8, 0.5, 6) = \boxed{0.1094}$

A fair coin is tossed 8 times. What is the probability of obtaining **up to 6** heads?

#Heads | 0 1 2 3 4 5 6, 7 8

binompdf(8, .5, 0) binompdf(8, .5, 1) binompdf(8, .5, 2) ...... binompdf(8, .5, 6)

'OR' so add these together

binomcdf (n, p, k)
= cumulative total of binompdf from 0 to k (including k)

X = # Heads | 0 1 2 3 4 5 6, 7 8

$P(X \le 6) = \text{binomcdf}(8, 0.5, 6) = \boxed{0.9648}$

A fair coin is tossed 8 times. What is the probability of obtaining **at least** 6 heads?

X = #Heads | 0 1 2 3 4 5 6 7 8

binomcdf(8, 0.5, 5)

$P(X \ge 6) = 1 - P(X < 6)$
$= 1 - \text{binomcdf}(8, 0.5, 5)$
$= 1 - 0.8555$
$= \boxed{0.1445}$

What is the probability that in a family of 7 children:

(a) 4 are girls?

Y = girl
p = 0,5

X = #Girls | 0 1 2 3 4 5 6 7

$P(X=4) = \text{binompdf}(7, 0.5, 4) = \boxed{0.2734}$

(b) At least 2 are girls?

X = #Girls | 0 1, 2 3 4 5 6 7

$P(X \ge 2) = 1 - \text{binomcdf}(7, 0.5, 1)$
$= 1 - .0625$
$= \boxed{.9375}$

(c) At least 2 and not more than 4 are girls?

X = #Girls | 0 1 2 3 4, 5 6 7

$P(X = 2 \text{ or } 3 \text{ or } 4) = \text{binompdf}(7, 0.5, 2) + \text{binompdf}(7, 0.5, 3) + \text{binompdf}(7, 0.5, 4)$
$= \boxed{0.709}$

A television manufacturer tests a random sample of 15 picture tubes to determine whether any are defective. The probability that a picture tube is defective has been found from past experience to be .03.

X = #defects | 0 1 2 3 4 15

(a) What is the probability that there are no defective tubes in the sample? success = "defect" p = .03

P(X=0) = binompdf(15, .03, 0)
= $\boxed{.6332}$

(b) What is the probability that more than 2 of the tubes are defective?

X = #defects | 0 1 2 3 4 5 ...... 13 14 15

P(X≥3) = 1 − P(X≤2)
= 1 − binomcdf(15, .03, 2)
= 1 − .9906
= $\boxed{.0094}$

43. A supposed coffee connoisseur claims she can distinguish between a cup of instant coffee and a cup of drip coffee 80% of the time. You give her 6 cups of coffee and tell her that you will grant her claim if she correctly identifies at least 5 of the 6 cups.

(a) What are her chances of having her claim granted if she is in fact guessing? success = id cup correctly
p = 0.5 (guessing)

X = #cups correct | 0 1 2 3 4 (reject) 5 6 (grant)

P(X≥5) = 1 − binomcdf(6, .5, 4)
= 1 − .8906
= $\boxed{.1094}$

(b) What are her chances of having her claim rejected when in fact she really does have the ability she claims?

p = 0.8 (really an expert)

X = #cups correct | 0 1 2 3 4 5 6

P(X≤4) = binomcdf(6, .8, 4)
= $\boxed{.3446}$

A fair coin is tossed 8 times:
What is the probability of obtaining exactly 3 heads if it is known that at least 1 head appeared.

P(ex 3H | at least 1H) (conditional probability)
P(A∩B) = P(A)·P(B|A)
P(B|A) = P(A∩B) / P(A)
P(ex 3H | at least 1H) = P(ex 3H ∩ at least 1H) / P(at least 1H)

success = Head
p = .5

X = #Heads | 0 1 2 3 4 5 6 7 8

P(at least 1H) = 1 − binomcdf(8, .5, 0)
= 1 − .0039
= .9961

X = Heads | 0 1 2 3 4 5 6 7 8
intersection

P(ex 3H ∩ at least 1H) = binompdf(8, .5, 3)
= .21875

= .21875 / .9961
= $\boxed{.2196}$