Unit 3, (Ch 4)

Honors Finite Mathematics – Lesson Notes: ~~Chapter 3~~

4.1 ~~3.1~~ – *Linear Inequalities*

Linear Programming:
Used when minimizing and maximizing a linear expression within a set of linear inequality restrictions.

We will study linear programing in two variables in this chapter.

Ex. Determine whether $P_1=(7,5)$, and $P_2=(9,12)$, and $P_3(3,1)$ are solutions to the following system without graphing.

$$\begin{cases} 10x - y \geq 0 \\ -x + 2y \geq 0 \\ x + y \leq 15 \end{cases}$$

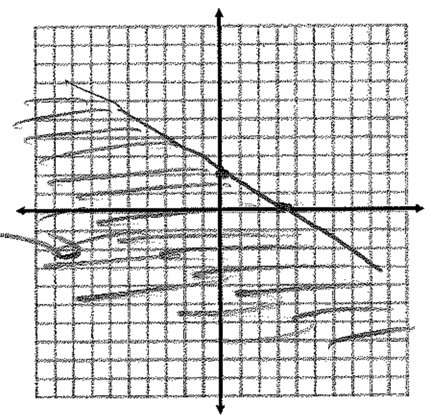
(7,5)	(9,12)	(3,1)
$10(7)-(5)\geq 0$ yes	$10(9)-12\geq 0$ yes	$10(3)-(1)\geq 0$ yes
$-(7)+2(5)\geq 0$ yes	$-(9)+2(12)\geq 0$ yes	$-(3)+2(1)\geq 0$ no
$(7)+(5)\leq 15$ no	$(9)+(12)\leq 15$ no	
$\boxed{\text{NO}}$	$\boxed{\text{NO}}$	$\boxed{\text{No}}$

Review: Graph $2x + 3y \leq 6$

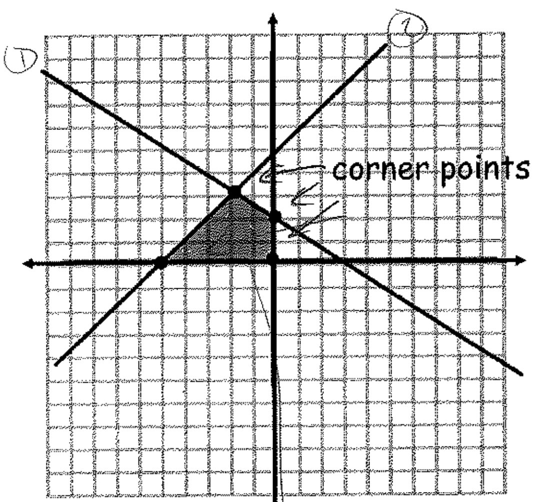
test (0,0) $0 \leq 6$ true

'half plane'

1 inequality: solution set is all points in the half plane



1 2
$$\begin{cases} 2x + 3y \leq 6 \\ x - y \leq -5 \\ x \leq 0 \\ y \geq 0 \end{cases}$$



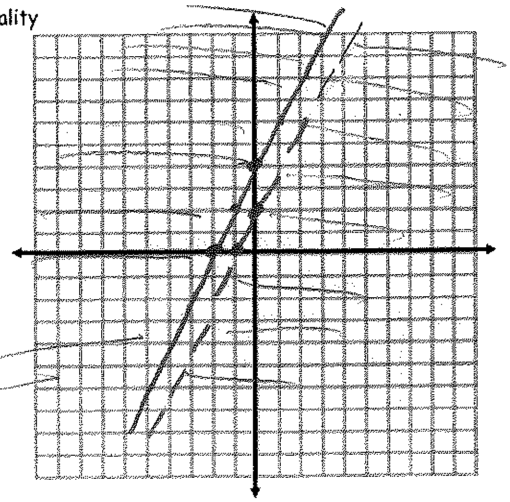
feasible region, solution

System of inequalities: solution set is all points common to all half planes (overlapped region).

$$\begin{cases} 2x - y \le -4 \\ 2x - y > -2 \end{cases}$$

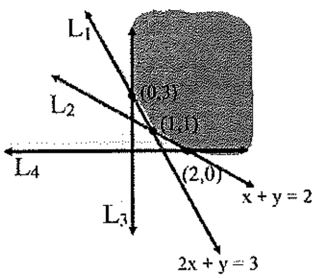
"nonstrict" inequality

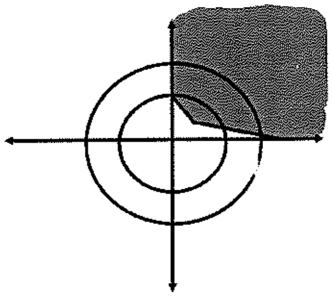
"strict" inequality

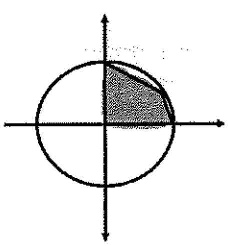


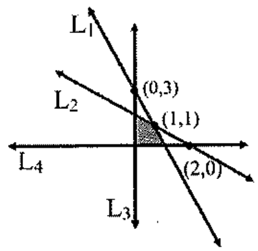
no solution

Definition: Unbounded regions - extend infinitely for in some direction (cannot be enclosed by a circle).









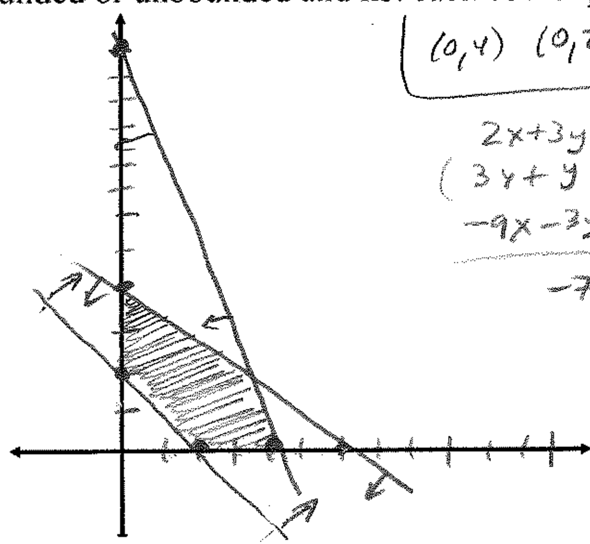
Definition: Bounded regions - can be enclosed by some circle.

~~students~~

Ex. Graph the system of linear inequalities.
Tell whether the graph is bounded or unbounded and list each corner point.

$$\begin{cases} x + y \ge 2 \\ 2x + 3y \le 12 \\ 3x + y \le 12 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

$\boxed{(0,4)\ (0,2)\ (2,0)\ (4,0)\ \left(\frac{24}{7}, \frac{12}{7}\right)}$



$2x + 3y = 12$

$(3x + y = 12) \cdot -3$

$-9x - 3y = -36$

$-7x = -24$

$x = \frac{24}{7}$

$2\left(\frac{24}{7}\right) + 3y = 12$

$48 + 21y = 84$

$21y = 36$

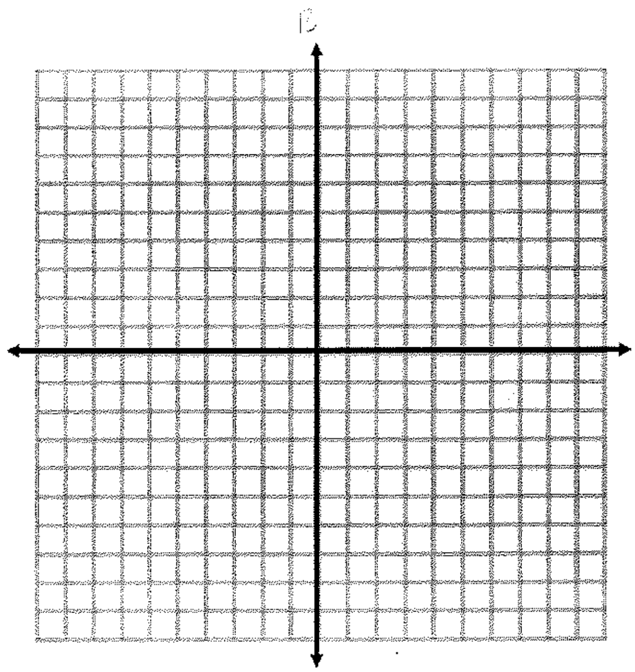
$y = \frac{36}{21} = \frac{12}{7}$

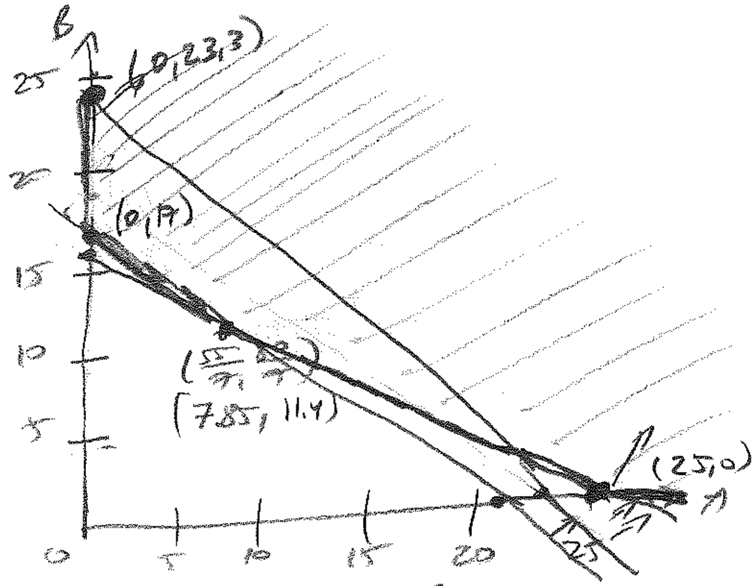
Ex. Nutrition. To maintain an adequate daily diet, nutritionists recommend the following: at least 85g of carbohydrate, 70g of fat, and 50g of protein. An ounce of food A contains 5 g of carbohydrate, 3 g of fat, and 2g of protein, while an ounce of food B contains 4g of carbohydrate, 3g of fat, and 3g of protein.

a) Write a system of linear inequalities that describes the possible quantities of each food.

b) Graph the system and list the corner points.

	A B	
C	$5A+4B$	≥ 85
F	$3A+3B$	≥ 70
P	$2A+3B$	≥ 50





$(0,17)$ $(0, 23.3)$ $(25,0)$ $(\frac{55}{7}, \frac{80}{7})$

$-5(2A+3B=50)$

$2(5A+4B=85)$

$-10A-15B=-250$

$10A+8B=170$

$-7B=-80$

$B=\frac{80}{7}$

$2A+3\cdot\frac{80}{7}=50$

$14A+240=350$

$14A=110$

$A=\frac{110}{14}=\frac{55}{7}$

~~3.2~~ 4.2 – Optimizing an Objective Function

Mike's Famous Toy Trucks manufactures two kinds of toy trucks - a standard model (x) and a deluxe model (y). In the manufacturing process, each standard model requires 2 hours of grinding and 2 hours of finishing, and each deluxe model needs 2 hours of grinding and 4 hours of finishing. The company has two grinders and three finishers, each of whom works at most 40 hours per week.

x y

grind, finish
$$\begin{cases} 2x + 2y \le 80 \\ 2x + 4y \le 120 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

Create a system of inequalities representing this scenario, graph, and find corner points.

$$\begin{array}{r} -(2x+2y=80) \\ 2x+4y=120 \\ \hline 2y=40 \end{array}$$

$y=20$

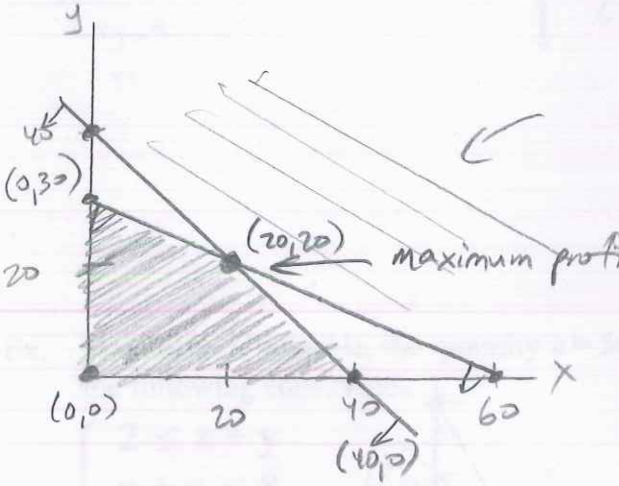
$2x+2(20)=80$

$2x=40$

$x=20$

Each standard toy truck brings a profit of \$3 and each deluxe toy truck brings a profit of \$4. Assuming that every truck made will be sold, how many of each should be made to maximize profit?

$p=3x+4y$



20 of each for maximum profit

max profit $= 3(20)+4(20)$

$60+80$

\$140

the objective: maximize profit

p=3x+4y   objective function

<u>A linear programming problem generally has 2 components</u>:

- An objective function to be maximized or minimized.
- A collection of inequalities representing conditions or constraints.

<u>Terms</u>:

- **Objective function**: the linear expression to by optimized.
- **Feasible point**: any point which obeys all constraints (lies in the solution region of the system of inequalities).
- **Solution**: a feasible point which maximized (or minimizes) the objective function.

<u>Fundamental Theorem of Linear Programming</u>:

If a linear programming problem has a solution, it is located at a corner point of the set of feasible points. If there are multiple solutions, at least one of them is located at a corner point of the set of feasible points. In either case, the corresponding value of the objective function is unique.

<u>Solving a Linear Programming Problem</u>:

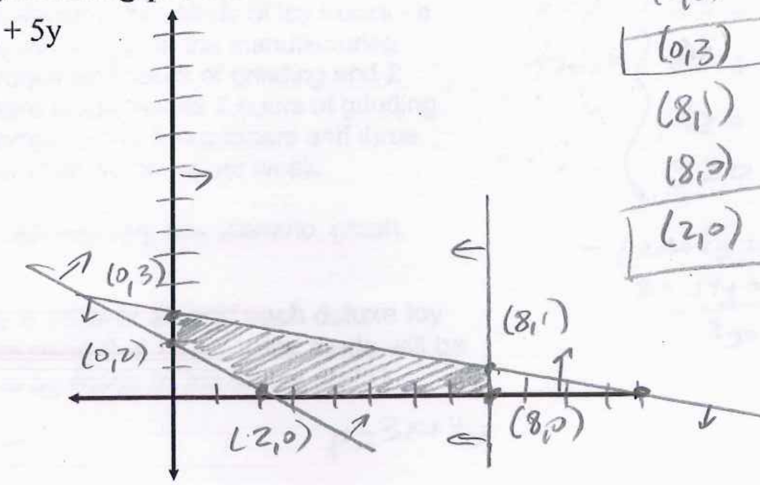
1) Write an expression for the objective function.
2) Determine all constraints and graph the set of feasible points.
3) List the corner points of the set of feasible points.
4) Determine the value of the objective function at each corner point.
5) Select the optimal solution.

Ex. Solve the linear programming problem

Objective function: $z = x + 5y$

Constraints:

$$\begin{cases} x + 4y \le 12 \\ x \le 8 \\ x + y \ge 2 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



$x+4y=12$

$(8)+4y=12$

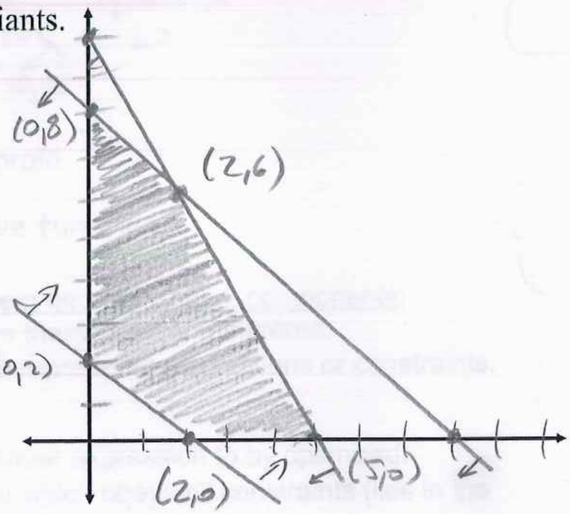
$4y=4$

$y=1$

corner	z	
(0,2)	10	
(0,3)	15	max
(8,1)	13	
(8,0)	8	
(2,0)	2	min

Ex. Maximize, if possible, the quantity $z = 5x + 7y$ subject to the following constriants.

$$\begin{cases} 2 \le x + y \\ x + y \le 8 \\ 2x + y \le 10 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



corner	z	
(0,2)	14	
(0,8)	56	max
(2,6)	52	
(5,0)	25	
(2,0)	10	min

$2x+y=10$

$x+y=8$

$y=8-x$

$2x+(8-x)=10$

$2x+8-x=10$

$x+8=10$

$x=2$

$y=6$

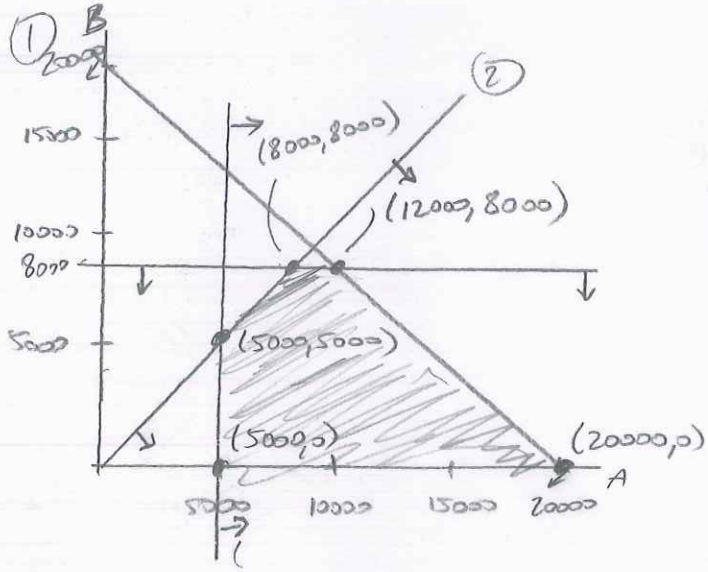
~~3.3~~ 4.3 – Applications of Linear Programming

Investment Strategy. An investment broker wants to invest up to \$20,000. She can purchase a type A bond yielding a 10% return on the amount invested and she can purchase a type B bond yielding a 15% return on the amount invested. She also wants to invest at least as much in the type A bond as the type B bond. She will also invest at least \$5000 in the type A bond and no more than \$8000 in the type B bond. How much should she invest in each type of bond to maximize her return?

$A+B=20000$
$B=8000$
$A=12000$

$$\begin{cases} ① \; A+B \le 20000 \\ ② \; A \ge B \\ A \ge 5000 \\ B \le 8000 \\ A \ge 0 \\ B \ge 0 \end{cases}$$

return $= 0.1A + 0.15B$
(maximize)



Corner	return
(5000,0)	500
(5000,5000)	1250
(8000,8000)	2000
$\boxed{(12000,8000)}$	$\boxed{2400}$
(20000,0)	2000

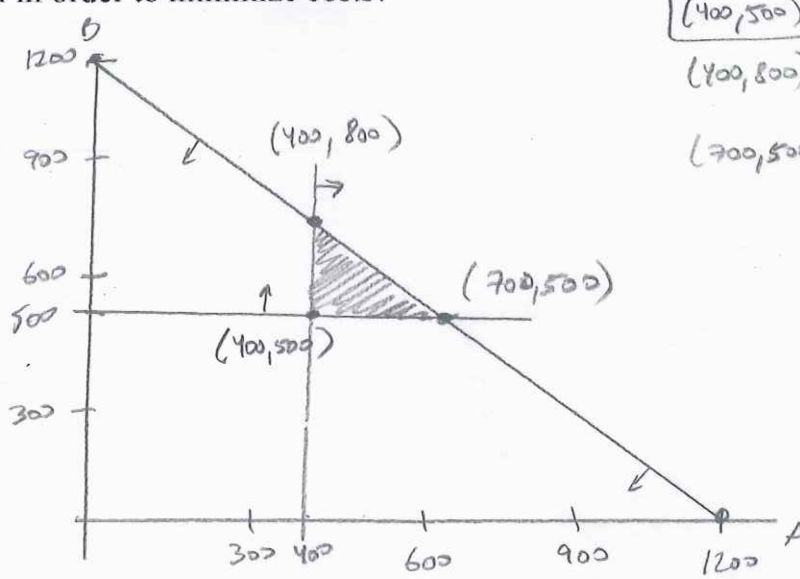
Transportation: An appliance company has a warehouse and two terminals. To minimize shipping costs, the manager must decide how many appliances should be shipped to each terminal. There is a total supply of 1200 units in the warehouse and a demand for 400 units in terminal A and 500 units in terminal B. It costs \$12 to ship each unit to terminal A and \$16 to ship to terminal B. How many units should be shipped to each terminal in order to minimize costs?

$$\begin{cases} A+B \le 1200 \\ A \ge 400 \\ B \ge 500 \end{cases}$$

cost $= 12A + 16B$

$(400)+B=1200$
$B=800$

$A+(500)=1200$
$A=700$



Corner	Cost
$\boxed{(400,500)}$	$\boxed{12800}$
(400,800)	17600
(700,500)	16400