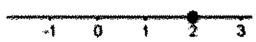
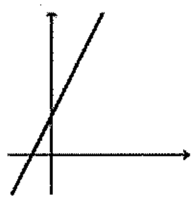
Honors Finite Mathematics – Lesson Notes: Chapter 2

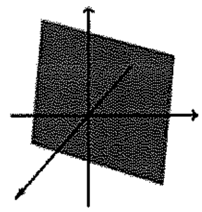
2.1 – Systems of Equations: Substitution, Elimination, Gaussian Elimination

Multivariable Linear Systems:

1-dimension	2-dimensions	3-dimensions
x=2	4x - 2y = -2	4x - 2y + 3z = 5

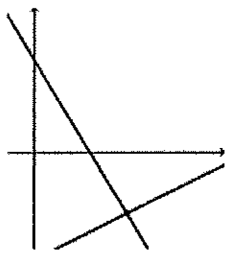




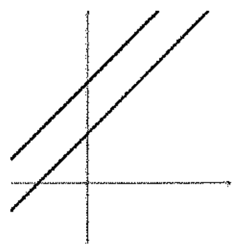


2-D systems of equations:

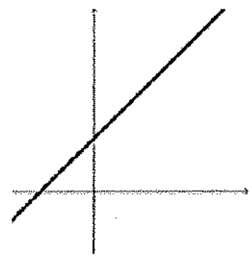
$$\begin{cases} 5x+3y=9 \\ 2x-4y=14 \end{cases}$$



consistent
solution: (5, -2)
equations independent



inconsistent
no solution



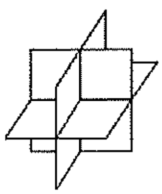
consistent
solution: (x, 2x+3)
equations dependent

consistent: system has at least one solution

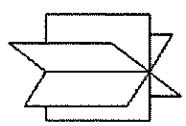
inconsistent: system has no solutions

3-D systems of equations:

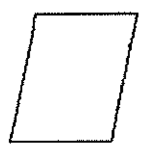
$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad\quad =-4 \\ 2x-5y+5z=17 \end{cases}$$



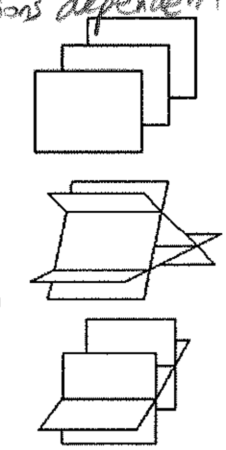
solution: (1, -1, 2)



solution:
(5z+1, 3z-3, z)

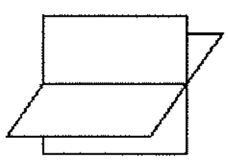


solution:
(3t+2, t-4, 5t+8)



No solution

$$\begin{cases} x-2y+3z=9 \\ -x+3y+z=10 \end{cases}$$



solution:
(5z+1, 3z-3, z)

consistent

inconsistent

Substitution Method:

Solve the system: $\begin{cases} 3x=24 \\ x+2y=0 \end{cases}$.

$\frac{3x}{3}=\frac{24}{3}$

$x=8$

$x+2y=0$

$(8)+2y=0$

$2y=-8$

$y=-4$

$\boxed{(8,-4)}$

Elimination Method:

Solve the system: $\begin{cases} 2x+3y=-4 \\ 3x-4y=11 \end{cases}$) 3) −2

$6x+9y=-12$
$-6x+8y=-22$
$17y=-34$
$y=-2$

$2x+3(-2)=-4$
$2x-6=-4$
$2x=2$
$x=1$

$\boxed{(1,-2)}$

Gaussian Elimination (w/Equations)Method: (row operations)

Solve the system: $\begin{cases} x-y+z=-4 \\ 2x-3y+4z=-15 \\ 5x+y-2z=12 \end{cases}$.

$\begin{cases} x-y+z=-4 \\ -y+2z=-7 \\ 6y-7z=32 \end{cases}$ $-2R_1+R_2$, $-5R_1+R_3$

$\begin{cases} x-y+z=-4 \\ -y+2z=-7 \\ 5z=-10 \end{cases}$ $6R_2+R_3$

$5z=-10$
$z=-2$
Back substitute:
$-y+2(-2)=-7$
$-y=-3$
$y=3$
$x-(3)+(-2)=-4$
$x=1$

$\boxed{(1,3,-2)}$

Solve the system: $\begin{cases} x-y-z=1 \\ -x+2y-3z=-4 \\ 3x-2y-7z=0 \end{cases}$.

$\begin{cases} x-y-z=1 \\ y-4z=-3 \\ y-4z=-3 \end{cases}$ R_1+R_2, $-3R_1+R_3$

$\begin{cases} x-y-z=1 \\ y-4z=-3 \\ 0=0 \end{cases}$ $-R_2+R_3$

consistent
(only 2 equations)
infinitely many solutions; use z as parameter

$y-4z=-3$
$y=4z-3$

$x-(4z-3)-z=1$
$x-4z+3-z=1$
$x-5z=-2$
$x=5z-2$

$\boxed{(5z-2,4z-3,z)}$

example points:
$(-2,-3,0)$
$(3,1,1)$
etc.

Solve the system: $\begin{cases} 2x-3y-z=0 \\ -x+2y+z=5 \\ 3x-4y-z=1 \end{cases}$.

$\begin{cases} -x+2y+z=5 \\ 2x-3y-z=0 \\ 3x-4y-z=1 \end{cases}$ $R_2 \leftrightarrow R_1$

$\begin{cases} -x+2y+z=5 \\ y+z=10 \\ 2y+2z=16 \end{cases}$ $2R_1+R_2$, $3R_1+R_3$

$\begin{cases} -x+2y+z=5 \\ y+z=10 \\ 0=-4 \end{cases}$ $-2R_2+R_3$

inconsistent

$\boxed{\text{No solution}}$

Diet Preparation A farmer prepares feed for livestock by combining two grains. Each unit of the first grain contains 2 units of protein and 5 units of iron, while each unit of the second grain contains 4 units of protein and 1 unit of iron. Determine the number of units of each kind of grain the farmer needs to feed each animal daily if each animal must have 10 units of protein and 16 units of iron each day.

x: 2 protein, 5 iron
y: 4 protein, 1 iron

protein: $2x+4y=10$
iron: $5x+y=16$) −4

$2x+4y=10$
$-20x-4y=-64$
$-18x=-54$
$x=3$

$2(3)+4y=10$
$6+4y=10$
$4y=4$
$y=1$

3 unit of X grain
1 unit of Y grain

2.2 – Systems of Equations: Augmented Matrix, Row Operations, Gauss-Jordan

Augmented Matrix Representation of a System

$$\begin{cases} x+2y=0 \\ 3x-4y=-10 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & -4 & -10 \end{array}\right]$$

$$\begin{cases} x-2y+z=7 \\ 2x-3y+2z=12 \\ 3x+y+z=3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 2 & -3 & 2 & 12 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

Row operations:

- Interchange any two rows
- Replace any row by a nonzero constant multiple of that row.
- Replace any row by the sum of that row and a constant multiple of another row.

Gaussian Elimination

$$\begin{array}{cc} x & y \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & -4 & -10 \end{array}\right]$$

$$-3R_1+R_2 \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -10 & -10 \end{array}\right]$$

$-10y=-10$ $\quad$ $x+2y=0$

$y=1$ $\quad$ $x+2(1)=0$

$x=-2$

$\boxed{(-2,1)}$

Calculator: ref

$$\left[\begin{array}{cc|c} 1 & -\frac{4}{3} & -\frac{10}{3} \\ 0 & 1 & 1 \end{array}\right]$$

$y=1$ $\quad$ $x-\frac{4}{3}y=\frac{-10}{3}$

$x-\frac{4}{3}=\frac{-10}{3}$

$x=\frac{4}{3}-\frac{10}{3}=\frac{-6}{3}=-2$

Gauss-Jordan Elimination

- work a column at a time
- get a 1 on the diagonal
- use it to cancel the rest of the column

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 2 & -3 & 2 & 12 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 7 & -2 & -18 \end{array}\right]$$

$$\begin{array}{r} 2R_2+R_1 \\ \\ -7R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{2}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Calculator rref

$$-R_3+R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x=1$

$y=-2$

$z=2$

$\boxed{(1,-2,2)}$

Three Possible Outcomes of Gauss-Jordan Elimination

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right] \qquad \left[\begin{array}{cc|c} a & b & c \\ 0 & 0 & non-zero \end{array}\right] \qquad \left[\begin{array}{cc|c} a & b & c \\ 0 & 0 & 0 \end{array}\right]$$

consistent
one solution
(point of intersection)

inconsistent
no solution
(no intersection)

consistent
many solutions
(a line of points)

$$\begin{cases} 2x+2y+z=6 \\ x-y-z=-2 \\ x-2y-2z=-5 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

consistent
(1, 1, 2)

$$\begin{cases} 3x-y+2z+3 \\ 3x+3y+z=3 \\ 3x-5y+3z=12 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{12} & 0 \\ 0 & 1 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

inconsistent
no solution

Applications

Cost of Fast Food One group of people purchased 10 hotdogs and 5 soft drinks at a cost of $12.50. A second group bought 7 hot dogs and 4 soft drinks at a cost of $9.00. What is the cost of a single hot dog? A single soft drink?

h s

group 1: $10h + 5s = 12.50$
group 2: $7h + 4s = 9.00$

$$\left[\begin{array}{cc|c} 10 & 5 & 12.50 \\ 7 & 4 & 9.00 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & .5 \end{array}\right]$$

$\boxed{\text{hot dog} = \$1.00 \quad \text{soft drink} = \$0.50}$

General Applications Problem Procedure

1. Use variables to represent what is being asked and put these at the top of the matrix horizontally.
2. Put the other information vertically at the left side of the matrix.
3. Fill in the information.
4. Solve the matrix.
5. Answer the question.

Mixture Suppose that a store has three sizes of cans of nuts. The *large* size contains 2 pounds of peanuts and 1 pound of cashews. The *mammoth* size contains 1 pound of walnuts, 6 pounds of peanuts, and 2 pounds of cashews. The *giant* size contains 1 pound of walnuts, 4 pounds of peanuts, and 2 pounds of cashews. Suppose that the store receives an order for 5 pounds of walnuts, 26 pounds of peanuts, and 12 pounds of cashews. How can it fill this order with the given sizes of cans?

$$\begin{array}{l} \\ \text{walnuts} \\ \text{peanuts} \\ \text{cashews} \end{array} \begin{array}{c} \begin{array}{ccc} L & M & G \end{array} \\ \left[\begin{array}{ccc|c} 0 & 1 & 1 & 5 \\ 2 & 6 & 4 & 26 \\ 1 & 2 & 2 & 12 \end{array}\right] \end{array}$$

(rref)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad \boxed{\begin{array}{l} L = 2 \\ M = 1 \\ G = 4 \end{array}}$$

100 pounds

Mixture A store sells almonds for \$6 per pound, cashews for \$5 per pound, and peanuts for \$2 per pound. One week the manager decides to prepare 100 16-ounce packages of nuts by mixing 40 pounds of peanuts with some almonds and cashews. Each package will be sold for \$4. How many pounds of almonds and cashews should be mixed with the peanuts so that the mixture will produce the same revenue as selling the nuts separately?

$$\begin{array}{l} \\ \text{pounds} \\ \text{cost} \end{array} \begin{array}{c} \begin{array}{cc} a & c \end{array} \\ \left[\begin{array}{cc|c} 1 & 1 & 60 \\ 6 & 5 & 320 \end{array}\right] \end{array}$$

(rref)

$$\left[\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 40 \end{array}\right]$$

$\boxed{\begin{array}{l} \text{20 lbs Almonds} \\ \text{40 lbs Cashews} \end{array}}$

pounds

$a + c + 40 = 100$

$a + c = 60$

Cost

100 packages @ \$4 each = \$400 revenue

Sold separately:

$6a + 5c + 2(40) = 400$

$6a + 5c + 80 = 400$

$6a + 5c = 320$

$$\begin{cases} a + c = 60 \\ 6a + 5c = 320 \end{cases}$$

2.3 – n x m systems, Matrix addition and substraction

What are the equations in these systems? What is the solution?

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -4 \end{array}\right] \end{array}$$

$x = 3$

$y = 8$

$z = -4$

$\boxed{(3, 8, -4)}$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x + 3z = 0$

$y + 2z = 0$

$0 = 1$

$0 = 0$

contradiction

inconsistent

$\boxed{\text{no solution}}$

(no way to select x, y, z that would make $0 = 1$)

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right] \end{array}$$

$x_1 + 2x_4 = 5$

$x_2 + x_4 = 2$

$x_3 + 3x_4 = 4$

too few equations, x_1, x_2, x_3 can be written in terms of x_4:

$x_1 = -2x_4 + 5$

$x_2 = -x_4 + 2$

$x_3 = -3x_4 + 4$

many solutions (points on 4D line)

general:

$(-2x_4 + 5, -x_4 + 2, -3x_4 + 4, x_4)$

some specific solutions

x_1	x_2	x_3	x_4
25	12	34	-10
17	8	22	-6
9	4	10	-2
5	2	4	0
-11	-6	-20	8

#54. **Mixing Chemicals** A chemistry laboratory has available three kinds of hydrochloric acid (HCl): 10%, 30%, and 50% solutions. How many liters of each should be mixed to obtain 100 liters of **40%** HC1? Provide a table showing at least six of the possible solutions.

liters → $x \quad y \quad z$

total qty: $x + y + z = 100$

amt of acid: $.1x + .3y + .5z = .4(100)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ .1 & .3 & .5 & 40 \end{array}\right]$$

rref

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -50 \\ 0 & 1 & 2 & 150 \end{array}\right]$$

$x - z = -50$

$y + 2z = 150$

$x = z - 50$

$y = -2z + 150$

(50%) z	(10%) $x = z - 50$	(30%) $y = -2z + 150$
50	0	50
60	10	30
70	20	10
75	25	0

#55. **Cost of Fast Food** One group of customers bought 8 deluxe hamburgers, 6 orders of large fries, and 6 large sodas for \$26.10. A second group ordered 10 deluxe hamburgers, 6 large fries, and 8 large sodas and paid \$31.60. Is there sufficient information to determine the price of each food item? If not, construct a table showing the various possibilities. Assume the hamburgers cost between \$1.75 and \$2.25, the fries between \$0.75 and \$1.00, and the sodas between \$0.60 and \$0.90.

$h \quad f \quad s$

group1: $8h + 6f + 6s = 26.10$

group2: $10h + 6f + 8s = 31.60$

$$\left[\begin{array}{ccc|c} 8 & 6 & 6 & 26.10 \\ 10 & 6 & 8 & 31.60 \end{array}\right]$$

rref

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 11/4 \\ 0 & 1 & -1/3 & 41/60 \end{array}\right]$$

$h + s = 11/4$

$f - \frac{1}{3}s = 41/60$

$h = -s + 11/4$

$f = \frac{1}{3}s + 41/60$

Soda s	hamb. $-s + 11/4$	fries $\frac{1}{3}s + 41/60$
0.60	2.15	0.883
0.65	2.10	0.90
0.70	2.05	0.917
0.75	2.00	0.933
0.80	1.95	0.95
0.85	1.90	0.967
0.90	1.85	0.983

Ex: A retired couple has \$30,000 available to invest. They require a return on their investment of \$2500 per year. As their financial consultant, you recommend they invest some money in Treasury bills that yield 7%, some money in corporate bonds that yield 9%, and some junk bonds that yield 11%. Prepare a table that shows various ways this couple can achieve their goal.

$t \quad c \quad j$

total: $t + c + j = 30000$

interest: $.07t + .09c + .11j = 2500$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 30000 \\ .07 & .09 & .11 & 2500 \end{array}\right]$$

rref

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 10000 \\ 0 & 1 & 2 & 20000 \end{array}\right]$$

$t - j = 10000$

$c + 2j = 20000$

$t = j + 10000$

$c = -2j + 20000$

junk bonds j	t-bills $j + 10000$	corp bonds $-2j + 20000$
0	10000	20000
2000	12000	16000
5000	15000	10000
8000	18000	4000
10000	20000	0

2.4 –Matrix addition and substraction

Matrix Terms

Dimensions: number of rows (m) and number of columns (n) m x n.
Square matrix: same number of rows and columns.
Equality of matrices: when each corresponding set of entries is the same.
Addition of matrices: add each corresponding set of entries.
Zero matrix: all entries are zero.
Scalar Multiplication: multiply each entry by the scalar.

42. If $A=\begin{bmatrix}2 & -3 & 4\\ 0 & 2 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & -2 & 0\\ 5 & 1 & 2\end{bmatrix}$ $C=\begin{bmatrix}-3 & 0 & 5\\ 2 & 1 & 3\end{bmatrix}$ perform the indicated operation:

$2A-5(B+C)$

$$2\begin{bmatrix}2 & -3 & 4\\ 0 & 2 & 1\end{bmatrix}-5\left(\begin{bmatrix}1 & -2 & 0\\ 5 & 1 & 2\end{bmatrix}+\begin{bmatrix}-3 & 0 & 5\\ 2 & 1 & 3\end{bmatrix}\right)$$

$$\begin{bmatrix}4 & -6 & 8\\ 0 & 4 & 2\end{bmatrix}-5\begin{bmatrix}-2 & -2 & 5\\ 7 & 2 & 5\end{bmatrix}$$

$$\begin{bmatrix}4 & -6 & 8\\ 0 & 4 & 2\end{bmatrix}-\begin{bmatrix}-10 & -10 & 25\\ 35 & 10 & 25\end{bmatrix}$$

$$\boxed{\begin{bmatrix}14 & 4 & -17\\ -35 & -6 & -23\end{bmatrix}}$$

54. Find x, y, and z so that: $\begin{bmatrix}x-2 & 3 & 2z\\ 6y & x & 2y\end{bmatrix}=\begin{bmatrix}y & z & 6\\ 18z & y+2 & 6z\end{bmatrix}$

$x-2=y$ $\quad$ $z=3$ $\quad$ $2z=6$ $\quad$ $\frac{6y}{6}=\frac{18(3)}{6}$

$6y=18z$ $\quad$ $x=y+2$ $\quad$ $2y=6z$ $\quad$ $\boxed{y=9}$

$x-2=9$

$x=11$

$$\boxed{\begin{matrix}x=11\\ y=9\\ z=3\end{matrix}}$$

58. Katy, Mike, and Danny go to the candy store. Katy buys 5 sticks of gum, 2 ice cream cones, and 20 jelly beans. Mike buys 2 sticks of gum, 15 jelly beans, and 3 candy bars. Danny buys 1 stick of gum, 1 ice cream cone, and 4 candy bars. Write a matrix depicting this situation.

$$\begin{array}{c|c} & \begin{matrix}\text{gum} & \text{ice cream} & \text{jelly beans} & \text{candy bars}\end{matrix}\\ \begin{matrix}\text{Katy}\\ \text{Mike}\\ \text{Danny}\end{matrix} & \begin{bmatrix}5 & 2 & 20 & 0\\ 2 & 0 & 15 & 3\\ 1 & 1 & 0 & 4\end{bmatrix}\end{array}$$

2.5/2.6 – Matrix multiplication, Matrix Inverses, Solving equations with inverses

- Multiplication of matrices is only possible is the number of columns of the first matrix is equal to the number of rows of the second matrix:

$$\underbrace{\begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 0 \\ 7 & -3 & 5 \\ -2 & 0 & 4 \end{bmatrix}}_{4\times 3}\underbrace{\begin{bmatrix} 4 & 8 \\ 3 & 0 \\ 7 & 1 \end{bmatrix}}_{3\times 2}$$

ok

size of result

- Given A and B are matrices, does AB=BA?
- Identity Matrix (square matrix, equivalent to multiplying by 1):

$$I_{2}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_{3}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ I_{4}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Identity Property: $AI_n = I_nA = A$

Multiplying 2 Matrices:

use calculator

$$\begin{bmatrix} \{2 & 4\} \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}=\begin{bmatrix} \{11\} & \\ & \end{bmatrix}\quad 2\cdot 1+4\cdot 5$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}=\begin{bmatrix} 11 & 8 \\ 15 & 3 \end{bmatrix}$$

Inverse of a Matrix:

- Only square matrices have inverses.
- Two matrices are inverses if $AB = BA = I_n$
- For 2x2 matrix, if $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1}=\dfrac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- Use calculator

2 Ways to Solve Systems of Equations with Matrices: $\begin{cases} 3x-2y=-1 \\ -2x+y=-1 \end{cases}$

Augmented matrix, rref

$$\left[\begin{array}{cc|c} 3 & -2 & -1 \\ -2 & 1 & -1 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 5 \end{array}\right]$$

$x=3$

$y=5$

Matrix equation, inverse

$$\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$A\ X = B$$

$$A^{-1}A\,X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ -1 \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$A=\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$ $B=\begin{bmatrix}1 & 2 & 3\\ -1 & 4 & -2\end{bmatrix}$ $C=\begin{bmatrix}3 & 1\\ 4 & -1\\ 0 & 2\end{bmatrix}$ $D=\begin{bmatrix}1 & 0 & 4\\ 0 & 1 & 2\\ 0 & -1 & 1\end{bmatrix}$ $E=\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}$

#1) $DC+C$

$$\begin{bmatrix}1 & 0 & 4\\ 0 & 1 & 2\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\ 4 & -1\\ 0 & 2\end{bmatrix}+\begin{bmatrix}3 & 1\\ 4 & -1\\ 0 & 2\end{bmatrix}$$

$3\times\underline{3 \cdot 3}\times 2$

$$\begin{bmatrix}3 & 9\\ 4 & 3\\ -4 & 3\end{bmatrix}+\begin{bmatrix}3 & 1\\ 4 & -1\\ 0 & 2\end{bmatrix}=\begin{bmatrix}6 & 10\\ 8 & 2\\ -4 & 5\end{bmatrix}$$

#2) $2EA-3BC$

$$2\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}-3\begin{bmatrix}1 & 2 & 3\\ -1 & 4 & -2\end{bmatrix}\begin{bmatrix}3 & 1\\ 4 & -1\\ 0 & 2\end{bmatrix}$$

$2\times\underline{3 \cdot 3}\times 2$

ok

$$\begin{bmatrix}-27 & -11\\ -31 & 59\end{bmatrix}$$

#3) Show that the given matrices are inverses of each other:

$$\underset{A}{\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 1 & 2 & 1\end{bmatrix}}\underset{B}{\begin{bmatrix}-\frac{5}{2} & 2 & -\frac{1}{2}\\ 1 & -1 & 1\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}}$$

$$AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

#4) Find the inverse:

$$\begin{bmatrix}3 & -2\\ 0 & 2\end{bmatrix}^{-1}=\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{2}\end{bmatrix}$$

#5) Solve using inverse matrices:

$$\begin{cases}x+y-z=3\\ 3x-y=-4\\ 2x-3y+4z=6\end{cases}$$

$$\begin{bmatrix}1 & 1 & -1\\ 3 & -1 & 0\\ 2 & -3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ -4\\ 6\end{bmatrix}$$

$A \quad X = B$

$X=A^{-1}B$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}\frac{4}{9} & \frac{1}{9} & \frac{1}{9}\\ \frac{4}{3} & -\frac{2}{3} & \frac{1}{3}\\ \frac{7}{9} & -\frac{5}{9} & \frac{4}{9}\end{bmatrix}\begin{bmatrix}3\\ -4\\ 6\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}14/9\\ 26/3\\ 65/9\end{bmatrix}$$

#6) **Factory Production** Suppose a factory is asked to produce three types of products, which we will call P_1, P_2, and P_3. Suppose the following purchase order was received: $P_1 = 7$, $P_2 = 12$, and $P_3 = 5$. Represent this order by a row vector and call it P: $P = [7 \quad 12 \quad 5]$. To produce each of the products, raw material of four kinds is needed. Call the raw material M_1, M_2, M_3, and M_4. The matrix below gives the amount of material needed for each product:

$$Q = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{cccc} M_1 & M_2 & M_3 & M_4 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 & 12 \\ 7 & 9 & 5 & 20 \\ 8 & 12 & 6 & 15 \end{bmatrix} \end{array}$$

Suppose the cost for each of the materials M_1, M_2, M_3, and M_4 is \$10, \$12, \$15, and \$20, respectively. The cost vector is:

$$C = \begin{bmatrix} 10 \\ 12 \\ 15 \\ 20 \end{bmatrix}$$

Compute each of the following and interpret: (a) PQ (b) QC (c) PQC

a)
$$\begin{array}{ccc} P_1 & P_2 & P_3 \end{array} \quad \begin{array}{cccc} M_1 & M_2 & M_3 & M_4 \end{array}$$
$$\begin{bmatrix} 7 & 12 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 & 12 \\ 7 & 9 & 5 & 20 \\ 8 & 12 & 6 & 15 \end{bmatrix}$$
$$= \begin{bmatrix} 138 & 189 & 97 & 399 \end{bmatrix}$$
total material of each type for P.O.

b)
$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{cccc} M_1 & M_2 & M_3 & M_4 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 & 12 \\ 7 & 9 & 5 & 20 \\ 8 & 12 & 6 & 15 \end{bmatrix} \end{array} \begin{bmatrix} 10 \\ 12 \\ 15 \\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} 311 \\ 653 \\ 614 \end{bmatrix}$$
cost of materials for 1 unit of each product

c)
$$\begin{bmatrix} 13083 \end{bmatrix}$$
total cost of purchase order

#7) **Department Store Purchases** Lee went to a department store and purchased 6 pairs of pants, 8 shirts, and 2 jackets. Chan purchased 2 pairs of pants, 5 shirts, and 3 jackets. If pants are \$25 each, shirts are \$18 each, and jackets are \$39 each, use matrix multiplication to find the amounts spent by Lee and Chan.

$$\begin{array}{c} \\ \text{Lee} \\ \text{Chan} \end{array} \begin{array}{c} \begin{array}{ccc} \text{pants} & \text{shirts} & \text{jackets} \end{array} \\ \begin{bmatrix} 6 & 8 & 2 \\ 2 & 5 & 3 \end{bmatrix} \end{array} \begin{bmatrix} 25 \\ 18 \\ 39 \end{bmatrix} \begin{array}{c} \text{pants} \\ \text{shirts} \\ \text{jackets} \end{array} = \begin{bmatrix} 372 \\ 257 \end{bmatrix} \begin{array}{c} \leftarrow \text{Lee spent} \\ \leftarrow \text{Chan spent} \end{array}$$

#8) Farmer Brown has 1000 acres of land on which he plans to grow corn (c), wheat (w), and soybeans (s). The cost of cultivating these crops is \$28 per acre for corn, \$40 per acre for wheat, and \$32 per acre for soybeans. If Farmer Brown wishes to use all his available land and his entire budget of \$30,000, and if he wishes to plant the same number of acres of corn as wheat and soybeans combined, how many acres of each crop can he grow?

$$\begin{cases} c + w + s = 1000 \\ 28c + 40w + 32s = 30000 \\ c - w - s = 0 \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 28 & 40 & 32 \\ 1 & -1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c \\ w \\ s \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1000 \\ 30000 \\ 0 \end{bmatrix}}_{B}$$

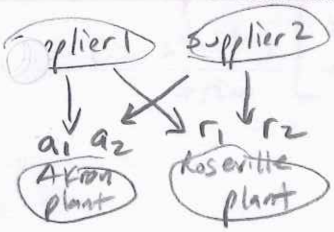
$$X = A^{-1}B$$

$c = w + s$

$$\begin{bmatrix} c \\ w \\ s \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 28 & 40 & 32 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 30000 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c \\ w \\ s \end{bmatrix} = \begin{bmatrix} 500 \\ 0 \\ 500 \end{bmatrix}$$

#9) **Transportation** A manufacturer purchases a part for use at both of its plants - one at Roseville, CA, the other at Akron, OH. The part is available in limited quantities from two suppliers. Each supplier has 75 units available. The Roseville plant needs 40 units, and the Akron plant requires 75 units. The first supplier charges \$70 per unit delvered to Roseville and \$90 per unit delivered to Akron. Corresponding costs from the second supplier are \$80 and \$120. The manufacturer wants to order a total of 75 units from the first, less expensive supplier, with the remaining 40 units to come from the second supplier. If the company spends \$10,750 to purchase the required number of units for the two plants, find the number of units that should be purchased from each supplier for each plant.



supplier 1 provides 75 units: $a_1 + r_1 = 75$

supplier 2 provides remaining 40 units: $a_2 + r_2 = 40$

Roseville needs 40 units: $r_1 + r_2 = 40$

Akron needs 75 units: $a_1 + a_2 = 75$

total spent: $70r_1 + 90a_1 + 80r_2 + 120a_2 = 10750$

more equations than variables, so use rref method:

$$\begin{array}{cccc} a_1 & r_1 & a_2 & r_2 \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 75 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 1 & 0 & 1 & 40 \\ 1 & 0 & 1 & 0 & 75 \\ 90 & 70 & 120 & 80 & 10750 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 35 \\ 0 & 1 & 0 & 0 & 40 \\ 0 & 0 & 1 & 0 & 40 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} a_1 \\ r_1 \\ a_2 \\ r_2 \\ \end{array}$$

Akron — 35 units from supplier 1, 40 units from supplier 2

Roseville — 40 units from supplier 1, no units from supplier 2

The Ace Novelty Company received an order from Magic World Amusement Park for 900 Grand Pandas, 1200 Saint Bernards, and 2000 Big Birds. Ace's management decided that 500 Giant Pandas, 800 Saint Bernards, and 1300 Big Birds could be manufactured in their Los Angeles plant, and the balance could be filled by their Seattle plant. Each Panda requires 1.5 square yards of plush, 30 cubic feet of stuffing, and 5 pieces of trim; each Saint Bernard requires 2 square yards of plush, 35 cubic feet of stuffing, and 8 pieces of trim; and each Big Bird requires 2.5 square yards of plush, 25 cubic feet of stuffing, and 15 pieces of trim. The plush costs \$4.50 per square yard, the stuffing costs 10 cents per cubic foot, and the trim costs 25 cents for each piece.

a. Find how much of each type of material must be purchased for each plant.

b. What is the total cost of materials incurred by each plant and the total cost of materials incurred by Ace Novelty in filling this order.

a)

$$A = \begin{array}{c} \\ \text{LA} \\ \text{Seattle} \end{array} \begin{array}{c} \begin{array}{ccc} \text{giant panda} & \text{Saint B} & \text{Big Bird} \end{array} \\ \begin{bmatrix} 500 & 800 & 1300 \\ 400 & 400 & 700 \end{bmatrix} \end{array} \qquad B = \begin{array}{c} \\ \text{giant panda} \\ \text{Saint B} \\ \text{Big Bird} \end{array} \begin{array}{c} \begin{array}{ccc} \text{plush} & \text{stuffing} & \text{trim} \end{array} \\ \begin{bmatrix} 1,5 & 30 & 5 \\ 2 & 35 & 8 \\ 2,5 & 25 & 15 \end{bmatrix} \end{array}$$

$$AB = \begin{bmatrix} 500 & 800 & 1300 \\ 400 & 400 & 700 \end{bmatrix} \begin{bmatrix} 1,5 & 30 & 5 \\ 2 & 35 & 8 \\ 2,5 & 25 & 15 \end{bmatrix} = \begin{array}{c} \\ \text{LA} \\ \text{Seattle} \end{array} \begin{array}{c} \begin{array}{ccc} \text{plush} & \text{stuff} & \text{trim} \end{array} \\ \begin{bmatrix} 5600 & 75500 & 28400 \\ 3150 & 43500 & 15700 \end{bmatrix} \end{array}$$

b)

$$C = \begin{array}{c} \\ \text{plush} \\ \text{stuffing} \\ \text{trim} \end{array} \begin{array}{c} \text{cost} \\ \begin{bmatrix} 4,50 \\ 0,10 \\ 0,25 \end{bmatrix} \end{array} \qquad (AB)C = \begin{array}{c} \\ \text{LA} \\ \text{Seattle} \end{array} \begin{array}{c} \begin{array}{ccc} \text{plush} & \text{stuff} & \text{trim} \end{array} \\ \begin{bmatrix} 5600 & 75500 & 28400 \\ 3150 & 43500 & 15700 \end{bmatrix} \end{array} \begin{array}{c} \text{plush} \\ \text{stuff} \\ \text{trim} \end{array} \begin{bmatrix} 4,50 \\ 0,10 \\ 0,25 \end{bmatrix}$$

$$= \begin{array}{c} \text{LA} \\ \text{Seattle} \end{array} \begin{bmatrix} 39850 \\ 22450 \end{bmatrix}$$

total cost $= 39850 + 22450 = \boxed{\$62{,}300}$