

Honors Finite Mathematics – Lesson Notes: Chapter 1

1.1 – Rectangular Coordinates, Lines

Terms to Recall:

Ordered pair, x-coordinate (abscissa, independent variable), y-coordinate (ordinate, dependent variable), origin, quadrant.

Forms of an equation of a line:

Slope-intercept: $y = mx + b$

General/Standard: $Ax + By = C$

Point-slope: $(y - y_1) = m(x - x_1)$
where (x_1, y_1) is any point on the line

special cases ↗

Slope formula: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$

Vertical line: $x = a$ where $(a, 0)$ is the x-int.

Horizontal line: $y = b$ where $(0, b)$ is the y-int.

#10) Find the slope and y-intercept of this line, and graph the line. $2x - 3y = 6$



$$\begin{array}{r} -2x \quad -2x \\ -3y = -2x + 6 \\ \hline -3 \quad -3 \quad -3 \end{array}$$

$$\begin{array}{l} y = \frac{2}{3}x - 2 \\ \text{slope: } \frac{2}{3} \\ \text{y-int: } (0, -2) \end{array}$$

#11) Find a general form equation for the y-axis.

$$x = 0$$

#12) Find the equation of the line given the following:

x-intercept = $(-4, 0)$ and y-intercept = $(0, 4)$

$$\text{slope} = \frac{4-0}{0-(-4)} = \frac{4}{4} = 1 \quad \text{point-slope: } (y-4) = 1(x-0)$$

$$y-4 = x$$

$$\begin{array}{l} x - y = -4 \\ y = x + 4 \end{array}$$

#13) **Electricity Rates:** Commonwealth Edison Company supplies electricity in the summer months to residential customers for a monthly charge of \$9.36 plus 10.494 cents per kilowatt-hour for up to 400 kilowatt-hours.

(a) Write an equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month.

(b) Graph this equation.

(c) What is the monthly charge for using 100 kilowatt-hours?

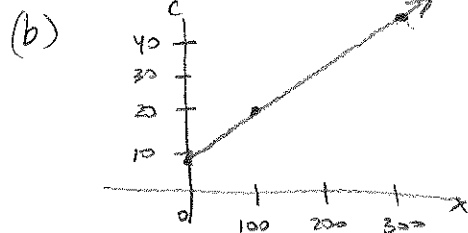
(d) What is the monthly charge for using 300 kilowatt-hours?

(e) Interpret the meaning of the 'slope' of the line in this problem.

(a) $C(x) = 9.36 + 0.10494x$

(c) $C = 9.36 + 0.10494(100)$ (d) $C = 9.36 + 0.10494(300)$

$$= 11.85 \quad = 40.83$$



(e) slope = rate cost increases per kilowatt-hr

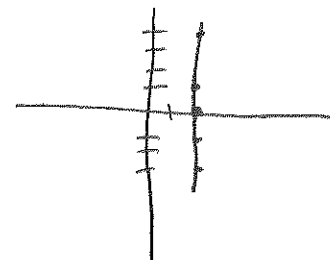
1.1 class problems

Key

#1) Plot the points $(2,0), (2,-3), (2,4), (2,1), (2,-1)$.

Describe the collection of all points of the form $(2,y)$ where y is a real number.

points on the vertical line $x=2$.



#2) Fill in the missing values if $2x - y = 6$

x	0	3	2	-2	4	-4
y	-6	0	-2	-10	10	-22
	$2(0) - y = 6$	$2x - (0) = 6$	$2(2) - y = 6$	$2(-2) - y = 6$	$2(4) - y = 6$	$2(-4) - y = 6$
	$y = -6$	$x = 3$	$4 - y = 6$	$-4 - y = 6$	$8 - y = 6$	$-8 - y = 6$
			$-y = 2$	$-y = 10$	$-y = -2$	$-y = 14$

#3) Find the x-intercept and the y-intercept and graph the equation: $3x + 2y = 0$

$$3(0) + 2y = 0$$

$$y = 0$$

$$(0, 0)$$

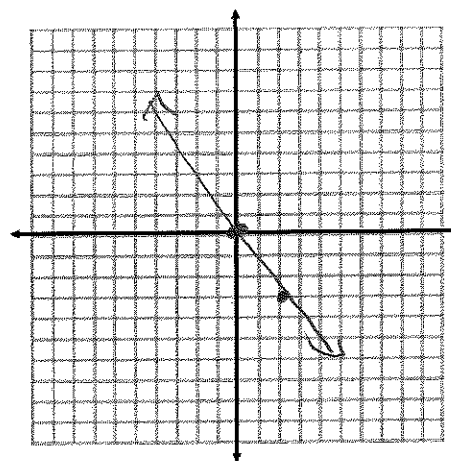
$$3x + 2(0) = 0$$

$$x = 0$$

$$3x + 2y = 0$$

$$2y = -3x$$

$$y = -\frac{3}{2}x$$



#4) Find the x-intercept and the y-intercept and graph the equation: $y = 5x - 7$

$$y = 5(0) - 7$$

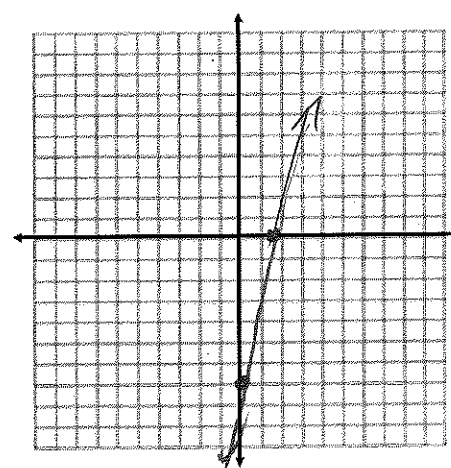
$$(0, -7)$$

$$0 = 5x - 7$$

$$5x = 7$$

$$x = \frac{7}{5}$$

$$\left(\frac{7}{5}, 0\right)$$



1.4

1.7

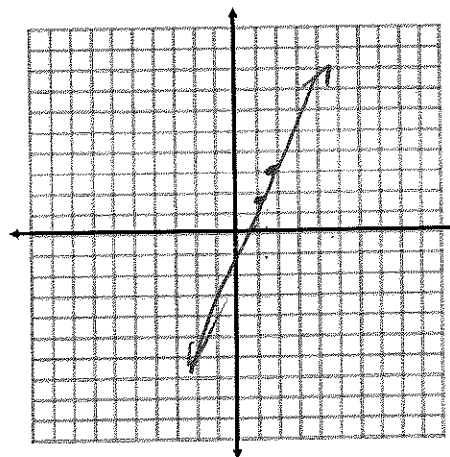
#5) Plot the points and determine the slope, then graph the line: $(\sqrt{2}, 3)$, $(1, \sqrt{3})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sqrt{3} - 3}{1 - \sqrt{2}}$$

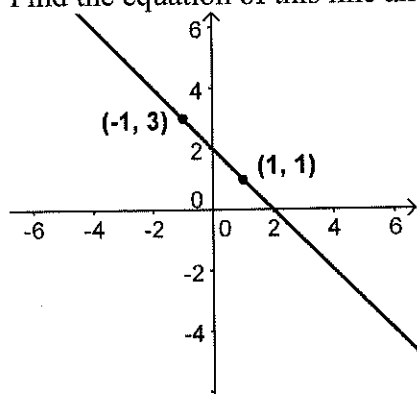
$$\frac{(\sqrt{3} - 3)(1 + \sqrt{2})}{(1 - \sqrt{2})(1 + \sqrt{2})}$$

$$\frac{\sqrt{3} + \sqrt{6} - 3 - 3\sqrt{2}}{1 - 2}$$

$$\boxed{-\sqrt{3} - \sqrt{6} + 3 + 3\sqrt{2}} \approx 3.0611$$



#6) Find the equation of this line and write it in general form:



$$m = \frac{3 - 1}{-1 - 1} = \frac{2}{-2} = -1$$

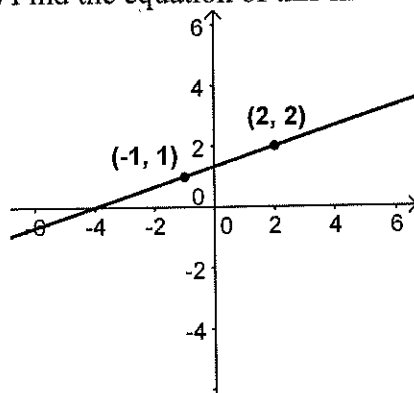
$$(y - 1) = -1(x - 1)$$

$$y - 1 = -x + 1$$

$$x + y - 1 = 1$$

$$\boxed{x + y = 2}$$

#7) Find the equation of this line and write it in point-slope form:



$$m = \frac{2 - 1}{2 - (-1)} = \frac{1}{3}$$

$$\boxed{y - 1 = \frac{1}{3}(x + 1)}$$

#8) Find the equation of the line (in general form $Ax + By = C$) that has $\text{slope} = -\frac{2}{3}$ and passes through $(1, -1)$.

$$(y + 1) = -\frac{2}{3}(x - 1)$$

$$3y + 3 = -2x + 2$$

$$2x + 3y + 3 = 2$$

$$\boxed{2x + 3y = -1}$$

9) **Profit from selling newspapers:** Each Sunday, a newspaper agency sells x copies of a certain newspaper for \$1.00 per copy. The cost to the agency of each newspaper is \$0.50. The agency pays a fixed cost for storage, delivery, etc. of \$100.00 per Sunday.

Write an equation that relates the profit, P (in dollars) to the number x of copies sold. Graph this equation.

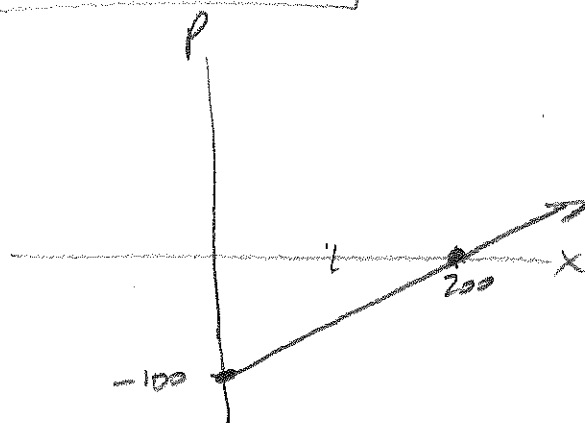
$$C(x) = 100 + 0.5x$$

$$R(x) = 1.00x$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 1.00x - (100 + 0.5x)$$

$$P(x) = 0.5x - 100$$



1.2 – Parallel and Perpendicular Lines

Groups try these...

#1) Determine whether the given pair of lines are parallel, intersecting, or coincident:

$$2x + y = 8$$

$$2x - y = -4$$

#2) Find the point of intersection. Graph the pair of lines:

$$4x + 3y = 2$$

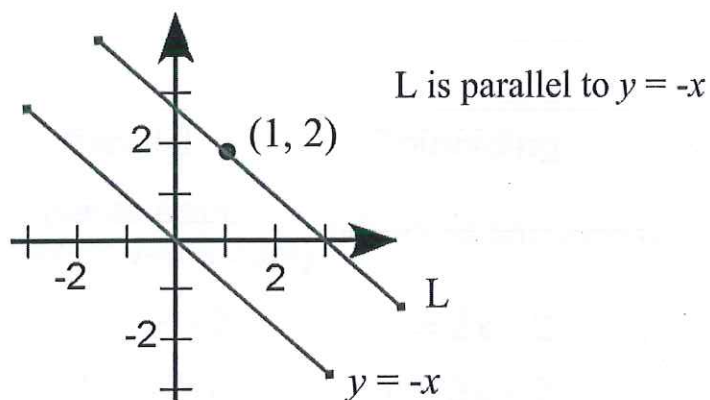
$$2x - y = -1$$

#3) Show that the lines are perpendicular:

$$20x - 2y = -7$$

$$x + 10y = 8$$

#4) Find an equation for the line L (write the equation in general form).



#5) Find an equation for the line. (Write the equation in slope-intercept form).

Perpendicular to the line $y = 3x - 15$ passing through $\left(-\frac{2}{3}, \frac{3}{5}\right)$

#6) Find the equation of the line passing through $(-2, -5)$ and **Perpendicular** to the line through $(-4, 5)$ and $(2, -1)$.

#1) Determine whether the given pair of lines are parallel, intersecting, or coincident:

$$2x + y = 8$$

$$y = -2x + 8$$

$$-y = -2x - 4$$

$$2x - y = -4$$

$$y = 2x + 4$$

intersecting

#2) Find the point of intersection. Graph the pair of lines:

$$4x + 3y = 2$$

$$4x + 3y = 2$$

$$4(-\frac{1}{10}) + 3y = 2$$

$$(2x - y = -1) \cdot 3$$

$$6x - 3y = -3$$

$$-4 + 3y = 2$$

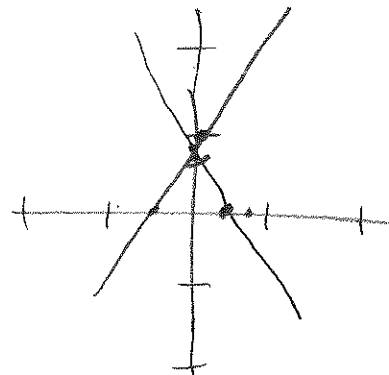
$$10x = -1$$

$$3y = 24$$

$$x = -\frac{1}{10}$$

$$y = \frac{24}{3} = 8$$

$$(-\frac{1}{10}, 8)$$



#3) Show that the lines are perpendicular:

$$20x - 2y = -7$$

$$-2y = -20x - 7$$

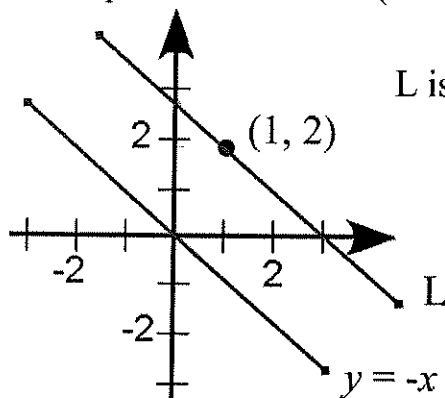
$$x + 10y = 8$$

$$y = -10x - \frac{7}{2}$$

$$10y = -x + 8$$

$$y = -\frac{1}{10}x + \frac{4}{5}$$

#4) Find an equation for the line L (write the equation in general form).



L is parallel to $y = -x$

$$y = mx + b$$

$$y = -x + 3$$

$$y = -x + b$$

$$(2) = -(1) + b$$

$$2 = -1 + b$$

$$+1 \quad +1$$

$$3 = b$$

$$x + y = 3$$

#5) Find an equation for the line. (Write the equation in slope-intercept form).

Perpendicular to the line $y = 3x - 15$ passing through $(-\frac{2}{3}, \frac{3}{5})$

$$m = -\frac{1}{3}$$

$$y = mx + b$$

$$y = -\frac{1}{3}x + b$$

$$\frac{3}{5} = -\frac{1}{3}(-\frac{2}{3}) + b$$

$$\frac{3}{5} = \frac{2}{9} + b$$

$$3 = \frac{10}{9} + 9b$$

$$27 = 10 + 81b$$

$$17 = 81b$$

$$y = -\frac{1}{3}x + \frac{17}{81}$$

$$y = -0.33x + 0.378$$

$$b = \frac{17}{81}$$

- #6) Find the equation of the line passing through $(-2, -5)$ and **Perpendicular** to the line through $(-4, 5)$ and $(2, -1)$.

$$m = \frac{5+1}{-4-2} = \frac{6}{-6} = -1$$

new $m = 1$ $y+5 = 1(x+2)$

$$y+5 = x+2$$

$$\boxed{y = x - 3}$$

- #7) Find the equation of the vertical line passing through $(-2, 5)$.

$$\boxed{x = -2}$$

- #8) Find the equation of the horizontal line passing through $(-2, 5)$.

$$\boxed{y = 5}$$

- #9) Find the equation of the line passing through $(-2, -5)$ and **Parallel** to the line through $(-4, 5)$ and $(2, -1)$.

$$m = \frac{5+1}{-4-2} = -1$$

new $m = -1$

$$(y+5) = -(x+2)$$

$$y+5 = -x-2$$

$$\boxed{y = -x - 7}$$

1.3 – Applications

Terms to know:

Cost, $C(x)$: The amount of money it takes to produce x units of something.

Revenue, $R(x)$: The amount of money you make from selling x units of something.

Break-even: When Revenue=Cost, when a business makes enough in revenue to exactly pay its costs (no profit yet).

Supply, $S(p)$: The amount of a commodity sellers are willing to offer at a given price, p .

Demand, $D(p)$: The amount of a commodity buyers are willing to buy at a given price, p .

Market price (or equilibrium): The price at which Supply=Demand, suppliers are willing to offer the same amount of a commodity that buyers are willing to buy.

#1) Profit for Selling Newspapers

Each Sunday a newspaper agency sells x copies of a certain newspaper for \$2.00 per copy. The cost to the agency for each newspaper is \$1.00. The agency pays a fixed cost for storage, delivery, and so on, of \$200 per Sunday. How many newspapers need to be sold for the agency to break even?

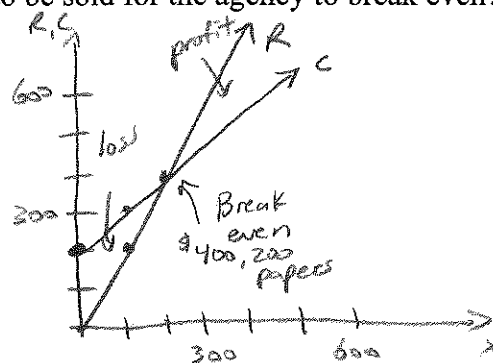
$$R(x) = 2x$$

$$R = C$$

$$C(x) = x + 200$$

$$2x = x + 200$$

$$\boxed{x = 200 \text{ papers}}$$



#2) Market Price of Sugar

The supply and demand equation for sugar have been estimated to be given by the equations

$$S = 0.7p + 0.4 \quad D = -0.5p + 1.6$$

Find the market price. What quantity of supply is demanded at this market price? Graph both the supply and demand equations.

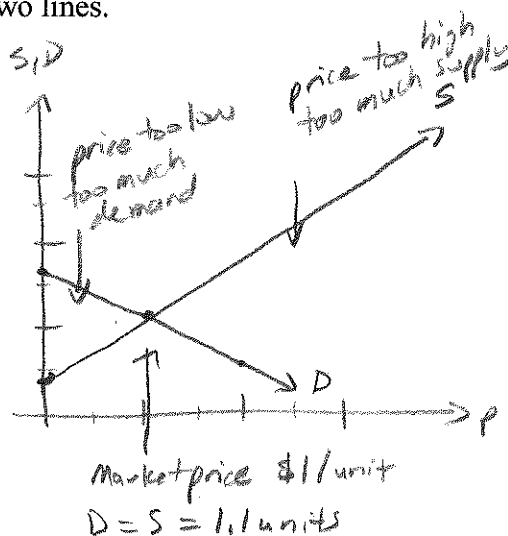
Interpret the point of intersection of the two lines.

$$S = D$$

$$0.7p + 0.4 = -0.5p + 1.6$$

$$1.2p = 1.2$$

$$\boxed{p = 1}$$



#3) Mixture Problem

The manager of Nutt's Nuts regularly sells cashews for \$6.50 per pound, pecans for \$7.50 per pound, and peanuts for \$2.00 per pound. How many pounds of cashews and pecans should be mixed with 40 pounds of peanuts to obtain a mixture of 100 pounds that will sell for \$4.89 so that the revenue is unchanged?

C - cashews

$$C + P + 40 = 100$$

P - pecans

$$6.5C + 7.5P + 2(40) = 4.89(100)$$

$$C + P = 60$$

$$6.5C + 7.5P = 409$$

$$-6.5C - 6.5P = -392$$

$$6.5C + 7.5P = 409$$

$$P = 19$$

$$C + (19) = 60$$

$$C = 41$$

19 lbs pecans, 41 lbs cashews

#4) Supply and Demand Problem

For a certain commodity the demand equation is given by

$$D = -3p + 20$$

At a price of \$1, four units of the commodity are supplied.

If the supply equation is linear and the market price is \$4, find the supply equation.

$$p = 4$$

$$D = -3(4) + 20$$

$$D = -12 + 20$$

$$D = 8 \text{ units}$$

So $S = 8$ units also
when $p = 4$

P	S
1	4
4	8

$$S = mp + b$$

$$S = \frac{4}{3}p + b$$

$$S = \frac{4}{3}p + \frac{8}{3}$$

$$(4) = m(1) + b \rightarrow m + b = 4$$

$$(8) = m(4) + b \rightarrow 4m + b = 8$$

solve simultaneously:

$$-m - b = -4$$

$$4m + b = 8$$

$$3m = 4$$

$$m = \frac{4}{3}$$

$$m + b = 4$$

$$\left(\frac{4}{3}\right) + b = 4$$

$$4 + 3b = 12$$

$$3b = 8$$

$$b = \frac{8}{3}$$

#5) A coffee manufacturer wants to market a new blend of coffee that will cost \$6.00 per pound by mixing \$5.00 per pound coffee and \$7.50 per pound coffee. What amounts of the \$5.00/pound coffee and \$7.50/pound coffee should be blended to obtain the desired mixture? HINT: ASSUME TOTAL WEIGHT OF THE DESIRED BLEND IS 100 POUNDS.

$$\begin{aligned} \text{amt of } \$5/\text{lb} &= x \\ \text{amt of } \$7.5/\text{lb} &= y \end{aligned}$$

$$x + y = 100$$

$$5x + 7.5y = 6(100)$$

$$x + y = 100$$

$$5x + 7.5y = 600$$

$$-5x - 5y = -500$$

$$5x + 7.5y = 600$$

$$2.5y = 100$$

$$y = 40$$

$$x + y = 100$$

$$x + 40 = 100$$

$$x = 60$$

60 lbs of \$5/lb coffee
40 lbs of \$7.5/lb coffee

#6) Predicting the Cost of a Compact Car

In 1995, the cost of a compact car averaged \$8000. In 1998, the cost of a compact car averaged \$9500.

Assuming that the relationship between time and cost is linear, develop a formula for predicting the

average cost of a compact car in the future. What do you predict the average cost of a compact car was in 2000?

$$t = 0 = 1990$$

t	C
5	8000
8	9500
10	?

$$C = mt + b$$

$$m = \frac{9500 - 8000}{8 - 5}$$

$$m = 500$$

$$C = 500t + b$$

$$(8000) = 500(5) + b$$

$$8000 = 2500 + b$$

$$5500 = b$$

$$C = 500t + 5500$$

$$\text{in 2000, } t = 10$$

$$C = 500(10) + 5500$$

$$C = 10500$$