Name_____Period_____

Spring Semester Open-Ended Final Exam Review – Honors Algebra 3-4

1. a. Convert to degrees: $\theta = \dfrac{5\pi}{8}$ radians.   b. Convert to radians: $\theta = 175°$, leave answer in terms of π.

2. If the cosecant of an acute angle is $\dfrac{25}{24}$, find the value of the cosine.

3. Find the reference angle for: a. $\theta = 435°$ b. $\theta = \dfrac{-17\pi}{3}$

4. Given the following, find θ, $0° \le \theta < 360°$, to the nearest degree:
   a. $\tan\theta = 3.8958$ b. $\sin\theta = .2654$ c. $\sec\theta = 2.0717$

5. Given $\cot\theta = \dfrac{-12}{5}$ and $\cos\theta < 0$, find $\csc\theta$.

6. Given $\sec\theta = \dfrac{-3}{2}$ and $\cot\theta > 0$, find $\sin\theta$.

7. Two 100' guy wires are attached to the top of a telephone pole, one on each side. The angle of elevation of each wire with the ground is $57°$. How tall is the telephone pole to the nearest foot?

8. From a 75-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The boat is 625 feet from the base of the tower. Find the angle of depression from the top of the tower to the boat to the nearest degree

9. Find the exact value of the expression: a. $\csc\left[\cos^{-1}\left(\dfrac{-3}{4}\right)\right]$ b. $\tan\left[\sin^{-1}\left(\dfrac{-8}{17}\right)\right]$

10. Find the period in degrees: a. $f(x) = \cos\left(\dfrac{1}{4}x\right)$ b. $f(x) = \tan(6x)$

11. Write the sine equation for the following: 12. Write the cosine equation for the following:
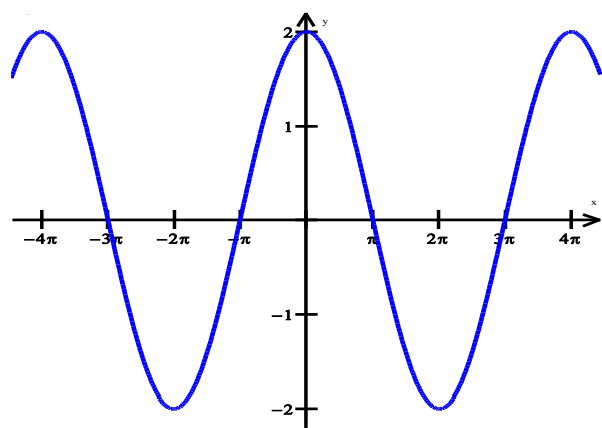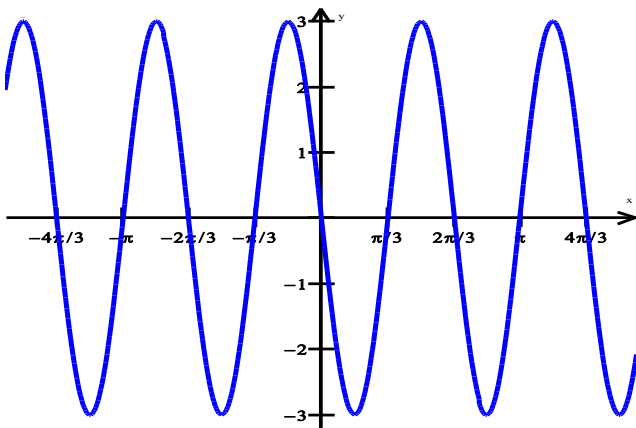
13. Graph: $f(x) = -3\cos\left(\dfrac{\pi}{2}x + \pi\right) + 1$

14. Graph: $f(x) = 4\csc(x)$

15. Simplify: $\tan^2 x - \sin^2 x \tan^2 x$

16. Simplify: $\dfrac{\sec^2 x - 1}{\sec x - 1}$

17. Perform the addition and simplify: $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$

18. Perform the subtraction and simplify: $\cos x \sec x - \dfrac{\cos x}{\sec x}$

19. Find all solutions in the interval $[0, 2\pi)$: $\quad 4\cos^2 x - 1 = 0$

20. Find all solutions in the interval $[0, 2\pi)$: $\quad 2\sin x \cos x = \sqrt{2}\sin x$

21. Find all solutions in the interval $[0, 2\pi)$: $\quad \sec^2 x - \sec x = 2$

22. Find all solutions in the interval $[0, 2\pi)$: $\quad 2\sin^2 x - 5\sin x + 2 = 0$

23. Evaluate: $\cos 255°$ (Use the fact that $255° = 315° - 60°$).

24. Evaluate: $\sin 105°$ (Use the fact that $105° = 150° - 45°$).

25. Given $\cos u = \dfrac{-12}{13}, \pi < u < \dfrac{3\pi}{2}$ and $\csc v = \dfrac{\sqrt{10}}{3}, \dfrac{\pi}{2} < v < \pi$, find $\cos(u + v)$.

26. Given $\cot u = \dfrac{2}{5}, 0 < u < \dfrac{\pi}{2}$ and $\cos v = \dfrac{-3}{5}, \pi < v < \dfrac{3\pi}{2}$, find $\tan(u + v)$.

27. Find the area of the triangle to the nearest tenth, with sides of length, 5, 9, and 10.

28. In $\triangle ABC$, BC = 90, AC = 52 and $m\angle C = 102°$. What is the area of $\triangle ABC$ to the nearest tenth?

29. Given a triangle with sides a = 6, b = 8, and c = 12, find $m\angle C$ to the nearest degree.

30. Given a triangle with $A = 115°, b = 15,$ and $c = 10,$ find a to the nearest tenth.

31. Solve for x, to the nearest tenth, in the given triangle.
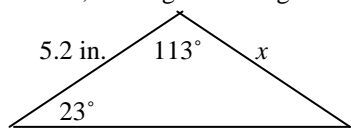
32. Find all remaining sides and angles of the triangle with B = 61°, c = 18, and b = 17.
    Find angles to the nearest degree and sides to the nearest tenth.

33. Rewrite the following in trigonometric form; express the answers in both degrees and radians:

 a. -3   b. $-2-2i\sqrt{3}$

34. Use DeMoivre's theorem to evaluate: $[2(\cos 300° + i\sin 300°)]^6$; express your answer in degrees.

35. Use DeMoivre's theorem to find the cube roots of: -27i; express your answers in degrees.

36. Write an explicit formula for the arithmetic sequence: 2, 5, 8, 11, 14, ….

37. Write an explicit formula for the arithmetic sequence with a first term of 100 and a common difference of -8.

38. Find the 83rd term of the **arithmetic** sequence with $a_1 = 1.8$ *and* $d = 2.4$.
 (Assume that n begins with 1)

39. Find the **sum** of the first 21 terms of the **arithmetic** sequence whose nth term is $a_n = 2n+5$
 (Assume that n begins with 1).

40. Find the sum: a. $\displaystyle\sum_{n=0}^{50}(1000-5n)$ b. $\displaystyle\sum_{n=2}^{5}\frac{4}{n+2}$ (express the answer as a fraction)

41. Evaluate: a. $\displaystyle\sum_{n=1}^{\infty}4\left(\frac{7}{3}\right)^{n-1}$ b. $\displaystyle\sum_{n=0}^{\infty}2\left(\frac{-2}{3}\right)^{n}$

42. Find the coefficient of: a. x^2y^3 in the expansion of $(2x-3y)^5$ b. x^8y^4 in the expansion of $(x+5y)^{12}$.

43. A card is drawn at random from a standard deck of 52 playing cards. Find the probability that the card is:
 a. red   b. a king.

44. A password is comprised of 4 letters followed by 3 digits. How many passwords are possible?

45. A password is comprised of 5 letters followed by 2 digits. To avoid confusion between "O" and "zero" and "I" and "one", the letters "O" and "I" are not used. How many passwords are possible?

46. A bag contains 10 quarters, 7 dimes, and 5 nickels. If three coins are selected without replacement, what is the probability of selecting three quarters?

47. A bag contains 10 quarters, 7 dimes, and 5 nickels. If three coins are selected without replacement, what is the probability of selecting one of each coin?

48. An employer interviews 12 people for four opportunities in the company. Five of the 12 people are women. If all 12 are qualified, in how many ways can the employer fill the four positions if:
 a. the selection is random?   b. exactly two women are selected?

49. Graph the following:

 a. $\begin{cases} x^2+y^2 \le 25 \\ y \ge x^2+2 \end{cases}$ b. $\begin{cases} (x-1)^2+(y-3)^2 \ge 16 \\ (x-1)^2+(y+3)^2 \le 36 \end{cases}$

50. Graph the system of inequalities.

$$5x - 3y \leq 15 \qquad\qquad 3x + 2y > 12$$

a. $x \geq 1$ b. $x \leq 6$

      $y \leq -2$ $y \leq 4$

51. A small business has an initial investment of $5000. The unit cost of the product is $21.60, and the selling price is $34.10. Find the sales necessary to break even.

52. Solve the following system:
$$x + y = 4$$
$$x^2 + y^2 = 4x$$

53. Solve the following system:
$$x^2 + y^2 = 169$$
$$x^2 - 8y = 104$$

54. Solve the system of linear equations:
$$2x + 3y + 3z = 3$$
$$6x + 6y + 12z = 13$$
$$12x + 9y - z = 2$$

55. Solve the system of linear equations:
$$-x + 2y = 1.5$$
$$2x - 4y = 3$$

56. Solve the system of linear equations:
$$2x + y - 7z = 4$$
$$2x - y - z = 0$$

57. Given: $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -3 & 2 \\ -4 & 1 & -2 \end{bmatrix}$ Find: a. $A + B$ b. $5A - 4B$

58. Find the product of matrices: a. $\begin{bmatrix} -5 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & 8 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$ c. $\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$

59. Given the following, Find A^{-1} a. $A = \begin{bmatrix} -2 & 7 \\ -4 & 11 \end{bmatrix}$ b. $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

60. Find the determinant of the following matrices: a. $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix}$ b. $\begin{vmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & 4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{vmatrix}$

Spring Open-Ended Answer Key

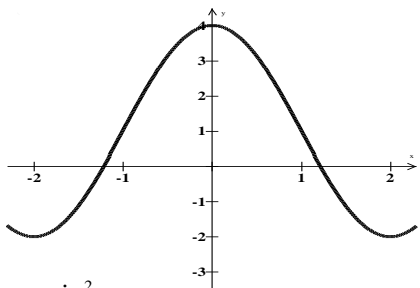**1.**   a. $112.5°$ b. $\dfrac{35\pi}{36}$

2. $\dfrac{7}{25}$

3.   a. $75°$ b. $\dfrac{\pi}{3}$

4.   a. $76°, 256°$ b. $15°, 165°$ c. $61°, 299°$

5. $\dfrac{13}{5}$

6. $\dfrac{-\sqrt{5}}{3}$

7.   84 feet

8. $7°$

9.   a. $\dfrac{4\sqrt{7}}{7}$ b. $\dfrac{-8}{15}$

10.   a. $1440°$ b. $30°$

11. $f(x) = -3\sin(3x)$

12. $f(x) = 2\cos\left(\dfrac{1}{2}x\right)$

13.

14.

15. $\sin^2 x$

16. $\sec x + 1$

17. $2\csc^2 x$

18. $\sin^2 x$

19. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

20. $0, \pi, \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

21. $\pi, \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

22. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

23. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

24. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

25. $\dfrac{27\sqrt{10}}{130}$

26. $\dfrac{-23}{14}$

27.   22.4

28.   2288.9

29. $117°$

30.   21.3

31.   2.9 inches

32.   **Case 1**   **Case 2**

       $\angle C = 68°$ $\angle C = 112°$

       $\angle A = 51°$ $\angle A = 7°$

       $a = 15.1$ $a = 2.4$

33.   a. $\begin{aligned} z &= 3(\cos 180° + i\sin 180°) \\ z &= 3(\cos \pi + i\sin \pi) \end{aligned}$

34. $64(\cos 0° + i\sin 0°)$

$z = 4\left(\cos 240° + i\sin 240°\right)$

b.
$$z = 4\left(\cos \frac{4\pi}{3} + i\sin \frac{4\pi}{3}\right)$$

35.
$r_1 = 3\left(\cos 90° + i\sin 90°\right)$
$r_2 = 3\left(\cos 210° + i\sin 210°\right)$
$r_3 = 3\left(\cos 330° + i\sin 330°\right)$

36. $a_n = 2 + (n-1)3$ or $a_n = 3n - 1$

37. $a_n = 100 + (n-1)(-8)$ or $a_n = -8n + 108$

38. 198.6

39. 567

40. a. 44,625   b. $\dfrac{319}{105}$

41. a. Does not exist   b. $\dfrac{6}{5}$

42. a. -1,080   b. 309,375

43. a. $\dfrac{1}{2}$ b. $\dfrac{1}{13}$

44. 456,976,000

45. 796,262,400

46. $\dfrac{6}{77}$

47. $\dfrac{5}{22}$

48. a. 495   b. 210

49. a.   b.

50. a.   b.

51. 400

52. (4, 0), (2, 2)

53. ($\pm$12, 5), (0, -13)

54. $\left(\dfrac{1}{2}, \dfrac{-1}{3}, 1\right)$
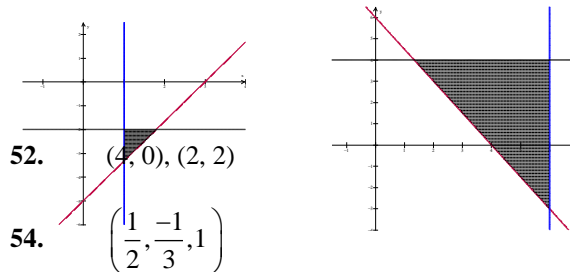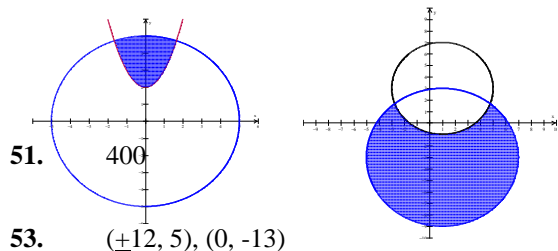
55. no solution

56. (2a+1, 3a+2, a)

57. a. $\begin{bmatrix} 8 & -2 & 3 \\ -5 & 0 & 3 \end{bmatrix}$ b. $\begin{bmatrix} -14 & 17 & -3 \\ 11 & -9 & 33 \end{bmatrix}$

58. a. $\begin{bmatrix} 9 & 37 \\ 48 & 54 \end{bmatrix}$ b. $\begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$ c. not possible

59. a. $\begin{bmatrix} \dfrac{11}{6} & \dfrac{-7}{6} \\ \dfrac{2}{3} & \dfrac{-1}{3} \end{bmatrix}$ b. $\begin{bmatrix} 1 & -1 & 0 \\ 7 & \dfrac{-17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

60. a. -75   b. 16