Worksheet 9.1-9.3

Name Key

Date _____ Period _____

$a_n = a_1 + (n-1)d$

$a_n = dn + c, \quad c = a_1 - d$

$a_n = a_1 r^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$

$S = a_1\left(\frac{1-r^n}{1-r}\right) \qquad S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = a_n(n+1)$.

$a_1 = 3$

$a_2 = a_1(1+1) = 3(2) = 6$

$a_3 = a_2(2+1) = 6(3) = 18$

$a_4 = a_3(3+1) = 18(4) = 72$

$a_5 = a_4(4+1) = 72(5) = 360$

$3, 6, 18, 72, 360$

2. What is the most apparent nth term of this sequence (assume that n begins with 1):

a) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ ($n = 1, 2, 3, 4$; $2, 4, 6, 8$)

$a_n = \frac{1}{2n-1}$

b) $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \ldots$ ($n = 1, 2, 3, 4$)

$a_n = (-1)^{n+1}\frac{n^2}{n+1}$

3. Use **sigma notation** to write the given sum (assume n begins with 1):

$\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + \frac{64}{729}$

$r = \frac{2}{3}$ $r = \frac{2}{3}$

$a_n = \frac{2}{3}\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^n$

$\frac{64}{729} = \left(\frac{2}{3}\right)^n$

$n = 6$

$\sum_{n=1}^{6}\left(\frac{2}{3}\right)^n$

4. Write in expanded form and then find the sum:

a) $\sum_{n=1}^{4}\frac{2}{n}$ (not arithm. or geo.)

$= 2 + 1 + \frac{2}{3} + \frac{1}{2}$

$\frac{12}{6} + \frac{6}{6} + \frac{4}{6} + \frac{3}{6}$

$= \frac{25}{6}$

b) $\sum_{x=1}^{5} x(x-1)(x-2)$

$= 0 + 0 + 6 + 24 + 60$

$= 90$

5. **The addends are terms of an arithmetic sequence. Find each sum.**

a) 36 terms of $21\sqrt{2} + 18\sqrt{2} + 15\sqrt{2} + \ldots$ ($-3\sqrt{2}$, $-3\sqrt{2}$)

arith. $d = -3\sqrt{2}$

$S = \frac{1}{2}n(a_1 + a_n)$

$a_n = a_1 + (n-1)d$

$a_{36} = 21\sqrt{2} + (35)(-3\sqrt{2}) = -84\sqrt{2}$

$S = \frac{1}{2}(36)(21\sqrt{2} - 84\sqrt{2})$

$S = -1134\sqrt{2}$

b) $\ln 2 + \ln 4 + \ln 8 + \ldots + \ln 1024$ ($n = 1$, $n = 10$)

$d = \ln 4 - \ln 2$

$d = \ln\frac{4}{2} = \ln 2$

$a_n = a_1 + (n-1)d$

$\ln 1024 = \ln 2 + (n-1)\ln 2$

$S = \frac{1}{2}n(\ln 2 + \ln 1024)$

$S = \frac{1}{2}(10)(\ln 2 + \ln 1024) = 5\ln 2048$

$= \ln(2048)^5$

$= 38,123$

$\frac{\ln 1024}{\ln 2} = 1 + n - 1 = n = 10$

6. A theater has 43 seats in the back row, 41 in the next, 39 in the next, and so on for 14 rows. How many seats are in the theater?

$d=-2 \quad a_1=43 \qquad a_{14}=43-(14-1)2=17$

$a_n=a_1+(n-1)d \qquad S=\frac{1}{2}(14)(43+17)$

$a_n=43-(n-1)2 \qquad S=$ 420 seats

7. The first three terms of a geometric sequence are –16, –8, –4,….
Write a rule that defines the sequence. $\times\frac{1}{2}$ $r=\frac{1}{2} \quad a_1=-16$

$a_n=a_1 r^{n-1}$

$a_n=-16\left(\frac{1}{2}\right)^{n-1}$

8. Find the fourth term of the geometric sequence whose first term is 4 and whose sixth term is $-\frac{243}{8}$.

$a_n=a_1 r^{n-1} \qquad a_4=4\left(-\frac{3}{2}\right)^{4-1}$

$-\frac{243}{8}=4r^{6-1} \qquad =4\cdot\frac{-27}{8}=\frac{-27}{2}$

$-\frac{243}{8}=4r^5$

$\frac{-243}{32}=r^5 \qquad r=\sqrt[5]{\frac{243}{32}}=\frac{3}{2}$

9. What is the common ratio of a geometric sequence whose second term is $\frac{2}{3}$ and whose sixth term is 54?

$a_2=\frac{2}{3} \qquad \frac{a_6}{a_2}=\frac{a_1 r^5}{a_1 r^1}=\frac{54}{\frac{2}{3}}$

$a_6=54 \qquad r^4=\frac{162}{2}=81$

$r=(81)^{1/4}=3$ $\quad r=3$

10. Find the sum of the first twelve terms of the geometric sequence whose first three terms are 240, -120, 60,…. (Use a formula).

$r=-\frac{1}{2}$

$S=a_1\left(\frac{1-r^n}{1-r}\right)=240\left(\frac{1-\left(-\frac{1}{2}\right)^{12}}{1+\frac{1}{2}}\right)=$ 159.96

11. Find the sum: $\sum_{n=1}^{\infty}2\left(\frac{2}{3}\right)^{n-1}$ converges. $S=\frac{a_1}{1-r}=\frac{2}{1-\frac{2}{3}}=\frac{2}{\frac{1}{3}}=6$

Worksheet 9.1-9.3

Name_____

Date_____ Period_____

$a_n = a_1 + (n-1)d$

$a_n = dn + c, \quad c = a_1 - d$

$a_n = a_1 r^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$

$S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad$ $S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = a_n(n+1)$.

2. What is the most apparent nth term of this sequence (assume that n begins with 1):

a) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ $\qquad$ b) $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \ldots$

3. Use **sigma notation** to write the given sum (assume n begins with 1):

$$\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + \frac{64}{729}$$

4. Write in expanded form and then find the sum:

a) $\sum_{n=1}^{4} \frac{2}{n}$ $\qquad$ b) $\sum_{x=1}^{5} x(x-1)(x-2)$

5. **The addends are terms of an arithmetic sequence. Find each sum.**

a) 36 terms of $21\sqrt{2} + 18\sqrt{2} + 15\sqrt{2} + \ldots$

b) $\ln 2 + \ln 4 + \ln 8 + \ldots + \ln 1024$

6. A theater has 43 seats in the back row, 41 in the next, 39 in the next, and so on for 14 rows. How many seats are in the theater?

7. The first three terms of a geometric sequence are –16, –8, –4,….
Write a rule that defines the sequence.

8. Find the fourth term of the geometric sequence whose first term is 4 and whose sixth term is $-\frac{243}{8}$.

9. What is the common ratio of a geometric sequence whose second term is $\frac{2}{3}$ and whose sixth term is 54?

10. Find the sum of the first twelve terms of the geometric sequence whose first three terms are 240, -120, 60,…. (Use a formula).

11. Find the sum: $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1}$