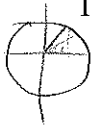


Show all supporting work. All answers must be exact, but you may use calculator to check.

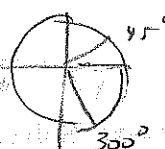
Sum and difference formulas:

(Hint: $105^\circ = 60^\circ + 45^\circ$)



1. $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$
 $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$
 $\cos 105^\circ = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$
 $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$
 $\tan 105^\circ = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$
 $\tan 60^\circ = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$
 $\tan 45^\circ = 1$

(Hint: $225^\circ = 300^\circ - 45^\circ$)

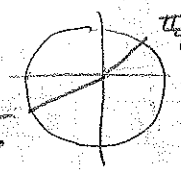


2. $\sin 225^\circ = \sin 300^\circ \cos 45^\circ - \cos 300^\circ \sin 45^\circ$
 $= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$
 $\cos 225^\circ = \cos 300^\circ \cos 45^\circ + \sin 300^\circ \sin 45^\circ$
 $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$
 $\tan 225^\circ = \frac{\tan 300^\circ - \tan 45^\circ}{1 + \tan 300^\circ \tan 45^\circ} = \frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$
 $\tan 300^\circ = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$
 $\tan 45^\circ = 1$

3. $\cos 32^\circ \cos 15^\circ - \sin 32^\circ \sin 15^\circ = \cos(32^\circ + 15^\circ) = \cos(47^\circ)$

4. $\frac{\tan 212^\circ - \tan 84^\circ}{1 + \tan 212^\circ \tan 84^\circ} = \tan(212^\circ - 84^\circ) = \tan 128^\circ$

5. (Hint: $\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$)



$\sin \frac{17\pi}{12} = \sin \frac{7\pi}{6} \cos \frac{\pi}{4} + \cos \frac{7\pi}{6} \sin \frac{\pi}{4}$
 $= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2} - \sqrt{6}}{4}$

$\cos \frac{17\pi}{12} = \cos \frac{7\pi}{6} \cos \frac{\pi}{4} - \sin \frac{7\pi}{6} \sin \frac{\pi}{4}$
 $= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6} + \sqrt{2}}{4}$

$\tan \frac{17\pi}{12} = \frac{\tan \frac{7\pi}{6} + \tan \frac{\pi}{4}}{1 + \tan \frac{7\pi}{6} \tan \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2} + 1}{1 + \frac{\sqrt{2}}{2}(1)} = \frac{\sqrt{2} + 2}{2 + \sqrt{2}}$

$\tan \frac{7\pi}{6} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
 $\tan \frac{\pi}{4} = 1$

6. $\sin 5\theta \cos 4\theta - \cos 5\theta \sin 4\theta = \sin(5\theta - 4\theta)$

7. Verify: $\sin(3\pi - x) = \sin x$

$\sin 3\pi \cos x - \cos 3\pi \sin x = \sin x$
 $(0) \cos x - (-1) \sin x = \sin x$
 $\sin x = \sin x$



Double Angles:

8. Solve: $\cos 2x + \sin x = 0$

$$1 - 2\sin^2 x + \sin x = 0$$

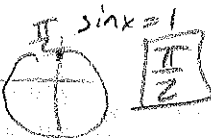
$$-2\sin^2 x + \sin x + 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$\frac{(2\sin x - 2)(\sin x + 1)}{2} = 0$$

$$(\sin x - 1)(2\sin x + 1) = 0$$

$$\sin x - 1 = 0$$



$$2\sin x + 1 = 0$$

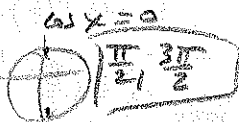
$$\sin x = -\frac{1}{2}$$



9. Solve: $\sin 2x + \cos x = 0$

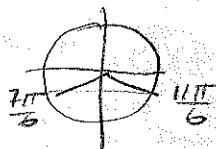
$$2\sin x \cos x + \cos x = 0$$

$$\cos x (2\sin x + 1) = 0$$



$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$



10. Given: $\cos \theta = -\frac{2}{3}$ and $\frac{\pi}{2} < \theta < \pi$

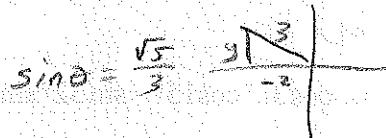
$$\text{Find: } \sin 2\theta = 2\sin \theta \cos \theta = 2 \left(\frac{\sqrt{5}}{3}\right) \left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$$

$$y^2 + 4 = 9$$

$$y^2 = 5$$

$$y = \sqrt{5}$$



Half Angles:

(Hint: $165^\circ = \frac{330^\circ}{2}$)



(Hint: $22.5^\circ = \frac{45^\circ}{2}$)



$$11. \sin 165^\circ = \pm \sqrt{\frac{1 - \cos 330^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$\cos 165^\circ = \pm \sqrt{\frac{1 + \cos 330^\circ}{2}} = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$12. \sin 22.5^\circ = \pm \sqrt{\frac{1 - \cos 45^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$\cos 22.5^\circ = \pm \sqrt{\frac{1 + \cos 45^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}}$$

(Hint: $\frac{\pi}{12} = \frac{\frac{\pi}{6}}{2}$)



$$13. \sin \frac{\pi}{12} = \pm \sqrt{\frac{1 - \cos \frac{\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$\cos \frac{\pi}{12} = \pm \sqrt{\frac{1 + \cos \frac{\pi}{6}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}}$$

Show all supporting work. All answers must be exact, but you may use calculator to check.

Sum and difference formulas:

(Hint: $105^\circ = 60^\circ + 45^\circ$)

1. $\sin 105^\circ =$

$\cos 105^\circ =$

$\tan 105^\circ =$

(Hint: $225^\circ = 300^\circ - 45^\circ$)

2. $\sin 225^\circ =$

$\cos 225^\circ =$

$\tan 225^\circ =$

3. $\cos 32^\circ \cos 15^\circ - \sin 32^\circ \sin 15^\circ =$

4. $\frac{\tan 212^\circ - \tan 84^\circ}{1 + \tan 212^\circ \tan 84^\circ} =$

5. (Hint: $\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$)

$\sin \frac{17\pi}{12} =$

$\cos \frac{17\pi}{12} =$

$\tan \frac{17\pi}{12} =$

6. $\sin 5\theta \cos 4\theta - \cos 5\theta \sin 4\theta =$

7. Verify: $\sin(3\pi - x) = \sin x$

Double Angles:

8. Solve: $\cos 2x + \sin x = 0$

9. Solve: $\sin 2x + \cos x = 0$

10. Given: $\cos \theta = -\frac{2}{3}$, and $\frac{\pi}{2} < \theta < \pi$

Find: $\sin 2\theta$

$\cos 2\theta$

Half Angles:

(Hint: $165^\circ = \frac{330^\circ}{2}$)

11. $\sin 165^\circ =$

$\cos 165^\circ =$

(Hint: $22.5^\circ = \frac{45^\circ}{2}$)

12. $\sin 22.5^\circ =$

$\cos 22.5^\circ =$

(Hint: $\frac{\pi}{12} = \frac{\left(\frac{\pi}{6}\right)}{2}$)

13. $\sin \frac{\pi}{12} =$

$\cos \frac{\pi}{12} =$

Honors Algebra 3-4
Ch. 5 Review Worksheet

Name _____
Period _____

(Do work on separate sheets of paper)

1. Simplify: $\sec x \cos\left(\frac{\pi}{2} - x\right)$

2. Simplify: $\frac{\csc x}{\tan x + \cot x}$

3. Perform the addition and simplify:

$$\frac{\tan x}{\csc x} + \frac{\sin x}{\tan x}$$

4. Simplify: $\frac{\sin^2 x}{\sec^2 x - 1}$

5. Simplify: $\frac{1}{\cot \theta} + \frac{1}{\tan \theta}$

6. Factor and simplify:

$$\sec^2 x \csc^2 x - \sec^2 x - \csc^2 x + 1$$

7. Factor and simplify: $\cot^4 x + 2 \cot^2 x + 1$

8. Rewrite the expression so that it is not in

fractional form: $\frac{\cos^2 x}{1 - \sin x}$

9. Use the substitution $x = 3 \cos \theta$ to write the algebraic expression $\sqrt{9 - x^2}$ as a trigonometric expression involving θ , where $0 < \theta < \frac{\pi}{2}$

10. Verify the identity: $\frac{\sec x - \cos x}{\tan x} = \sin x$

11. Verify the identity: $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

12. Verify the identity: $\frac{1 + \tan x}{\sin x} - \sec x = \csc x$

13. Verify the identity: $\sin\left(\frac{\pi}{2} - x\right) \cos(-x) = \cos^2 x$

14. Verify the identity: $\frac{\cos x}{1 - \sin^2 x} = \sec x$

15. Verify the identity: $1 + \frac{1}{\csc^2 x - 1} = \sec^2 x$

16. Find all the solutions in the interval $[0, 2\pi)$: $\tan 3t = \sqrt{3}$

17. Find all the solutions in the interval $[0, 2\pi)$: $\sec^2 x = \sec x + 2$

18. Find all the solutions in the interval $[0, 2\pi)$: $\cot^2 x - \tan^2 x = 0$

19. Find all the solutions in the interval $[0, 2\pi)$: $2 \sin x \cos x + \cos x = 0$

20. Given $\sin x = \frac{3}{10}$ and $\cos x = -\frac{\sqrt{91}}{10}$, find $\tan x$. (Draw the diagrams)

21. Evaluate: $\sin 105^\circ$. (Use the fact that $105^\circ = 60^\circ + 45^\circ$)

22. Evaluate: $\tan 165^\circ$. (Use the fact that $165^\circ = 210^\circ - 45^\circ$)

23. Evaluate: $\cos 285^\circ$. (Use the fact that $285^\circ = 330^\circ - 45^\circ$)

24. Simplify: $\sin 8x \cos 2x + \cos 8x \sin 2x$ (sum & difference formulas)

25. Given $\sin u = -\frac{5}{13}$, $0 < u < \frac{3\pi}{2}$ and $\csc v = \frac{\sqrt{10}}{3}$, $\frac{\pi}{2} < v < \pi$, find $\cos(u - v)$. (Draw the diagrams)

26. Find all solutions in the interval $[0, 2\pi)$: $\cos 2x + \sin x = 0$

Ch 5 Review worksheet

① $\sec x \cos\left(\frac{\pi}{2} - x\right)$

$$\frac{\sec x \sin x}{\cos x} = \tan x$$

② $\frac{\csc x}{\tan x + \cot x}$

$$\frac{1}{\sin x} \cdot \frac{\sin x \cos x}{\cos x + \sin x} = \frac{\cos x}{\sin^2 x + \cos^2 x} = \frac{\cos x}{1} = \cos x$$

③ $\frac{\tan x}{\csc x} + \frac{\sin x}{\tan x}$

$$\frac{\tan^2 x + \sin x \csc x}{\tan x \csc x} = \frac{\tan^2 x + 1}{\tan x \csc x} = \frac{\sec^2 x}{\tan x \csc x} = \frac{1 \cos x \sin x}{\cos^2 x \sin x} = \frac{1}{\cos x} = \sec x$$

④ $\frac{\sin^2 x}{\sec^2 x - 1}$

$$\frac{\sin^2 x}{\tan^2 x} \rightarrow \sin^2 x \cot^2 x = \frac{\sin^2 x \cos^2 x}{\sin^2 x} = \cos^2 x$$

⑤ $\frac{1}{\cot \theta} + \frac{1}{\tan \theta}$

$$\frac{\tan \theta + \cot \theta}{\cot \theta \tan \theta} = \tan \theta + \cot \theta$$

⑥ $\sec^2 x \csc^2 x - \sec^2 x - \csc^2 x + 1$

$$\sec^2 x (\csc^2 x - 1) - (\csc^2 x - 1) = (\csc^2 x - 1)(\sec^2 x - 1) = \cot^2 x \tan^2 x = \frac{\tan^2 x}{\tan^2 x} = 1$$

⑦ $\cot^4 x + 2\cot^2 x + 1$

$$u = \cot^2 x, \quad u^2 + 2u + 1 = (u+1)(u+1) = (\cot^2 x + 1)(\cot^2 x + 1) = \csc^2 x \csc^2 x = \csc^4 x$$

⑧ $\frac{\cos^2 x}{1 - \sin x}$

$$\frac{\cos^2 x (1 + \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{\cos^2 x (1 + \sin x)}{1 - \sin^2 x} = \frac{\cos^2 x (1 + \sin x)}{\cos^2 x} = 1 + \sin x$$

⑨ sub $x = 3 \cos \theta, \sqrt{9 - x^2}$

$$\sqrt{9 - (3 \cos \theta)^2} = \sqrt{9 - 9 \cos^2 \theta} = \sqrt{9(1 - \cos^2 \theta)} = \sqrt{9 \sin^2 \theta} = 3 \sin \theta$$

(10) $\frac{\sec x - \cos x}{\tan x} = \sin x$

$\frac{\sec x - \cos x}{\tan x} = \sin x$

$\frac{1}{\cos x \sin x} - \frac{\cos x \cos x}{\sin x} = \sin x$

$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} = \sin x$

$\frac{\sin^2 x}{\sin x} = \sin x$

$\sin x = \sin x$

(11) $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

$\frac{1}{\sin^2 x} - \frac{\cos x \cos x}{\sin x \sin x} = 1$

$\frac{1 - \cos^2 x}{\sin^2 x} = 1$

$\frac{\sin^2 x}{\sin^2 x} = 1$

$1 = 1$

(12) $\frac{1 + \tan x}{\sin x} - \sec x = \csc x$

$\frac{1}{\sin x} + \frac{\tan x}{\sin x} - \frac{1}{\cos x} = \csc x$

$\frac{1}{\sin x} + \frac{\sin x}{\cos x \sin x} - \frac{1}{\cos x} = \csc x$

$\frac{1}{\sin x} + \frac{1}{\cos x} - \frac{1}{\cos x} = \csc x$

$\frac{1}{\sin x} = \csc x$

$\csc x = \csc x$

(13) $\sin\left(\frac{\pi}{2} - x\right) \cos(-x) = \cos^2 x$

$\cos x \cos x = \cos^2 x$

$\cos^2 x = \cos^2 x$

(14) $\frac{\cos x}{1 - \sin^2 x} = \sec x$

$\frac{\cos x}{\cos^2 x} = \sec x$

$\frac{1}{\cos x} = \sec x$

$\sec x = \sec x$

(15) $1 + \frac{1}{\csc^2 x - 1} = \sec^2 x$

$\frac{\csc^2 x - 1 + 1}{\csc^2 x - 1} = \sec^2 x$

$\frac{\csc^2 x}{\csc^2 x - 1} = \sec^2 x$

$\frac{\csc^2 x}{\cot^2 x} = \sec^2 x$

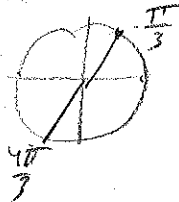
$\frac{1}{\sin^2 x} \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$

$\frac{1}{\cos^2 x} = \sec^2 x$

$\sec^2 x = \sec^2 x$

(16) $\tan 3t = \frac{\sqrt{3}}{2} \leftarrow \sin$
 $\frac{1}{2} \leftarrow \cos$

$3t = \frac{\pi}{3} \quad t = \frac{\pi}{9}$
 $3t = \frac{4\pi}{3} \quad t = \frac{4\pi}{9}$



(17) $\sec^2 x = \sec x + 2$

$\sec^2 x - \sec x - 2 = 0$

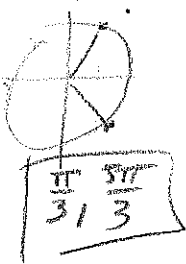
$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2 \quad \sec x = -1$

$\cos x = \frac{1}{2} \quad \cos x = 1$



(18) $\cot^2 x - \tan^2 x = 0$

$\frac{1}{\tan^2 x} - \tan^2 x = 0$

$\tan^2 x - \tan^4 x = 0$

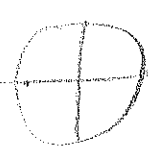
$\tan^4 x - \tan^2 x = 0$

$\tan^2 x (\tan^2 x - 1) = 0$

$\tan^2 x = 0 \quad \tan^2 x = 1$

$\sin^2 x = 0 \quad \tan x = 1$

$\sin x = 0 \quad \sin x = \cos x$



$0, \pi$

$\frac{\pi}{4}$

$0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

(19) $2\sin x \cos x + \cos x = 0$

$\cos x (2\sin x + 1) = 0$

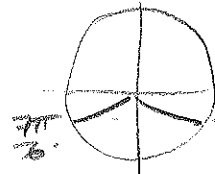
$\cos x = 0$



$2\sin x + 1 = 0$

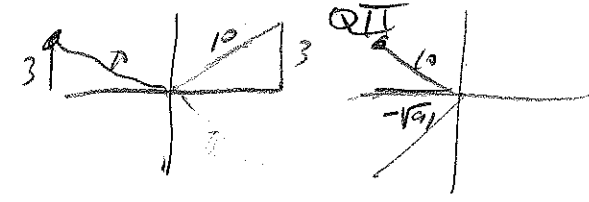
$2\sin x = -1$

$\sin x = -\frac{1}{2}$



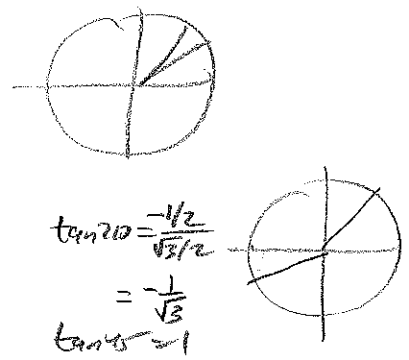
$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

20) $\sin x = \frac{2}{10}$, $\cos x = -\frac{\sqrt{91}}{10}$ find $\tan x$



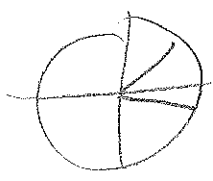
$$\tan x = \frac{\sin x}{\cos x} = \frac{2/10}{-\sqrt{91}/10} = \frac{-2}{\sqrt{91}} = \frac{-2\sqrt{91}}{91}$$

21) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$
 $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
 $= \frac{\sqrt{6} + \sqrt{2}}{4}$



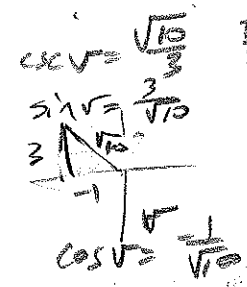
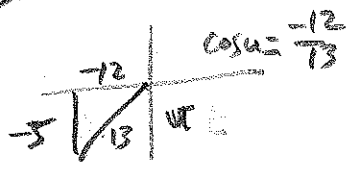
22) $\tan 165^\circ = \tan(210^\circ - 45^\circ) = \frac{\tan 210^\circ - \tan 45^\circ}{1 + \tan 210^\circ \tan 45^\circ}$
 $= \frac{-\frac{1}{\sqrt{3}} - 1}{1 + \frac{-1}{\sqrt{3}} \cdot 1} = \frac{-1 - \sqrt{3}}{\sqrt{3} - 1}$

23) $\cos 285^\circ = \cos(330^\circ - 45^\circ) = \cos 330^\circ \cos 45^\circ + \sin 330^\circ \sin 45^\circ$
 $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$



24) $\sin 8x \cos 2x + \cos 8x \sin 2x = \sin(8x + 2x) = \sin 10x$

25) $\sin u = \frac{-5}{13}$, $0 < u < \frac{3\pi}{2}$



$\frac{\pi}{2} < v < \pi$ find $\cos(u-v)$
 $3^2 + x^2 = 10$
 $9 + x^2 = 10$
 $x^2 = 1$
 $x = 1$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v = \left(\frac{-12}{13}\right)\left(\frac{-1}{\sqrt{10}}\right) + \left(\frac{-5}{13}\right)\left(\frac{3}{\sqrt{10}}\right) = \frac{12-8}{13\sqrt{10}} = \frac{4\sqrt{10}}{130} = \frac{2\sqrt{10}}{65}$$

26) $\cos 2x + \sin x = 0$
 $1 - 2\sin^2 x + \sin x = 0$
 $-2\sin^2 x + \sin x + 1 = 0$
 $2\sin^2 x - \sin x - 1 = 0$
 $2u^2 - u - 1 = 0$
 $(2u-2)(u+1) = 0$
 $(u-1)(u+1) = 0$
 $(\sin x - 1)(2\sin x + 1) = 0$

