HAlg3-4, 2.6 Notes – Rational Functions and Asymptotes

A **Rational Function** is a function in the form of a ratio of polynomials:

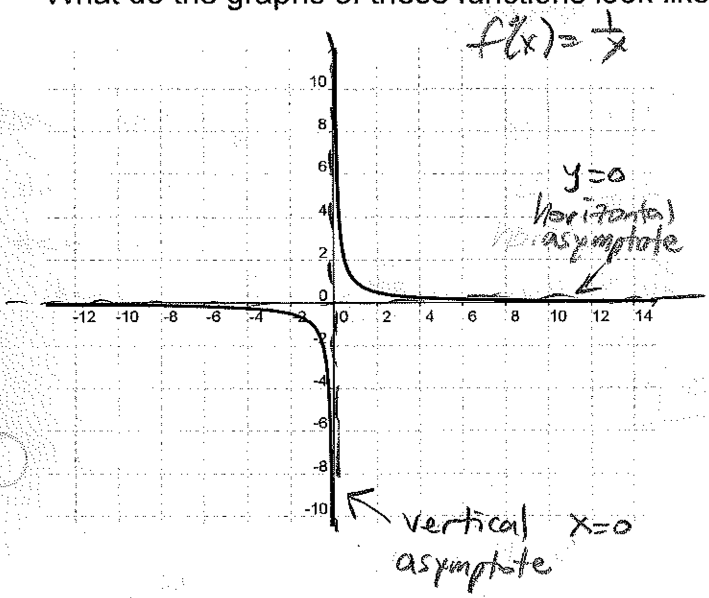
$$f(x) = \frac{N(x)}{D(x)}$$ example: $f(x) = \frac{3x^2-2}{4x^3-x^2+2x-1}$ or $f(x) = \frac{1}{x}$

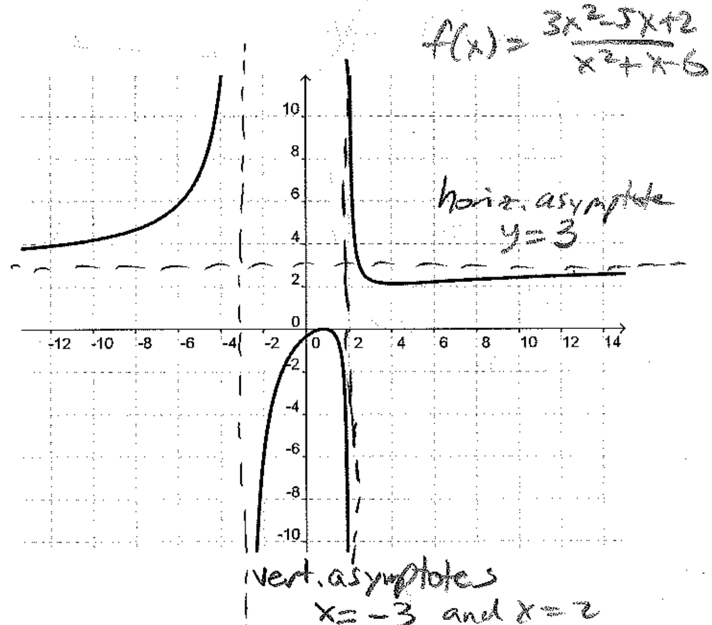
domain of a rational function = all real numbers except where denominator is zero

Find domain of: $f(x) = \frac{1}{x}$ $\mathbb{R}, x \neq 0$ $f(x) = \frac{3x^2-5x+2}{x^2+x-6}$ $\mathbb{R}, x \neq -3, x \neq 2$

$(x+3)(x-2)$

What do the graphs of these functions look like?





Asymptotes of a Rational Function: (book way to find asymptotes)

If $f(x) = \frac{N(x)}{D(x)}$

$$f(x) = \frac{a_n x^{n} + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}{b_m x^{m} + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0}$$ (degree n) (degree m)

Vertical asymptotes: graph of f has vertical asymptotes at the zeros of denominator $D(x)$

Horizontal asymptotes: graph of f has, at most, one horizontal asymptote.

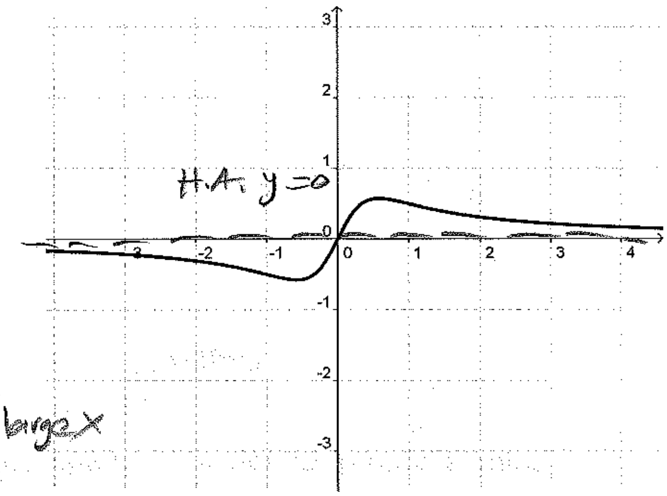
- If n < m, the line $y = 0$ (x-axis) is a horizontal asymptote.
- If n = m, the line $y = \frac{a_n}{b_m}$ is a horizontal asymptote.
- If n > m, the graph has no horizontal asymptote but may have a slant asymptote.

(another way to find horizontal asymptote)

Think about what happens when x is very large. The non-x terms become negligible and can be removed. Then simplify the result.

Examples: Find the asymptotes of...

$$f(x) = \frac{2x}{3x^2+1}$$

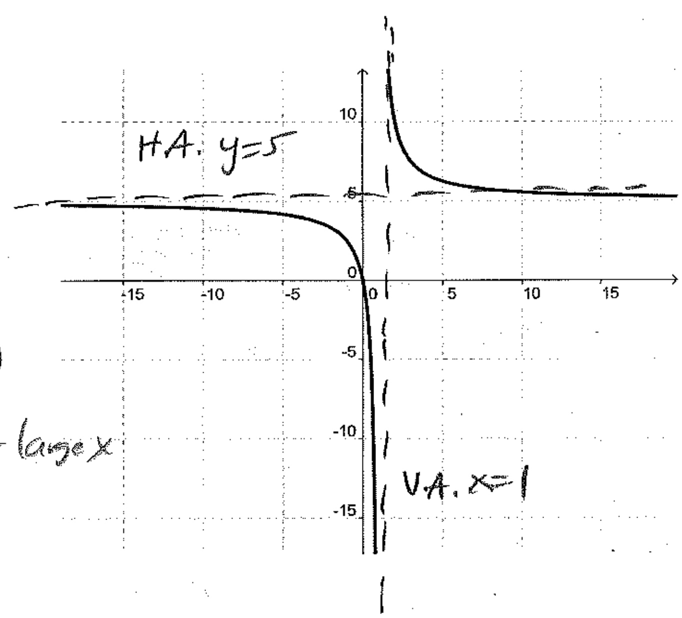


vertical: when denom $3x^2+1=0$
no x makes denom zero
so $\boxed{\text{no vertical asymptote}}$

horiz. (book way)
$n=1,\ m=2$
$n<m$, so
horiz asymptote: $\boxed{y=0}$

horiz (other way)
$\frac{2x}{3x^2+1} \approx \frac{2x}{3x^2}$ for large x
$\approx \frac{2}{3x}$
≈ 0
so $\boxed{y=0}$

$$f(x) = \frac{5x}{x-1}$$



vertical when $x-1=0$
$\boxed{x=1}$

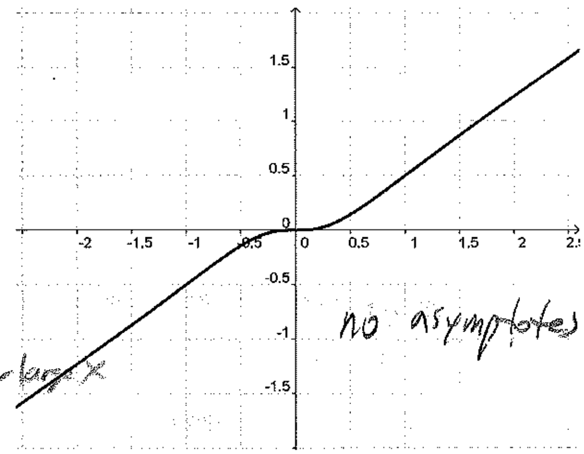
horiz. (book way)
$n=1,\ m=1$
$n=m$ so
$\frac{5x}{1x-1}$
horiz asympt: $y=\frac{5}{1}$
$\boxed{y=5}$

horiz. (other way)
$\frac{5x}{x-1} \approx \frac{5x}{x}$ for large x
≈ 5
$\boxed{y=5}$

$$f(x) = \frac{2x^3}{3x^2+1}$$

Vertical: $3x^2+1=0$

$\boxed{\text{none}}$



no asymptotes

horiz. (book way)

$n=3,\ m=2$

$n>m$, $\boxed{\text{no horiz. asymp.}}$

horiz. (other way)

$\frac{2x^3}{3x^2+1} \approx \frac{2x^3}{3x^2}$ for large x

$\approx \frac{2x}{3}$

$\approx \infty$

does not approach a number,

$\boxed{\text{no horiz asymptote}}$

Applications – many real-world problems exhibit 'asymptotic behavior' (approach a value)

A business has a cost function $C = 0.5x + 5000$ where C is cost in dollars and x is number of units produced.

- What is the average cost per unit when the number of units is 1000, and 10,000.
- What is the average cost per unit when a very large number of units is produced?

avg cost per unit $= A(x) = \frac{C(x)}{x} = \frac{0.5x+5000}{x}$

0
15000
0
10

vertical: when $\boxed{x=0}$

horiz. $n=1,\ m=1$

so horiz. asymptote at $\frac{0.5x+5000}{1x}$

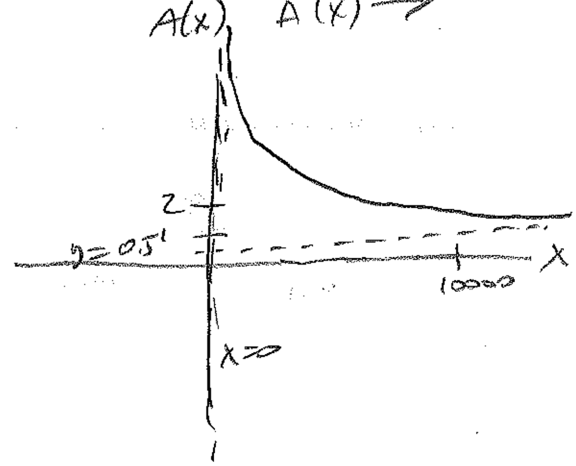
$y = \frac{0.5}{1}$

$\boxed{y=0.5}$

$A(1000)=5$

$A(10000)=1$

$A(x) \to 0.5$



HAlg3-4, 2.7 Notes – Graphs of Rational Functions

Finding slant asymptotes:

Example: Find horizontal asymptote of $f(x)=\dfrac{x^2-x}{x+1}$ $\quad n=2$, $m=1$.

$n>m$, no horiz. asymptote —or— for large x: $\approx \dfrac{x^2-x}{x}$
$\approx x-1$
$\approx \infty$

Slant asymptotes exist when degree of numerator is exactly one greater than degree of denominator. **Find equation of line of slant asymptote by dividing the polynomials.** The quotient (without remainder) is the equation of the slant asymptote.

$$\begin{array}{r} x-2 \\ x+1 \overline{)\, x^2-x+0} \\ \underline{x^2+x} \\ -2x+0 \\ \underline{-2x-2} \\ 2 \end{array}$$

or by synthetic division:

$$\begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & & -1 & 2 \\ \hline & 1 & -2 & 2 \end{array}$$

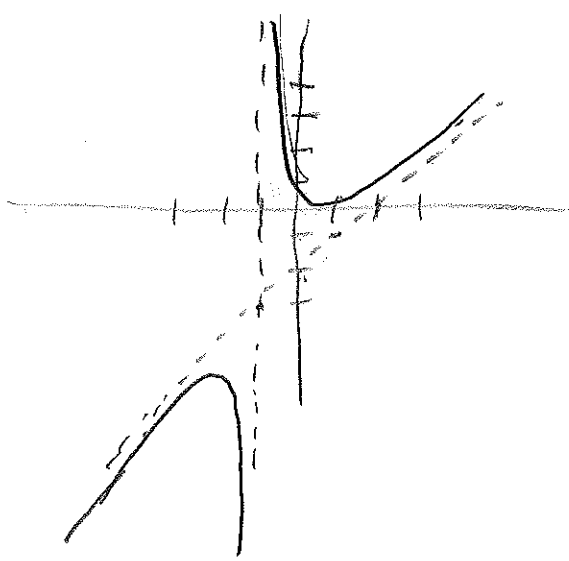
$x-2 \quad R\ 2$

$y=x-2$

$\underbrace{x-2}$ $+\dfrac{2}{x+1}$

slant asymptote:
$y=x-2$

vertical asymptote where denominator $=0$: $x=-1$



Sketching rational functions

1) Find f(0) (plug in 0 for x)…this gives y-intercept (if any).

2) Find zeros of numerator polynomial…this gives x-intercepts (if any).

3) Find zeros of denominator polynomial…this gives vertical asymptotes (if any).

4) Use entire rational function to find horizontal or ~~slope~~ slant asymptotes (if any).

5) Plot above on graph and find at least one point in each 'region'.

6) Finish sketch with smooth curve.

Example: sketch $f(x) = \dfrac{x}{x^2 - x - 2}$

1) $f(0) = \dfrac{0}{0^2 - 0 - 2} = \dfrac{0}{-2} = 0$ y-int: $(0,0)$

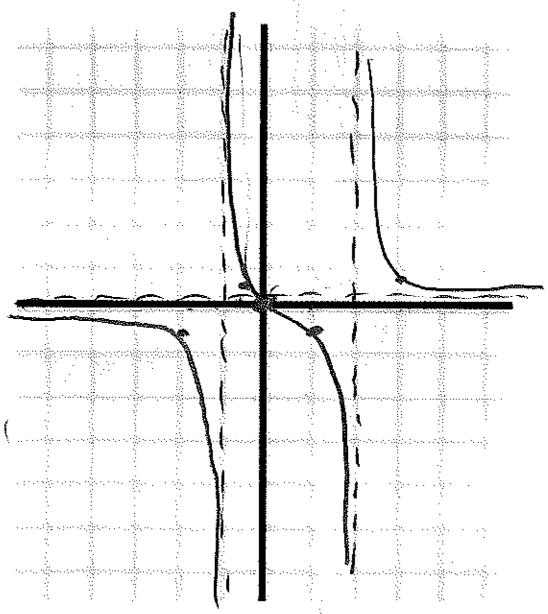
2) $x = 0$ x-int: $(0,0)$

3) $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

vertical asymptotes at: $x = 2, x = -1$

$\dfrac{x}{x^2 - x} = \dfrac{1}{x-1}$

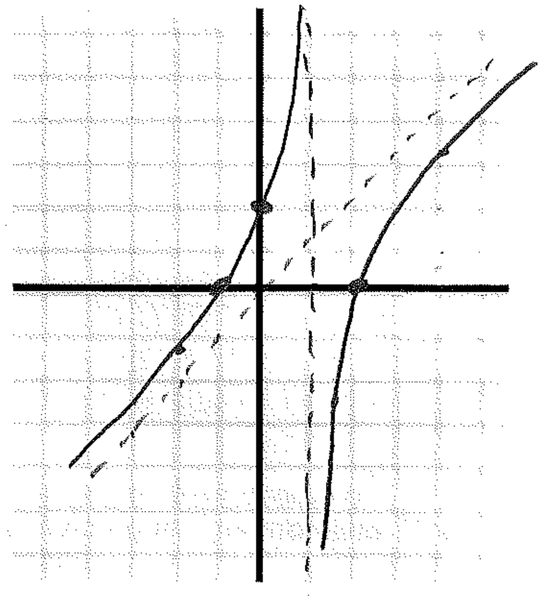


4) $n = 1$, $m = 2$ $\quad n < m$, horiz. asymptote $y = 0$

5)

x	$f(x)$
-2	$\dfrac{-2}{4+2-2} = \dfrac{-2}{4} = -\dfrac{1}{2}$ $\left(-2, -\dfrac{1}{2}\right)$
$-\dfrac{1}{2}$	$\dfrac{-\frac{1}{2}}{\frac{1}{4}+\frac{1}{2}-2} = \dfrac{-\frac{1}{2}}{\frac{1}{4}+\frac{2}{4}-\frac{8}{4}} = \dfrac{-\frac{1}{2}}{-\frac{5}{4}} = -\dfrac{1}{2}\left(-\dfrac{4}{5}\right) = \dfrac{2}{5}$ $\left(-\dfrac{1}{2}, \dfrac{2}{5}\right)$
1	$\dfrac{1}{1-1-2} = \dfrac{1}{-2} = -\dfrac{1}{2}$ $\left(1, -\dfrac{1}{2}\right)$
3	$\dfrac{3}{9-3-2} = \dfrac{3}{4}$ $\left(3, \dfrac{3}{4}\right)$

Example: sketch $f(x)=\dfrac{x^2-x-2}{x-1}$



1) $f(0)=\dfrac{0^2-0-2}{0-1}=\dfrac{-2}{-1}=2$ y-int $(0,2)$

2) $x^2-x-2=0$ x-int: $(2,0)$ and $(-1,0)$
$(x-2)(x+1)=0$

3) $x-1=0$
$x=1$ vertical asymptote

4) horiz. asymptote: for large $x \approx \dfrac{x^2-x}{x}$
$\approx x-1$
≈ 0 no horiz.

but n one greater than m, so slant asymptote:

$$\begin{array}{r} x \\ x-1 \overline{)\, x^2-x-2} \\ \underline{x^2-x} \\ 0\ -2 \end{array}$$

$\underbrace{x}\ -\dfrac{2}{x-1}$

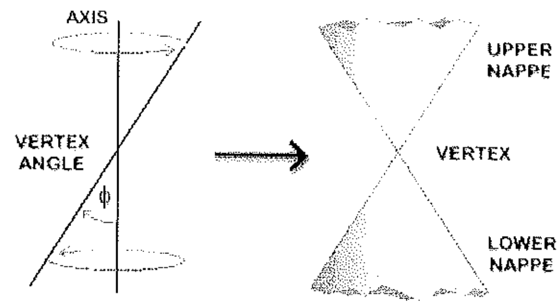
$y=x$ slant asymptote

5)

x	$f(x)$
-2	$\dfrac{4+2-2}{-2-1}=\dfrac{4}{-3}=-\dfrac{4}{3}$ $\left(-2,-\dfrac{4}{3}\right)$
4	$\dfrac{16-4-2}{4-1}=\dfrac{10}{3}=$ $\left(4,\dfrac{10}{3}\right)$
$\frac{3}{2}$	$\dfrac{\frac{9}{4}-\frac{3}{2}-2}{\frac{3}{2}-1}=\dfrac{\frac{1}{4}-\frac{6}{4}-\frac{8}{4}}{\frac{3}{2}-\frac{2}{2}}=\dfrac{-\frac{5}{4}}{\frac{1}{2}}=\left(-\frac{5}{4}\right)(2)=-\frac{5}{2}$ $\left(\frac{3}{2},-\frac{5}{2}\right)$

HAlg3-4, 10.1 Notes – Conic Sections and Parabolas

<u>Conic Sections</u> – curves formed by intersection of a double-napped cone with a plane.



The shape of the intersection curve depends upon the angle the plane makes with the vertical axis of the cone:

Circle - if the plane is perpendicular to the cone axis, the curve is a circle.

CIRCLE

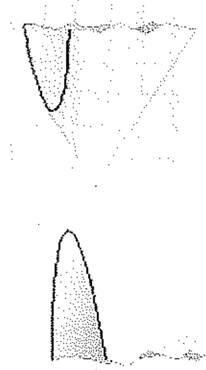
Ellipse – if the plane's angle is greater than the vertex angle, but not perpendicular to cone axis, the curve is an ellipse.

ELLIPSE

Parabola – if the plane's angle matches the vertex angle, the curve is a parabola.

PARABOLA

Hyperbola – if the plane's angle is smaller than the vertex angle, the curve cuts through both nappes and is a hyperbola.

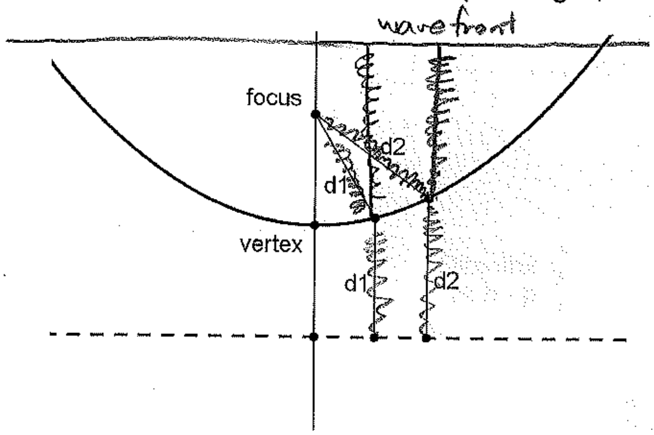


HYPERBOLA

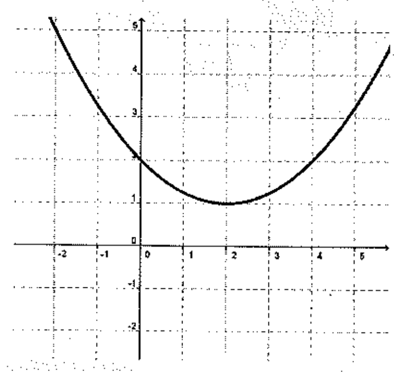
Conic Sections are defined by the general equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
...but each curve has a standard form that makes it easier to sketch.

Parabola = The set of all points (x,y) that are equidistant from a fixed line (called the directrix) and a fixed point (called the focus).

Applications – reflectors (flashlight, antenna dishes)



Let's look at an example: $y = \frac{1}{4}x^2 - x + 2$



How could we graph this by hand? Easier to graph if we change the equation into the standard form equation for a parabola:

$y = \frac{1}{4}x^2 - x + 2$

$\frac{1}{4}x^2 - x = y - 2$ (x terms on left, everything else on rt)

$x^2 - 4x = 4y - 8$ (make x^2 coefficient 1)

$x^2 - 4x + \underline{4} = 4y - 8 + \underline{4}$ (complete the square)

$(x-2)^2 = 4y - 4$

$(x-2)^2 = 4(y-1)$ (factor right side)

vertex: $(2,1)$

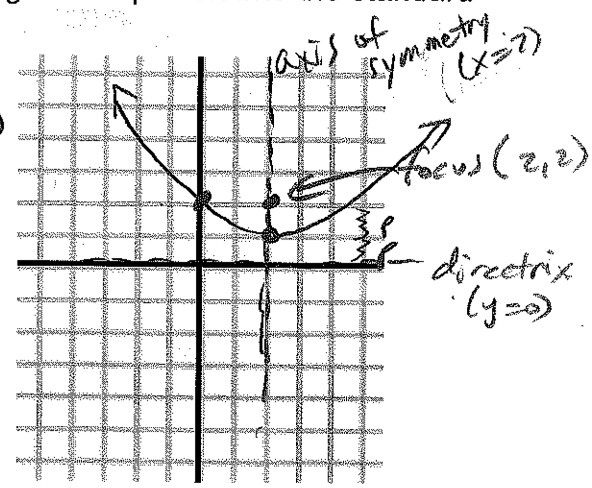
To find another point find an intercept

y-int $(x=0)$ $(0-2)^2 = 4(y-1)$

$(-2)^2 = 4(y-1)$

$4 = 4(y-1)$

$1 = y-1$ $y=2$ $(0,2)$



$(x-2)^2 = (4)(y-1)$

this number is $4p$

Standard form equation of a parabola:

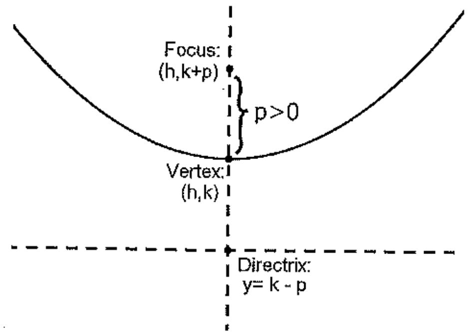
Vertical axis:

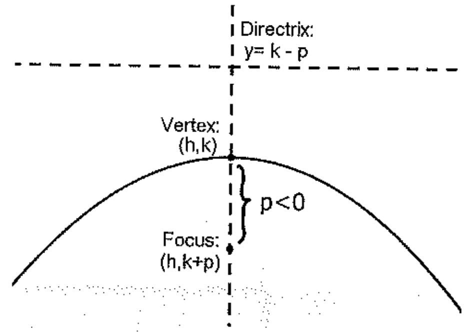
$(x-h)^2 = 4p(y-k)$

Vertex: (h,k)

Focus: $(h,k+p)$

Directrix: $y=k-p$





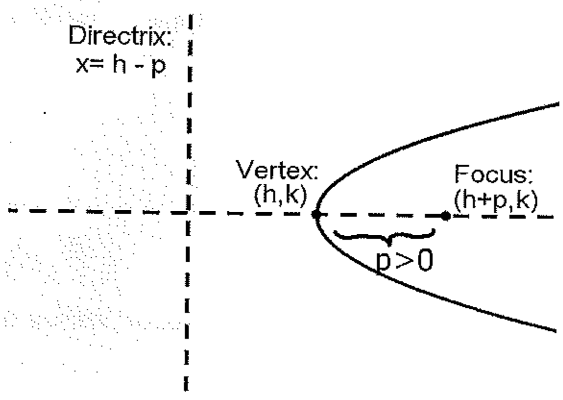
Horizontal axis:

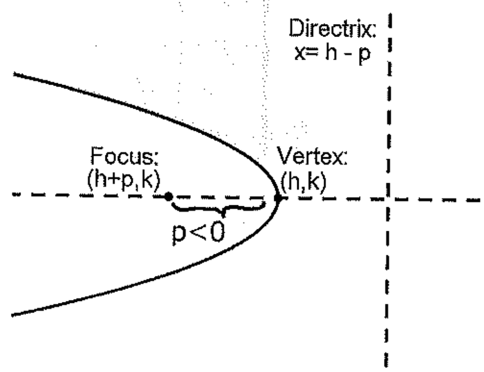
$(y-k)^2 = 4p(x-h)$

Vertex: (h,k)

Focus: $(h+p,k)$

Directrix: $x=h-p$





For all parabolas: p = distance between focus and vertex (and vertex to directrix)

To sketch, plug in x=0 or y=0 to find an intercept.
(Curve is symmetric about its axis)

(parabolas don't have asymptotes)

Example: Find the vertex, focus, directrix and sketch the graph of $(x+1)^2 = 4(y-2)$

$(x+1)^2 = 4(y-2)$

$(x-h)^2 = 4p(y-k)$

$h = -1$

$k = 2$

$4p = 4, \quad p = 1$

vertex: (h,k) $(-1,2)$

focus: $(h, k+p)$ $(-1,3)$

directrix: $y = k-p, \quad y = 1$

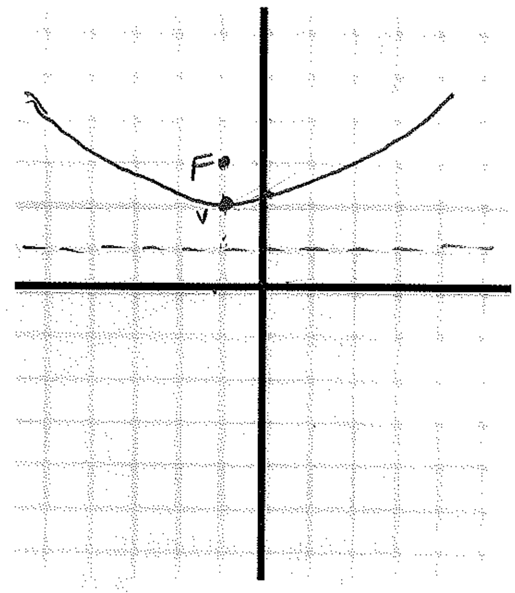
$(0+1)^2 = 4(y-2)$

$1 = 4y - 8$

$+8 \quad +8$

$9 = 4y$

$\frac{9}{4} = y \quad (0, \frac{9}{4})$



Example: Find the vertex, focus, directrix and sketch the graph of $y^2 = -2x$

$y^2 = -2x$

$(y-0)^2 = -2(x-0)$

$(y-k)^2 = 4p(x-h)$

$h = 0$

$k = 0$

$4p = -2, \quad p = -\frac{1}{2}$

vertex $(h,k) = (0,0)$

focus $(h+p, k) = (-\frac{1}{2}, 0)$

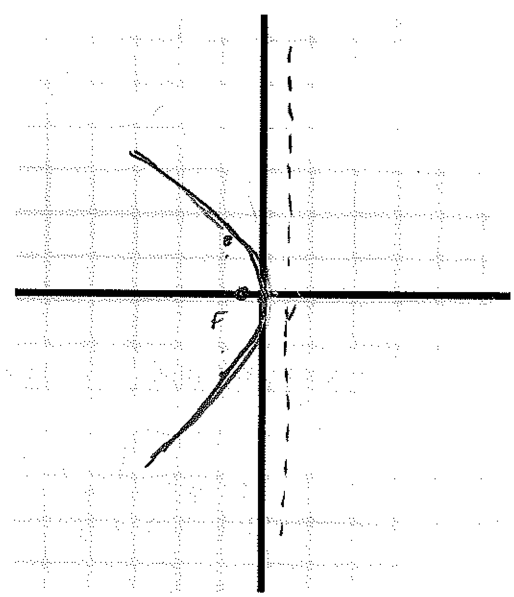
directrix: $x = h-p, \quad x = \frac{1}{2}$

plug in $x = -1$

$y^2 = -2(-1)$

$y^2 = 2$

$y = \pm\sqrt{2} \approx \pm 1.414$



Example: Find the vertex, focus, directrix and sketch the graph of $x^2-2x+8y+9=0$

$x^2-2x = -8y-9$

complete the square:

$(x^2-2x+1)-1 = -8y-9$

$(x-1)^2-1 = -8y-9$

$\quad +1 \qquad\quad +1$

$(x-1)^2 = -8y-8$

$(x-1)^2 = -8(y+1)$

$h=1$

$k=-1$

$4p=-8,\ p=-2$

vertex: (h,k) $(1,-1)$

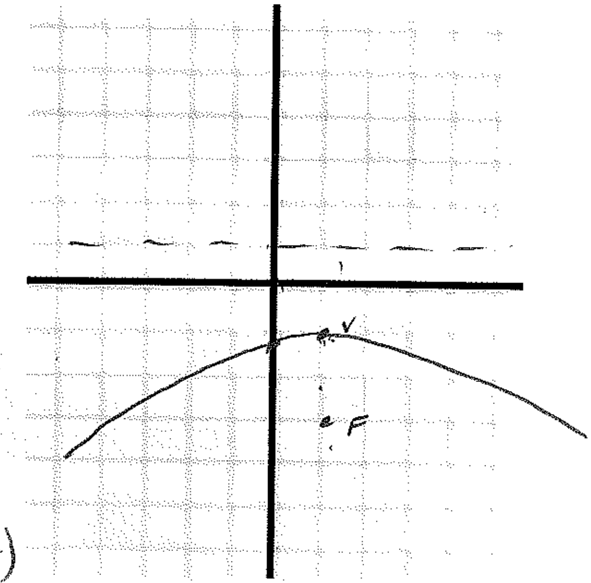
focus: $(h, k+p)$ $(1,-3)$

directrix: $y=k-p,\ y=1$

$(-1)^2 = -8y-8$

$1=-8y-8$

$9=-8y \quad y=-\frac{9}{8} \quad (0,-\frac{9}{8})$



Example: Find standard form equation of a parabola with vertex at (-1, 2) and focus at (-1, 0)

h k

h, k+p

$(x-h)^2 = 4p(y-k)$

$(x+1)^2 = 4p(y-2)$

$(x+1)^2 = 4(-2)(y-2)$

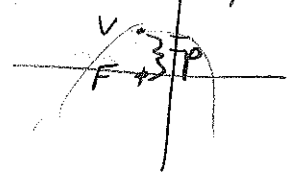
$\boxed{(x+1)^2 = -8(y-2)}$

$k=2$

$k+p=0$

$2+p=0$

$p=-2$



points down, $p<0$

Example: Find standard form equation of a parabola with vertex at (-2, 1) with directrix x=1

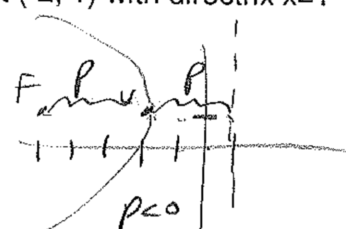
h k

$(y-k)^2 = 4p(x-h)$

$(y-1)^2 = 4p(x+2)$

$(y-1)^2 = 4(-3)(x+2)$

$\boxed{(y-1)^2 = -12(x+2)}$



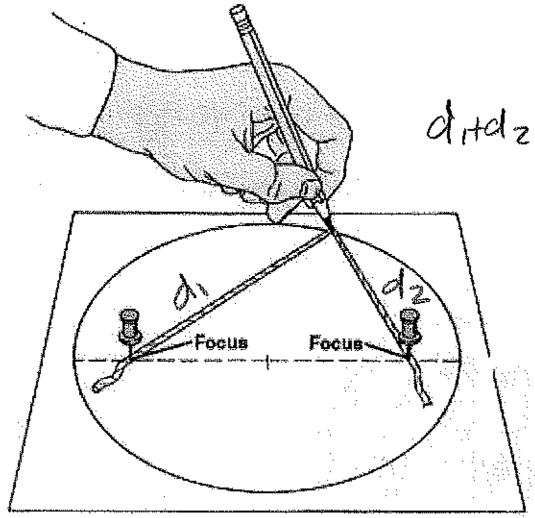
$p<0$

$p=-3$

points left, $p<0$

HAlg3-4, 10.2 Notes – Ellipses

Ellipse = the set of all points (x,y) the sum of whose distances from two fixed points (called foci) is constant.



$d_1 + d_2 = \text{constant}$

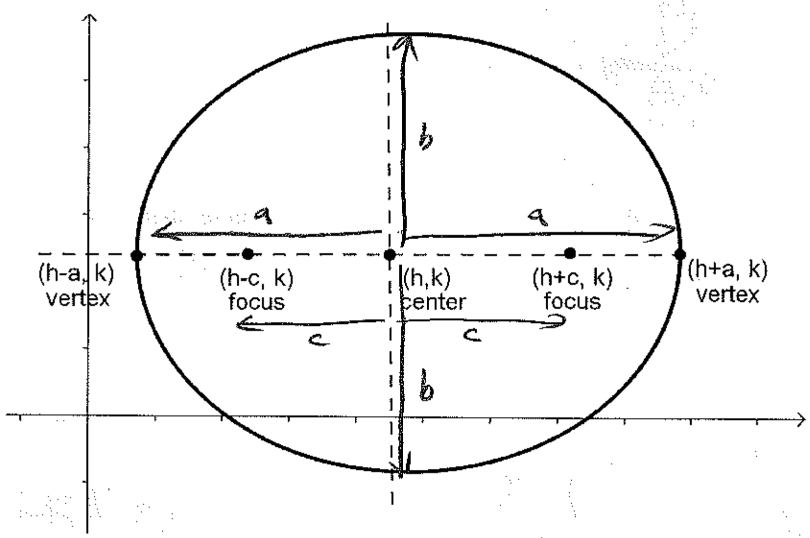
real-life:
- orbits of planets, comets
- circles in perspective
- tanker truck cross section (lower center of gravity)

Standard form equation of an ellipse

Horizontal major axis
(bigger number under x^2 term)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

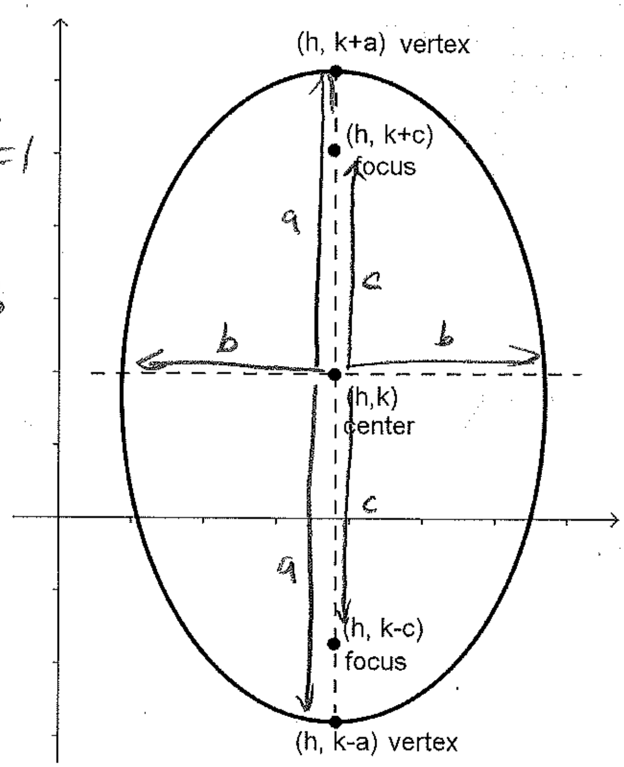
ex: $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$



Vertical major axis
(bigger number under y^2 term)

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

ex: $\frac{(x-1)^2}{4}+\frac{(y-2)^2}{9}=1$



For all ellipses: a is always bigger than b

(h,k) = center

a = distance from center to a vertex on major (longer) axis

b = distance from center to a point on minor (shorter) axis

c = distance from center to a focus

$c^2 = a^2 - b^2$

Since $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$ 'oval-ness' $0 < e < 1$ $e = 0$ when $c = 0$ when $a = b$

($a = b$ ellipse is a circle)

Examples...

#1 Find the center, vertices, foci and eccentricity and sketch: $\frac{(x+3)^2}{16} + \frac{(y-5)^2}{25} = 1$

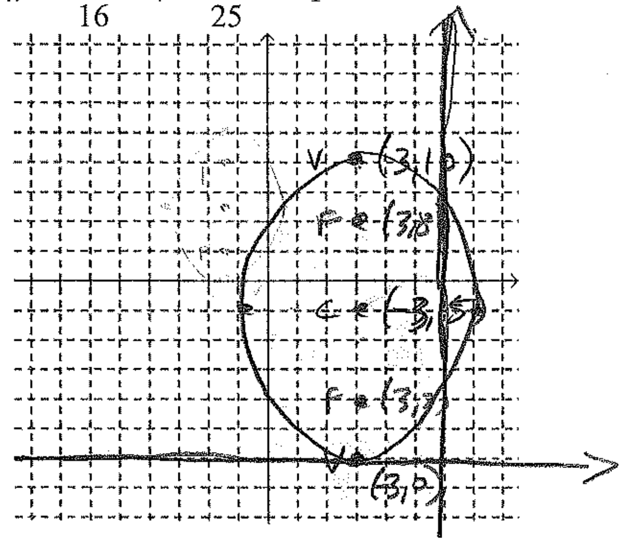
bigger # under y, vertical major axis

$a = \sqrt{25} = 5$

$b = \sqrt{16} = 4$

$c^2 = a^2 - b^2 = 25 - 16 = 9$

$c = 3$



Center: $(h,k): (-3,5)$

Vertices: $(h, k \pm a): (-3,10), (-3,0)$

Foci: $(h, k \pm c): (-3,8), (-3,2)$

Eccentricity: $e = \frac{c}{a} = \boxed{\frac{3}{5}}$

#2 Find the center, vertices, foci and eccentricity and sketch: $x^2 + 4y^2 + 6x - 8y + 9 = 0$

horiz. major axis

$(x^2 + 6x \quad) + (4y^2 - 8y \quad) = -9$

$a = \sqrt{4} = 2$

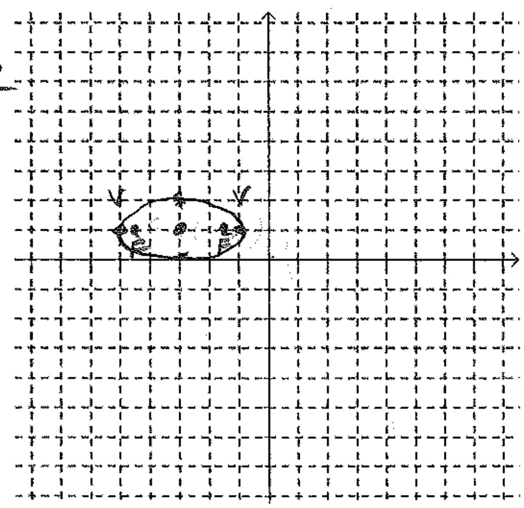
$(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = -9 + 9 + 4$

$b = 1$

$c^2 = 4 - 1 = 3 \quad c = \sqrt{3}$

$\frac{(x+3)^2}{4} + \frac{4(y-1)^2}{4} = \frac{4}{4}$

$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$



Center: $(h,k): (-3,1)$

Vertices: $(h \pm a, k): (-1,1), (-5,1)$

Foci: $(h \pm c, k): (-3+\sqrt{3}, 1)\ (-3-\sqrt{3}, 1)$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{3}}{2}$

Try it... Find the center, vertices, foci and eccentricity and sketch: $9x^2 + 4y^2 - 36x - 24y + 36 = 0$

vertical major axis

$(9x^2 - 36x) + (4y^2 - 24y) = -36$

$a = 3$

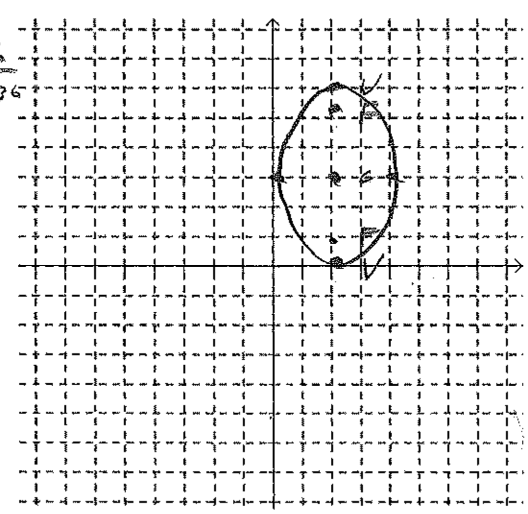
$9(x^2 - 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$

$b = 2$

$c^2 = 9 - 4 = 5 \quad c = \sqrt{5}$

$\frac{9(x-2)^2}{36} + \frac{4(y-3)^2}{36} = \frac{36}{36}$

$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$



Center: $(2,3)$

Vertices: $(2,6), (2,0)$

Foci: $(2, 3+\sqrt{5})\ (2, 3-\sqrt{5})$

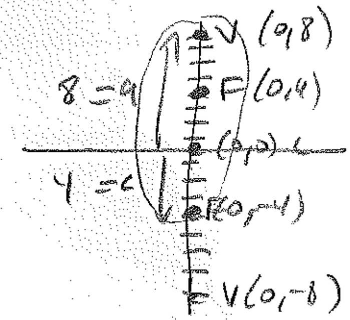
Eccentricity: $e = \frac{c}{a} = \boxed{\frac{\sqrt{5}}{3}}$

#3 Find the standard form of the equation of the ellipse if:

Vertices: (0,8) and (0,-8)
Foci: (0,4) and (0,-4)

$$\frac{(x-0)^2}{48} + \frac{(y-0)^2}{64} = 1$$

$$\boxed{\frac{x^2}{48} + \frac{y^2}{64} = 1}$$



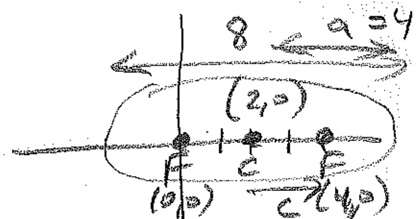
$c^2 = a^2 - b^2$
$16 = 64 - b^2$
$b^2 = 64 - 16 = 48$

#4 Find the standard form of the equation of the ellipse if:

Foci: (0,0) and (4,0)
Major axis of length 8

$$\frac{(x-2)^2}{16} + \frac{(y-0)^2}{12} = 1$$

$$\boxed{\frac{(x-2)^2}{16} + \frac{y^2}{12} = 1}$$

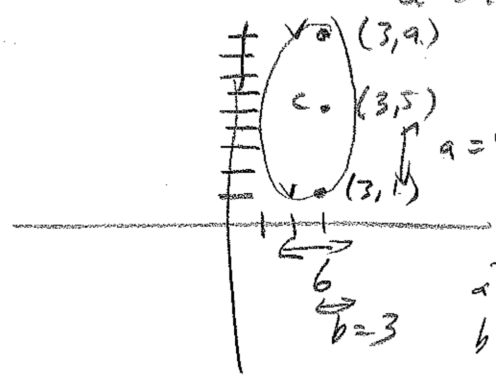


$c = 2$
$c^2 = a^2 - b^2$
$4 = 16 - b^2$
$b^2 = 16 - 4$
$b^2 = 12$
$a^2 = 16$

Try it... Find the standard form of the equation of the ellipse if:

Vertices: (3, 1) and (3, 9)
Minor axis of length 6

$$\boxed{\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1}$$



$a^2 = 16$
$b^2 = 9$

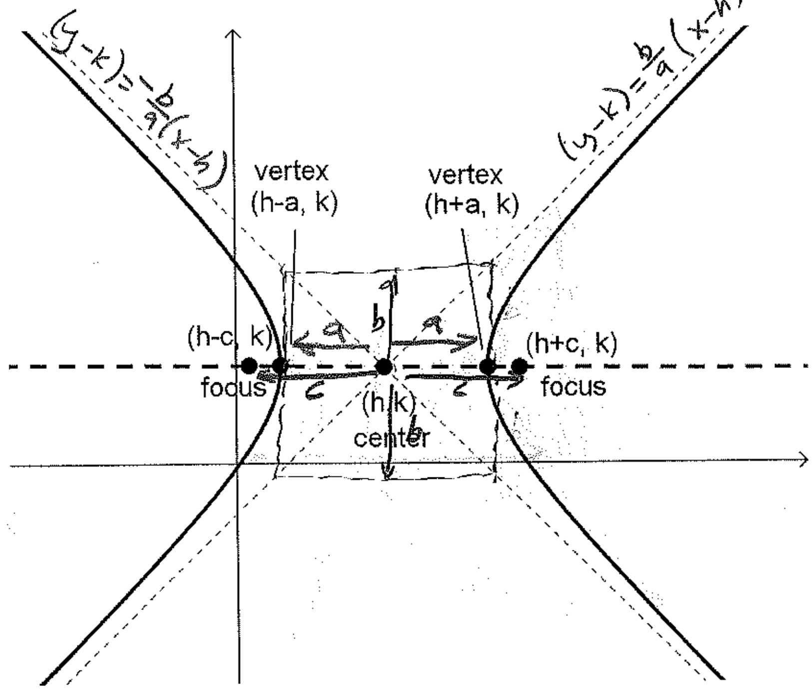
HAlg3-4, 10.3 Notes – Hyperbola

Hyperbola = the set of all points (x,y) the difference of whose distances from two fixed points (called foci) is a positive constant.

Standard form equation of a hyperbola (similar to ellipse, but – instead of + between terms)

Horizontal transverse axis (x^2 term is first)

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

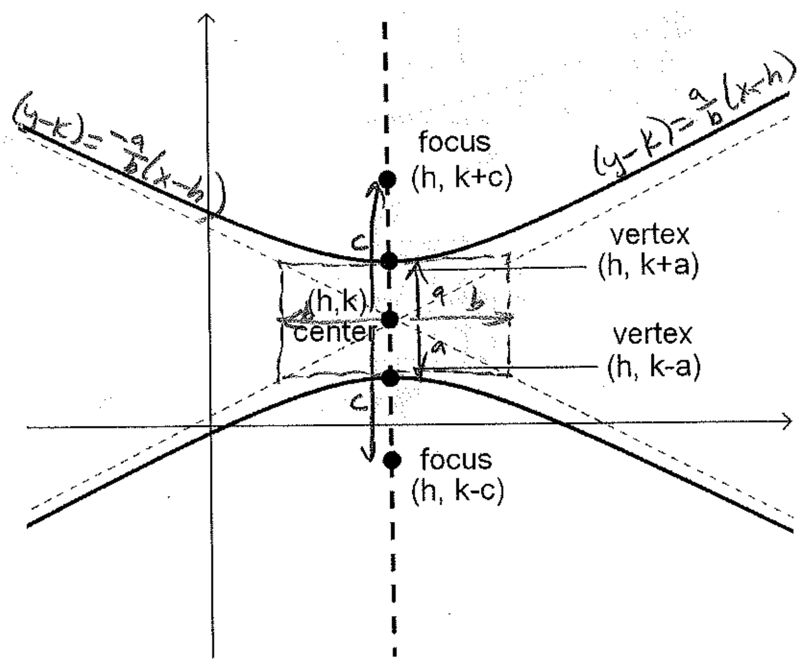


asymptotes at:

$$(y-k)=\pm\frac{b}{a}(x-h)$$

Vertical transverse axis (y^2 term is first)

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$



asymptotes at:

$$(y-k)=\pm\frac{a}{b}(x-h)$$

For all hyperbola:

(h,k) = center

a = distance from center to a vertex

c = distance from center to a focus.

$c^2=a^2+b^2$ therefore $c>a$

Eccentricity: $e = \frac{c}{a}$ like ellipse, but now $e > 1$ (higher hyperbola eccentricity = 'flatter' curves)

#1 Find the center, vertices, foci, asymptotes, eccentricity and sketch: $\frac{x^2}{9} - \frac{y^2}{25} = 1$

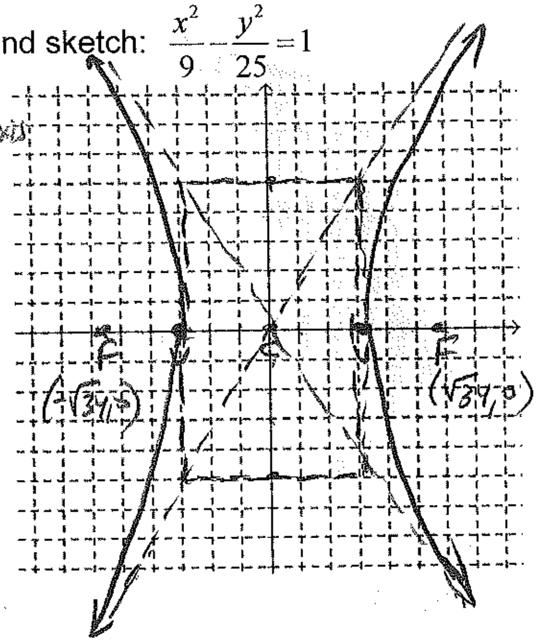
$\frac{x^2}{9} - \frac{y^2}{25} = 1$ x term 1st, horiz. transverse axis

$a^2 = 9, a = 3$ $\quad c^2 = a^2 + b^2$

$b^2 = 25, b = 5$ $\quad c^2 = 9 + 25 = 34$

$c = \sqrt{34}$

≈ 5.8



Center: (h,k) : $(0,0)$

Vertices: $(h \pm a, k)$: $(3,0)$ & $(-3,0)$

Foci: $(h \pm c, k)$; $(\sqrt{34},0)$ & $(-\sqrt{34},0)$

Asymptotes: $(y-k) = \pm\frac{b}{a}(x-h)$; $(y-0) = \pm\frac{5}{3}(x-0)$

$y = \pm\frac{5}{3}x$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$

#2 Find the center, vertices, foci, asymptotes, eccentricity and sketch:

$4y^2 - x^2 - 16y - 6x - 29 = 0$

$(4y^2 - 16y) + (-x^2 - 6x) = 29$

$4(y^2 - 4y + 4) - (x^2 + 6x + 9) = 29$

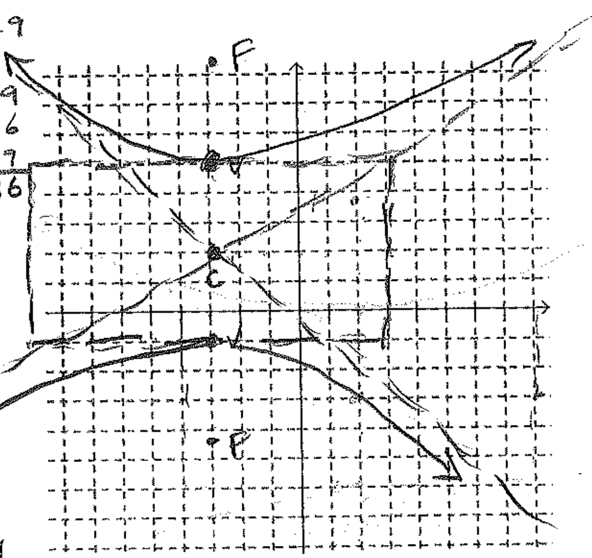
$+16$

-9

36

$\frac{4(y-2)^2}{36} - \frac{(x+3)^2}{36} = \frac{36}{36}$

$\frac{(y-2)^2}{9} - \frac{(x+3)^2}{36} = 1$



Center: $(-3, 2)$

Vertices: $(-3,5)$ & $(-3,-1)$

Foci: $(-3, 2+\sqrt{45})$, $(-3, 2-\sqrt{45})$

Asymptotes: $(y-2) = \pm\frac{1}{2}(x+3)$ $\quad a = 3$

$b = 6$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{45}}{3}$ $\quad c^2 = 9 + 36 = 45$

$c = \sqrt{45}$

vertical transverse axis

Try it... Find the center, vertices, foci, asymptotes, eccentricity and sketch:

$x^2 - 9y^2 + 36y - 72 = 0$

$x^2 + (-9y^2 + 36y) = 72$

$(x-0)^2 - 9(y^2 - 4y + 4) = 72 - 36$

$\frac{(x-0)^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$

$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$

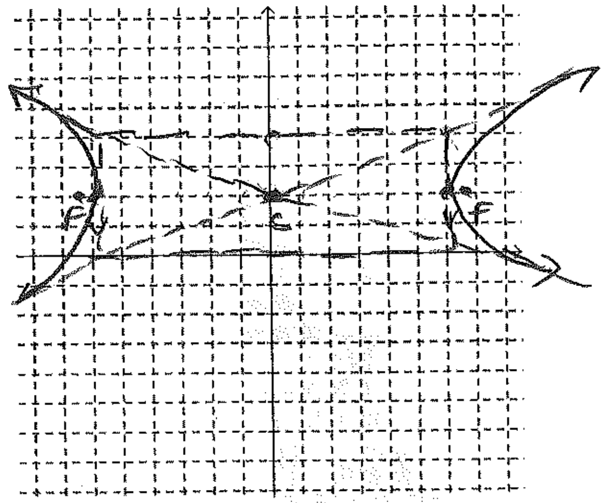
horiz. trans. axis

$a = 6$

$b = 2$

$c^2 = 40$

$c = \sqrt{40} \approx 6.3$



Center: $(0,2)$

Vertices: $(6,2)$ & $(-6,2)$

Foci: $(\sqrt{40},2)$ & $(-\sqrt{40},2)$

Asymptotes: $(y-2) = \pm\frac{1}{3}x$

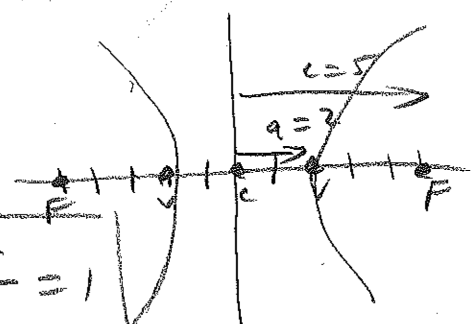
Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{40}}{6} = \frac{\sqrt{4}\sqrt{10}}{6} = \frac{2\sqrt{10}}{6} = \frac{\sqrt{10}}{3}$

#4 Find the standard form of the equation of the hyperbola if:

center $(0,0)$

Vertices: (2,0) and (-2,0)

Foci: (5,0) and (-5,0)



$c^2 = a^2 + b^2$

$25 = 4 + b^2$

$b^2 = 21$

$$\boxed{\frac{x^2}{4} - \frac{y^2}{21} = 1}$$

Try it... Find the standard form of the equation of the hyperbola if:

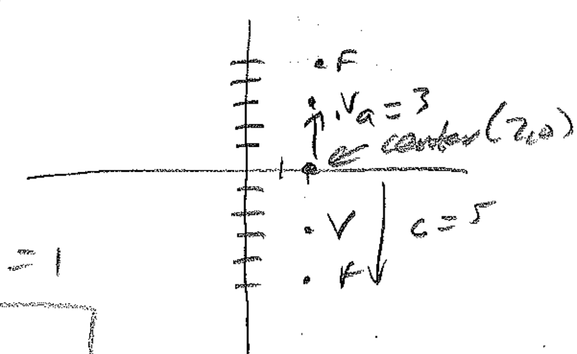
Foci: (2,5) and (2,-5)

Vertices: (2, 3) and (2, -3)

$c^2 = a^2 + b^2$

$25 = 9 + b^2$

$b^2 = 16$

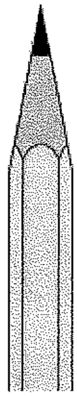


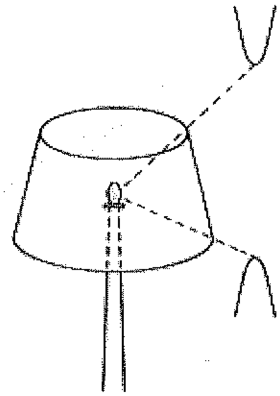
$\frac{(y-0)^2}{9} - \frac{(x-2)^2}{16} = 1$

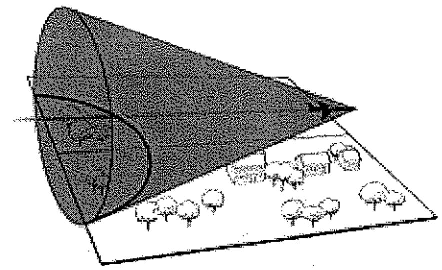
$$\boxed{\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1}$$

HAlg3-4, 10.3 day 2 Notes – Classifying Conic Sections by Equation

Where are hyperbolas found in the real-world? (where planes and cones intersect)







Classifying a conic section from the general equation:

$Ax^2 + Cy^2 + Dx + Ey + F = 0$ (no Bxy term — those are diagonal)

Look at the A and the C (the x^2 and the y^2 terms).... (different from book method)

Parabola	**1 squared term**
Circle	**2 squared terms, same sign, same coefficient**
Ellipse	**2 squared terms, same sign, different coefficients**
Hyperbola	**2 squared terms, opposite signs**

$4x^2 - y^2 - 4x - 3 = 0$ H

$4x^2 + 3y^2 + 8x - 24y + 51 = 0$ E

$4x^2 - 9x + y - 5 = 0$ P

$4x^2 - y^2 + 8x - 6y + 4 = 0$ H

$2x^2 + 4y^2 - 4x + 12y = 0$ E

$2x^2 + 2y^2 - 8x + 12y + 2 = 0$ C

$x^2 + y^2 - 6x + 4y + 9 = 0$ C

$x^2 + 4y^2 - 6x + 16y + 21 = 0$ E

$4x^2 - y^2 - 4x - 3 = 0$ H

$4y^2 - 2x^2 - 8x - 4y - 15 = 0$ H

$25x^2 - 10x - 200y - 119 = 0$ P

$4x^2 + 4y^2 - 16y + 15 = 0$ C

HAlg3-4, 10 review day – Comparing conics

Parabola	Ellipse	Hyperbola
$(x-h)^2 = 4p(y-k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$(y-k)^2 = 4p(x-h)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
x^2 like y= x^2 y^2 'other one'	a is always bigger, a under term of major axis	a not always bigger, a always under first term first term is transverse axis
p = dist. vertex to focus and dist. vertex to directrix	$c^2 = a^2 - b^2$	$c^2 = a^2 + b^2$

a = dist. center to vertex
c = dist. center to focus

	b = dist. center to point on minor axis	b = dist. to 'other side of box' asymptotes from center through corners of box: $(y-k) = \pm\frac{b}{a}(x-h)$ $(y-k) = \pm\frac{a}{b}(x-h)$ (look at box to see which)

eccentricity $e = \frac{c}{a}$

1st Semester Final Exam Formulas

Compound Interest:

$$A = P\left(1+\frac{r}{n}\right)^{nt} \qquad A = Pe^{rt}$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for } ax^2 + bx + c = 0$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$c^2 = a^2 - b^2$$

Hyperbola:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$