Combinatorics Worksheet – Simple Cases, Pascal's Triangle

Vame	Key

Period

1. For dinner, you must choose pizza (P) or chicken (C), and either milk (M) or water (W). Use a tree diagram, and list the outcome set (set of all possible dinners).

2. In a certain state the license plates are 3 numerical digits followed by one letter. How many license plates are possible?

3. Suppose you toss a coin twice. What are the possible outcomes? Draw a tree diagram to determine the outcome set.

4. You roll two dice. What are all the possible outcomes?

$$\frac{6}{114} \times \frac{6}{114} = \boxed{36}$$

5. If you roll two dice and add the numbers together, how many different sums are possible?

6. Alex has 6 pairs of pants, 10 shirts and 3 pairs of shoes that he wears for work. How many different combinations of a pair of pants, a shirt and a pair of shoes can Alex wear to work?

7. In how many ways can Jordan, Shelby, Sean, Nathan, and Steven stand in line?

8. Danielle, Connor, and Isaac ran in a race. In how many different orders can they finish the race?

9. From 6 students (4 boys and 2 girls) a teacher wishes to pick a boy and a girl. Find the number of possible outcomes.

10. If a store has three kinds of sweaters each in eight different colors, how many different sweaters can you buy?

11. Write the first ten rows of Pascal's Triangle:

1. How many four-person committees can be chosen from a group of seven people?

$$C_{4} = \frac{7!}{(4-4)! \cdot 4!} = \frac{7!}{3!4!} = \frac{7!}{3!4!} = \frac{7!}{3!2!} \cdot \frac{7!}{3!2!} = \frac{7!}{3!2!} =$$

2. Find the number of possible choices when you choose one item from each category: 3 cars, 5 colors

rs, 5 colors 
$$3.5 = 15$$

3. How many four-digit numbers can you make by arranging the numbers 1, 3, 7, and 9?

(4.) In how many ways can Jordan, Shelby, Sean, Nathan, and Steyen stand in line?

5. There are ten players on the basketball team. How many ways can a starting lineup of five players be chosen?

players be chosen? 
$$\frac{10.9 \cdot 9.017.6}{5.5} = \frac{10.9 \cdot 9.017.6}{5.5} = \frac{12.72}{5.5}$$

6. How many ways can a president and vicepresident be selected in a class of seventeen
students? Order matter  $P_2 = \frac{17!}{15!} = 17.66$ 



Danielle, Connor, and Isaac ran in a race. In how many different orders can they finish the race?

3 (0)

8. Find the number of possible choices when you choose one item from each category: 3 drinks, 4 vegetables, 2 desserts.

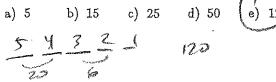
9. If there are three seats available on the bus and two people who want to sit down, how many different ways can those two people arrange themselves in the empty seats?

10. Ms. Crump always includes a few bonus questions on her tests. Since if you get them wrong they don't count against your score, you might as well try them. However, on the last test she said you were only allowed to answer two of the five bonus questions offered. How many ways could this happen?

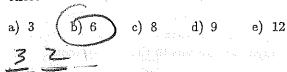
11. Andrew has six different kinds of cars to use on his model railroad. In how many different orders can he arrange the cars to be pulled by the engine?

12. It is student council election time again!
Your principle has asked you to vote for two representatives from your math class. Since your class is small (8 students) it should not be too hard to figure out how many ways two students could be selected in this process.

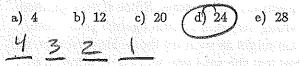
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	1	V
	e) 1	À
		(e) 1



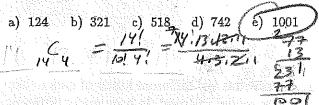
15. How many different two-digit numbers can be formed using each of the digits 3, 5, and 7 only once?



17. How many different four-digit numbers can be formed from using the digits 2, 5, 6, and 8, if each digit is used only once in each arrangement?



19. How many combinations of 4 CD's can be chosen from 14 CD's offered by a CD club?



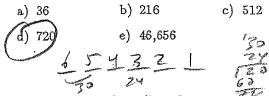
1.) From 6 students, 4 boys and 2 girls, a teacher wishes to pick a boy and a girl. Find the number of possible outcomes.

a) 4 b) 7. (c) 8 d) 9 e) 15  

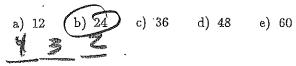
$$\frac{4}{2} = 7$$

New license plates for passenger cars in Massachusetts display three digits followed by three letters. What is the number of different plates possible?

14. How many ways are there to arrange 6 chairs in a row?



16. A box contains four slips of paper, each with one of the letters m, a, t, or h written on it. What is the number of three-letter outcomes that may be selected?



18. How many 4-digit numbers are possible using the digits 1, 2, 3, 4, 5, and 6, no digit used twice ( > 2 )

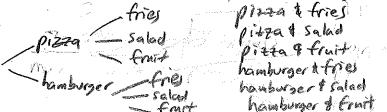
20. A book club offers your choice of 3 bestsellers for \$10. How many combinations of 3 bestsellers can you choose from among the 15 offered by the club?

a) 90 b) 365 c) 256 d) 
$$455$$
 e) 1025  
 $C_{15} = \frac{15!}{12!3!} = \frac{50.0071}{7.2.1}$ 

22. Seth is to select a center and guard for his basketball team from a group of 7 people. Find the number of possible outcomes.

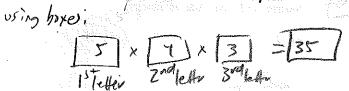
24. Authorization codes used by company consist four digits followed by two letters. How many different authorization codes are possible?

Suppose you're at a restaurant and you can choose between pizza and a hamburger as a main meal, 1 fries, salad, and fruit as a side dish. What are the possible meal combinations you could make?



You roll two dice. What are all the possible outcomes?

In how many ways can three letters be chosen from the letters A, B, C, D, and E? (ABC is different from ACB, etc.) < 61der matters = permutation



$$5^{2}_{3} = \frac{n!}{(n-7)!} = \frac{5!}{2!}$$

$$= 5.4.3 + \frac{35!}{24!}$$

4 Use Pascal's Triangle to evaluate

a 
$$_3C_2 = \boxed{3}$$

$$d \quad {}_{6}C_{2} = \sqrt{15}$$

. ", ,	
2 1 2 2 m 1 m 2	
331	
1 6 4 1	
15-10 13 5	
6 18 20 15 6	

A pizza shop offers seven toppings. How many different "four item" pizzas can be made?
$$\frac{7}{3} = \frac{7!}{3! \cdot 3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 4}{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 4} = \frac{7 \cdot 8 \cdot 5}{3 \cdot 2} = \frac{135}{3 \cdot 2}$$

Six of your favorite songs are not among the 17 chosen by a radio station. If you want to play three of your favorite songs, in how many ways can you choose three songs from the six?

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Combinations/Permutations

ermutations Per/Sec. \_\_\_\_\_ Date \_\_\_\_

1.) Each of two dice is marked with a different integer from 1 to 6 inclusive. The two dice are rolled and the numbers on the top face of each die are added. How many different sums are possible?

(a) 11 b) 12 c) 24 d) 34 e) 36

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13
3. How many possible outcomes are there if you roll 2 dice?

a) 12 (b) 36 c) 48 d) 60 e) 72

assuming order marters: 6 6 = 36

5. Eight boys and twelve girls attend a party. How 6. many different dancing pairs of one girl and one boy can be formed?

a) 4 b) 20 c) 40 d) 80 6) 96

7. A student has a choice of 4 math classes and 5 history classes. In how many ways can she choose one math and one history class?

a) 9 b) 10 c) 20 d) 24 e) 54

4 5 = 25

Worth Warfsey

9. Patrick picks a marble from a bag containing 8 differently colored marbles, then replaces the marble, and picks another. What is the number of possible outcomes?

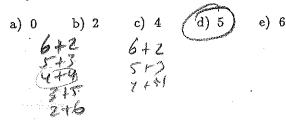
a) 56 (b) 64 c) 128 d) 256 e) 512  $\frac{8}{15}$  = 64

11. There are 5 candidates for senior class president, 3 candidates for vice president, and 4 candidates for secretary. How many different ways can the offices be filled?

a) 12 b) 30 c) 35 (d) 60 e) 75 order mattery fermulation.

5 · 3 · 4 = 60

Two dice are tossed. What is the number of ways 8 can appear from the throw of the dice?



Two coins are tossed, a penny and a dime. What is the number of possible outcomes?

a) 1 b) 2 c) 3 (d) 4 e) 5

6. Emily has 5 blouses and 3 pair of slacks. Find the number of possible outfits consisting of one blouse and one pair of slacks.

a) 2 b) 4 c) 8 d) 15 e)  $\frac{3}{5}$ how have

8. There are 5 candidates for governor and 6 candidates for controller. In how many ways can these offices be filled?

a) 4 b) 11 c) 18 d) 24 (e) 30  $\frac{5}{g^{2}}$ ,  $\frac{6}{6}$ ,  $\frac{32}{32}$ 

10. A room has 5 doors. In how many ways can Miguel enter the room and leave by any door?

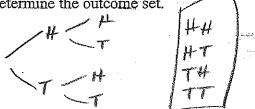
a) 5 b) 10 c) 15 d) 20 (e)

12. Alex has 6 pairs of pants, 10 shirts, and 3 pairs of shoes that he wears for work. How many different combinations of a pair of pants, a shirt, and a pair of shoes can Alex wear to work?

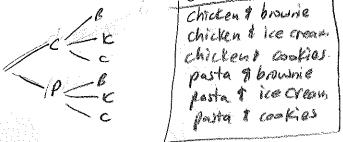
a) 140 b) 155 (5) 180 d) 200 e) 225  $\frac{6}{pan^{4}} \cdot \frac{1}{shirt} \cdot \frac{3}{shirt} = 180$ 

## I. Trees

Suppose you toss a coin twice. What are the possible outcomes? Draw a tree diagram to determine the outcome set.

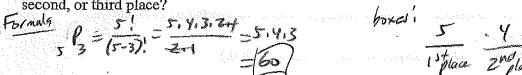


2 Suppose you are on an airplane, and you have a choice of chicken or pasta for the main meal and a choice of a brownie, ice cream, or cookies for dessert. What are all the possible meals you could select? Draw a tree diagram to determine the answer.



# II Permutations (No repetition, order matters)

1 Five horses are running a race. In how many different ways can these horses come in first, second, or third place?



2 Find the number of different ways to arrange four people in a line.

#### III Combinations

At Coldstone Creamery you can choose between nuts, Oreos, and sprinkles as your toppings.

How many different ways can you choose two of the toppings? Draw a tree diagram to
determine the answer. nuts to ores, nuts to pinkles ores that, (Crossed out diplicates)

We call choosing r items from a possible n items <u>combinations</u>, and the notation is  ${}_{n}C_{r}$ —"n choose r." In our last example we had to choose 2 topping from a total of 3 toppings. We would say " ${}_{3}C_{2}$ " or "3 choose 2."

We can always make a tree diagram for combinations, but sometimes it gets a little confusing, especially if we have many items to choose from. There is no repetition and order does <u>not</u> matter.

It can be easier to evaluate  ${}_{n}C_{r}$  by using Pascal's Triangle:

Hree: ores

yuts sprinkly

oreg sprinkly

sprinkly norts

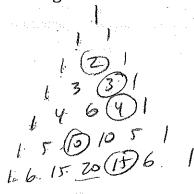
sprinkly norts

oregs

strinkly choice

choice

Pascal's Triangle



Let's refer back to our Coldstone Creamery example where we needed to evaluate  ${}_{3}C_{2}$ .

Count down to row 3.

Go over to column 2. (Don't forget to start counting with 0.)

The answer for  ${}_{3}C_{2}$  is  $\underline{3}$ 

Now try the following examples:

$$_4C_3 =$$

$$_{2}C_{1} = \underline{Z}$$

$$_5C_2 = 10$$

$$_{6}C_{4} = 15$$

2 Suppose that there is a club that has six members. How many ways can a committee of 3 members be formed?

Geometry	Name	
Combinatorics Worksheet – Simple Cases, Pascal's Triangle		Period
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11. Write the first ten rows of Pascal's Triangle:

Geometry
Counting Worksheet

Name
------

- 1. How many four-person committees can be chosen from a group of seven people?
- 8. Find the number of possible choices when you choose one item from each category: 3 drinks, 4 vegetables, 2 desserts.
- 2. Find the number of possible choices when you choose one item from each category: 3 cars, 5 colors
- 9. If there are three seats available on the bus and two people who want to sit down, how many different ways can those two people arrange themselves in the empty seats?
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  Your principle has asked you to vote for two representatives from your math class. Since your class is small (8 students) it should not be too hard to figure out how many ways two students could be selected in this process.
  Can you figure it out?
- 7. Danielle, Connor, and Isaac ran in a race. In how many different orders can they finish the race?

13.	How man 5 student			there to	arrange	14.	How ma	any ways a	re there to	o arrange	6 chairs in
	a) 5	b) 15	c) 25	d) 50	e) 120		a) 36	Ъ	) 216	c) {	512
				. •			d) 720	e	46,656		
15.	How man formed us once?	ing each	of the dig	nts 3, 5, a	nd 7 only	16.	of the le	etters m, a umber of t	, t, or h v	vritten on	
4.1.	a) 3	b) 6	c) 8 	<b>d) 9</b> 	e) 12		a) 12	b) 24	c) 36	d) 48	e) 60
17.	How many	om using	the digits	2, 5, 6, 2		18.		ny 4-digit 2, 3, 4, 5,			e using the
	a) 4	b) 12	c) 20	d) 24	e) 28		a) 256	Ъ) 360	c) 512	d) 720	e) 1024
19.	Approxim	combina	ations of 4	. CD's can	be chosen	20.	for \$10.	club offers How many choose fro	y combina	tions of 3	bestsellers
	a) 124	b) 321	c) 518	d) 742	e) 1001		the club	<b>?</b> ~			,
	office to the	, <sup>3</sup> 63 t	Frå (Venes Jervi)	·. ·	an ar mangara			Ь) 365	**	15	e) 1025
21.	From 6 str wishes to number of	udents, 4 pick a bo	boys and by and a	2 girls, a girl. Find	teacher the	22.	Seth is the basketba	to select a ll team fro ber of poss	center an m a grou	d guard for peo	ple. Find
				d) 9	·			b) 42	11 1 ±		2.0
23.	New licens Massachus three lette	se plates etts displ rs. What	for passer ay three of t is the nu	ligits follo imber of c	n wed by lifferent	, 24.	four digi	ation code ts followed authorizat	by two le	etters. Ho	w many
	plates poss	sible!	api ba, sr.	ng kawar	and who is		a) 920,0	00 b)	5,560,000	с) 5	,600,000
	a) 16,986,0				3,824,000	•	d) 6,084	,000 e)	6,760,000	) )	e de la
	d) 20,124,0	000 e)	22,980,00	00							. , , , ,

Suppose you're at a restaurant and you can choose between pizza and a hamburger as a main meal, fries, salad, and fruit as a side dish. What are the possible meal combinations you could make?

2 You roll two dice. What are all the possible outcomes?

In how many ways can three letters be chosen from the letters A, B, C, D, and E? (ABC is different from ACB, etc.)

4 Use Pascal's Triangle to evaluate

- a  $_3C_2$
- b 5C
- $c = {}_6C_4$
- $d_{6}C_{2}$

5 A pizza shop offers seven toppings. How many different "four item" pizzas can be made?

Six of your favorite songs are not among the 17 chosen by a radio station. If you want to play three of your favorite songs, in how many ways can you choose three songs from the six?

#### Geometry

Combinations	Permutations
COMPUTATION	

1. Each of two dice is marked with a different integer from 1 to 6 inclusive. The two dice are rolled and the numbers on the top face of each die are added. How many different sums are possible?

- a) 11
- ъ) 12
- c) 24
- d) 34

- 2. Two dice are tossed. What is the number of ways 8 can appear from the throw of the dice?
  - a) 0
- b) 2

Per/Sec. \_

- c) 4
- d) 5

Date

e) 6

3. How many possible outcomes are there if you roll 2 dice?

- a) 12
- b) 36
- c) 48
- d) 60
- e) 72

e) 36

4. Two coins are tossed, a penny and a dime. What is the number of possible outcomes?

- a) 1
- b) 2
- c) (
- d) 4
- e) 5

5. Eight boys and twelve girls attend a party. How many different dancing pairs of one girl and one boy can be formed?

- a) 4
- ъ) 20
- c) 40
- d) 80
- e) 96

6. Emily has 5 blouses and 3 pair of slacks. Find the number of possible outfits consisting of one blouse and one pair of slacks.

- a) 1
- b) 4
- c) 8
- d) 15
- e)  $\frac{3}{5}$

7. A student has a choice of 4 math classes and 5 history classes. In how many ways can she choose one math and one history class?

- a) 9
- b) 10
- c) 20
- d) 24
- e) 54

8. There are 5 candidates for governor and 6 candidates for controller. In how many ways can these offices be filled?

- a) 4
- b) 11
- c) 18
- d) 24
- e) 30

9. Patrick picks a marble from a bag containing 8 differently colored marbles, then replaces the marble, and picks another. What is the number of possible outcomes?

- a) 56
- ь) 64
- c) 128
- d) 256
- e) 512

10. A room has 5 doors. In how many ways can Miguel enter the room and leave by any door?

- a) 5
- b) 1
- c)
- d) 20
- e) 25

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- a) 12
- b) 30
- c) 35
- d) 60
- e) 75

12. Alex has 6 pairs of pants, 10 shirts, and 3 pairs of shoes that he wears for work. How many different combinations of a pair of pants, a shirt, and a pair of shoes can Alex wear to work?

- a) 140
- ъ) 155
- c) 180
- d) 200
- e) 225

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1	Suppose you toss a coin twice.	What are the possible outcomes?	Draw a tree diagram to
	determine the outcome set.		

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## Pascal's Triangle

Let's refer back to our Coldstone Creamery example where we needed to evaluate  ${}_{\scriptstyle 3}C_{\scriptstyle 2}$  .

Count down to row 3.

Go over to column 2. (Don't forget to start counting with 0.)

The answer for  ${}_{3}C_{2}$  is \_\_\_\_\_

Now try the following examples:

$$_{4}C_{3} =$$

$$_{2}C_{1}=$$

$$_{5}C_{2} =$$

$$_{5}C_{4}=$$

2 Suppose that there is a club that has six members. How many ways can a committee of 3 members be formed?