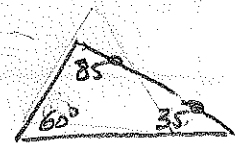
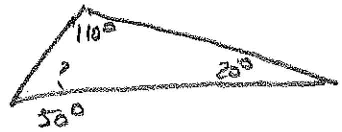
Geometry, 5.2 Notes –Proving lines are parallel

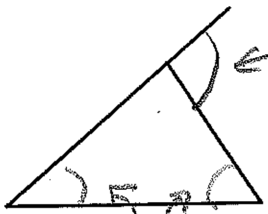
(Book doesn't show this until 7.1) - **The sum of the angles in a triangle = 180°**





$$\begin{array}{r} 110 \\ +\ 20 \\ \hline 130 \end{array} \qquad \begin{array}{r} 180 \\ -\ 130 \\ \hline 50° \end{array}$$

Exterior Angle Inequality Theorem: The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

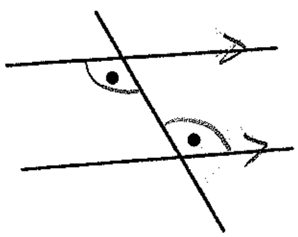


← exterior angle (formed by extending one side)

remote interior angles

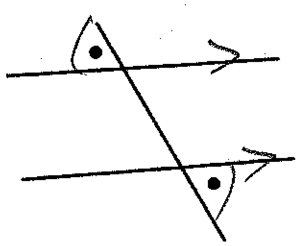
6 ways to prove two lines are parallel:

Alternate interior angles are congruent



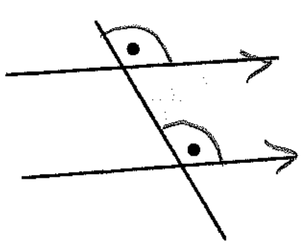
Alt. int $\angle$s $\cong \Rightarrow \parallel$ lines

Alternate exterior angles are congruent



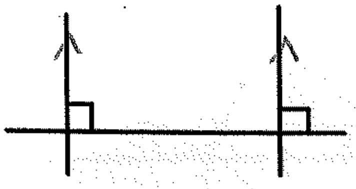
Alt. ext $\angle$s $\cong \Rightarrow \parallel$ lines

Corresponding angles are congruent



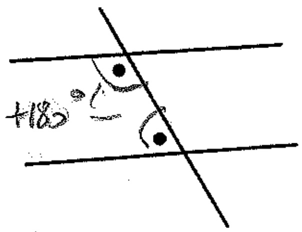
Corr. $\angle$s $\cong \Rightarrow \parallel$ lines

. lines both perpendicular to a 3rd line



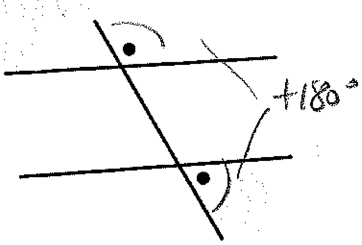
2 lines ⊥ to 3rd line ⇒ ∥ lines

Same side interior angles are supplementary



Same side int ∠s supp. ⇒ ∥ lines

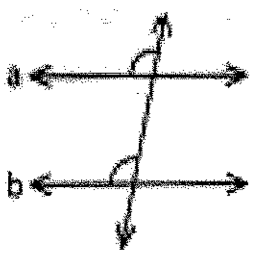
Same side exterior angles are supplementary



Same side ext ∠s supp. ⇒ ∥ lines

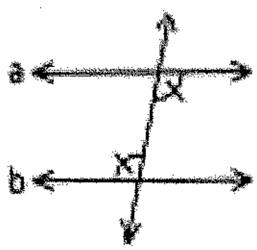
Practice: Which theorem proves the lines are parallel?

a



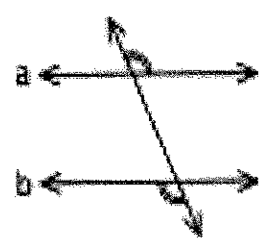
Corr. ∠s ≅ ⇒ ∥ lines

b



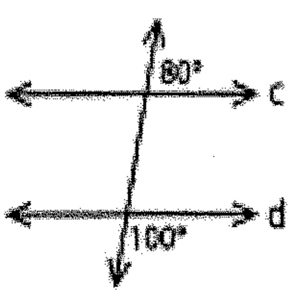
Alt. int ∠s ≅ ⇒ ∥ lines

c



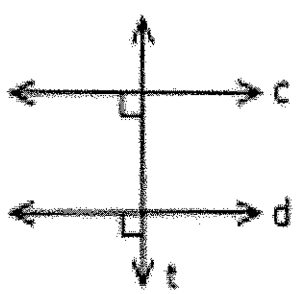
Alt ext ∠s ≅ ⇒ ∥ lines

a



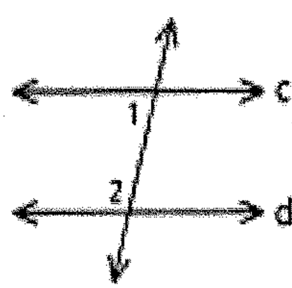
Same side ext ∠s supp ⇒ ∥ lines

b



2 lines ⊥ to 3rd lines ⇒ ∥ lines

c

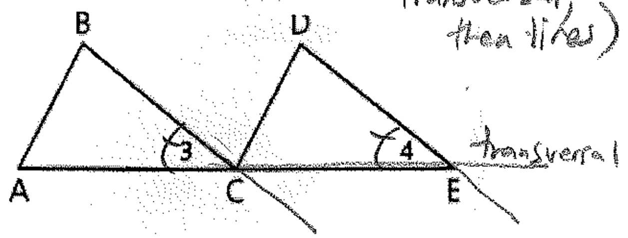


(∠1 supp. to ∠2)

Same side int ∠s supp ⇒ ∥ lines

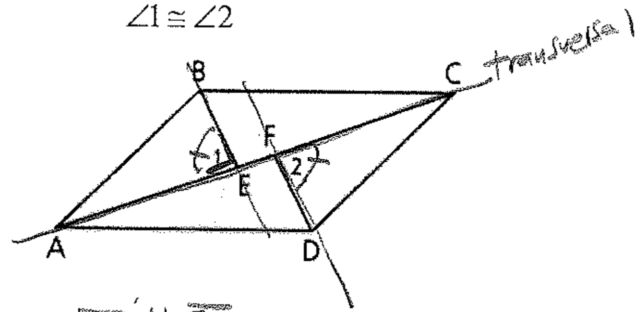
$\angle 3 \cong \angle 4$:

(identify common transversal, then lines)



$\overline{BC} \parallel \overline{DE}$ corr ∠s ≅ ⇒ ∥ lines

$\angle 1 \cong \angle 2$



$\overline{BE} \parallel \overline{FD}$ Alt ext ∠s ≅ ⇒ ∥ lines

Geometry, 5.3 Notes –Euclid's 5 Postulates, Congruent angles and parallel lines

Euclid (~300 B.C.) – the 'Father of Geometry' stated 5 postulates that form the basis of geometry.

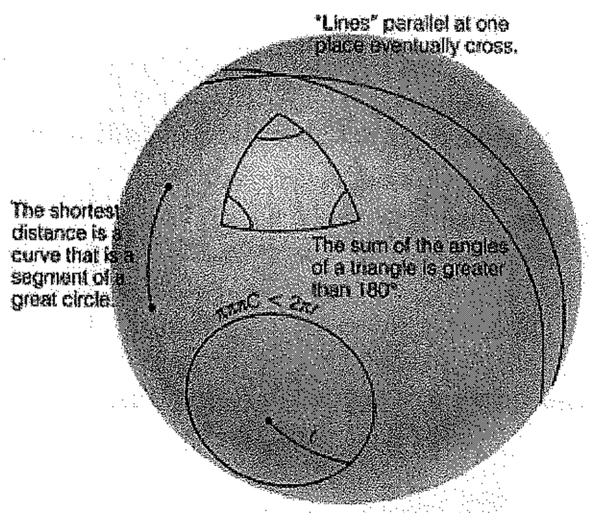
1) A straight line segment can be drawn joining any two points.
2) Any straight line segment can be extended indefinitely in a straight line.
3) Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4) All right angles are congruent.
5) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines must intersect each other on that side if extended far enough.

The last one is known as 'the parallel postulate', because it can be restated as:

Parallel postulate: Through a point not on a line there is exactly one parallel to the given line.

These postulates form the basis of geometry and the first 4 are provable, but the 5th postulate has never been proven. Instead, you get different 'geometries' depending upon if you assume it is true or not true. In our class, we assume it is true, so we are studying 'Euclidean Geometry' or 'planar Geometry', because this is the geometry of lines and shapes on a plane.

Other geometries are possible – for example, spherical geometry (geometry on a sphere):



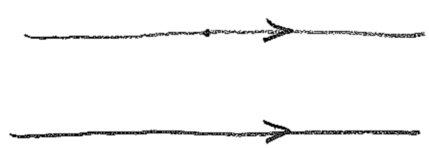
In spherical geometry, rules are different....

A triangle can have 3 right angles.

The shortest distance isn't a straight line it is a curve (known as a great circle)...these are the paths pilots fly when crossing the ocean.

But...in our class, we assume we are not on a sphere, but on a plane, so for us the parallel postulate is true.

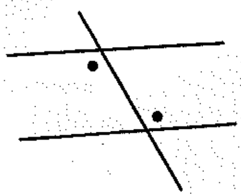
Parallel postulate: Through a point not on a line there is exactly one parallel to the given line.



Tick mark for showing 2 lines are parallel: 

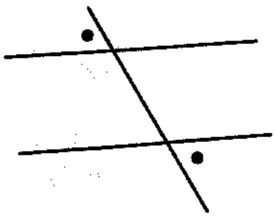
5 theorems about parallel lines cut by a transversal

Alternate interior angles are congruent



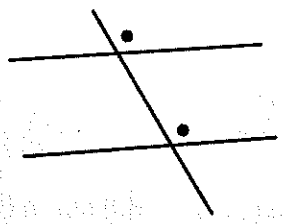
$\parallel$ lines $\Rightarrow$ alt. int. $\angle$'s $\cong$

Alternate exterior angles are congruent



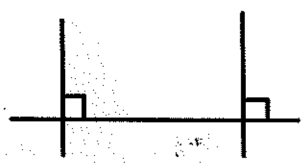
$\parallel$ lines $\Rightarrow$ alt. ext. $\angle$'s $\cong$

Corresponding angles are congruent



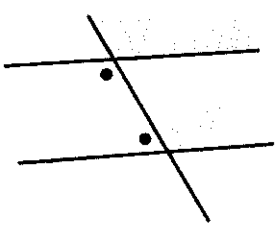
$\parallel$ lines $\Rightarrow$ corr. $\angle$'s $\cong$

A line that is perpendicular to one of two parallel lines is perpendicular to the other



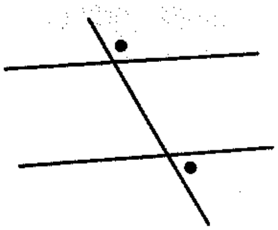
line $\perp$ to 1 of 2 $\parallel$ lines $\Rightarrow$ $\perp$ other

Same side interior angles are supplementary



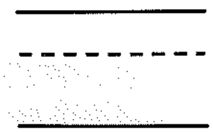
$\parallel$ lines $\Rightarrow$ same side int $\angle$'s supp.

Same side exterior angles are supplementary



$\parallel$ lines $\Rightarrow$ same side ext $\angle$'s supp.

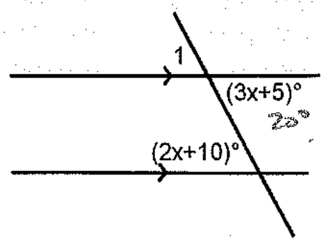
One more theorem: If 2 lines are parallel to a 3rd line, they are parallel to each other.



2 lines ∥ 3rd ⇒ ∥ each other

Practice:

1) Find $m\angle 1$



∥ lines ⇒ alt int ∠'s ≅ so

$3x+5 = 2x+10$

$-2x \quad -2x$

$x+5 = 10$

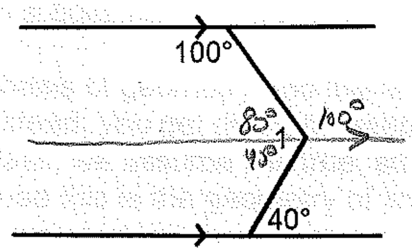
$-5 \quad -5$

$x = 5$

$3x+5 = 3(5)+5 = 20$

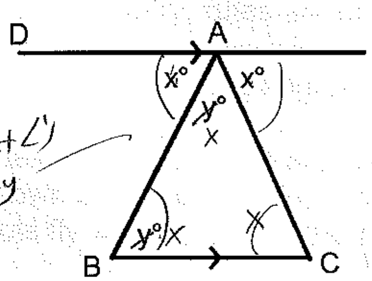
$m\angle 1 = 20°$ (vertical ∠'s)

2)Find $m\angle 1$

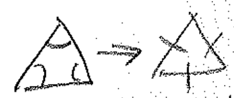


$m\angle 1 = 120°$

3) If $\overline{DA} \parallel \overline{BC}$, is ΔABC equilateral? Find $m\angle DAB$



alt int ∠'s so $x = y$



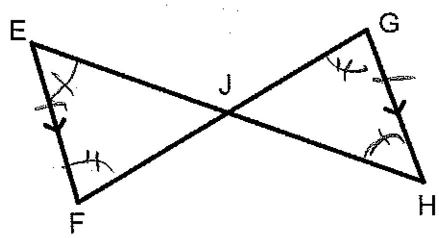
yes, equilateral.

$m\angle DAB = x = \frac{180}{3} = 60°$

4)Given: $\overline{EF} \parallel \overline{GH}$

$\overline{EF} \cong \overline{GH}$

Prove: $\overline{EJ} \cong \overline{JH}$

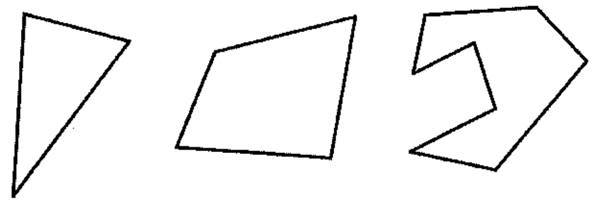


1. $\overline{EF} \parallel \overline{GH}$, $\overline{EF} \cong \overline{GH}$	1. Given
2. $\angle E \cong \angle H$	2. ∥ lines ⇒ alt int ∠'s ≅
3. $\angle F \cong \angle G$	3. ∥ lines ⇒ alt int ∠'s ≅
4. $\Delta EJF \cong \Delta HJG$	4. ASA
5. $\overline{EJ} \cong \overline{JH}$	5. CPCTC

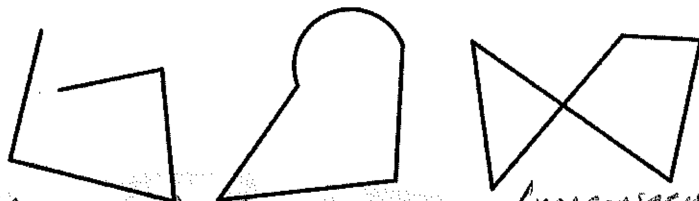
Geometry, 5.4 ~~day 1~~ Notes –4-sided Polygons

Polygon – a closed, plane (flat) figure with straight sides (consecutive sides intersect at endpoints.)

Examples of polygons:



Not polygons:

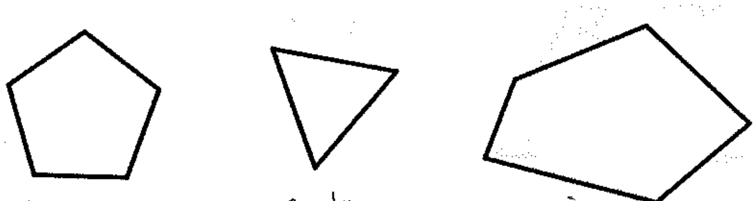


(not closed) (side not a line segment) (nonconsecutive sides intersect)

Convex and Concave polygons:
Convex = interior angles all < 180°.

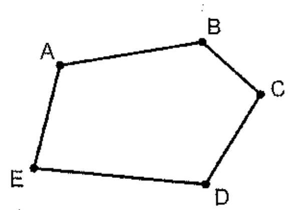
convex concave

Regular polygons – a polygon whose sides are all congruent.



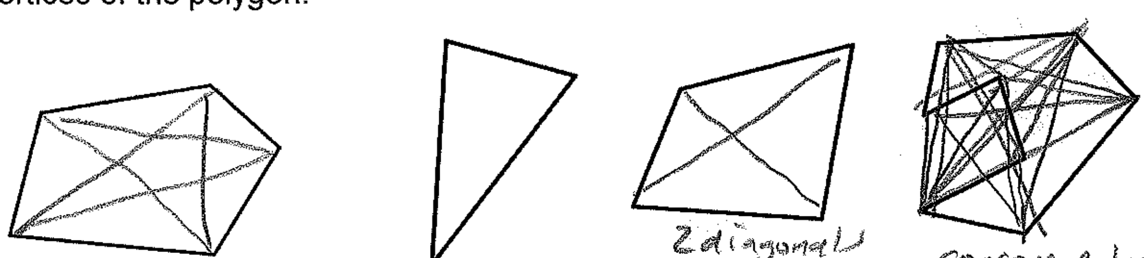
regular polygon regular polygon irregular polygon

Naming polygons: Start at a vertex and list each vertex in consecutive order (can go either clockwise or counterclockwise.)



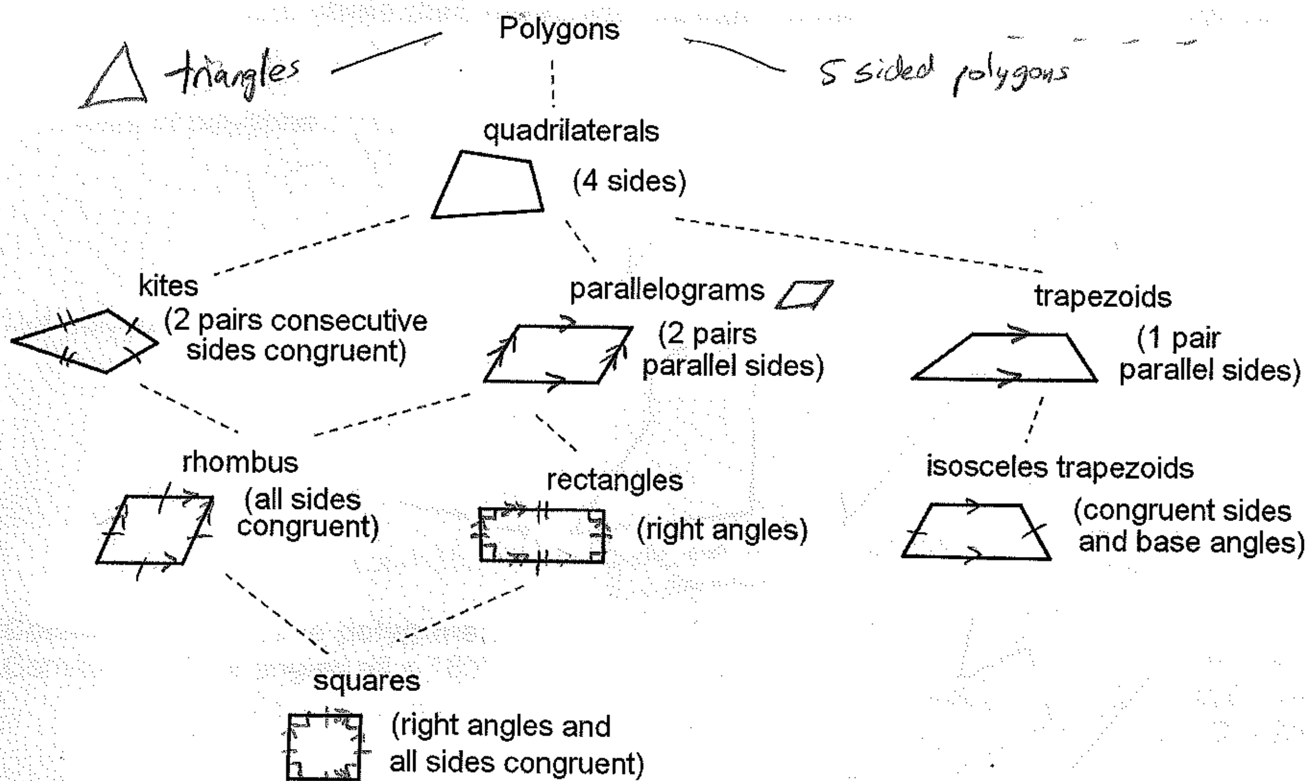
Possible names: polygon ABCDE
polygon CBAED
polygon EABCD

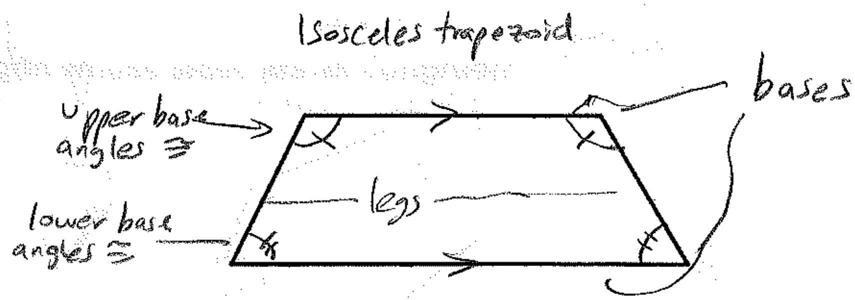
Diagonals of polygons: A line segment connecting two non-consecutive (non-adjacent) vertices of the polygon.



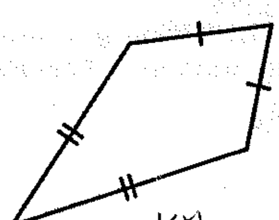
5 diagonals no diagonals 2 diagonals concave polygon diagonals cross 'outside' the polygon

Geometry, 5.4 ~~day~~ Notes –Classifying 4-sided Polygons

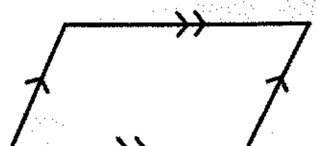




Practices: Identify the polygon…

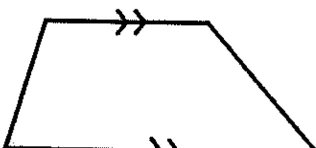


Kite

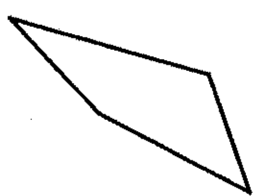


parallelogram

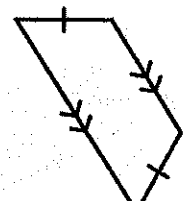
rhombus



trapezoid



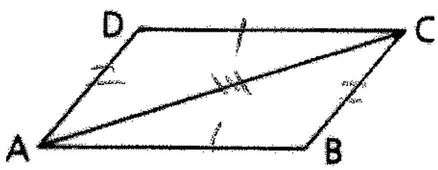
quadrilateral



Isosceles trapezoid

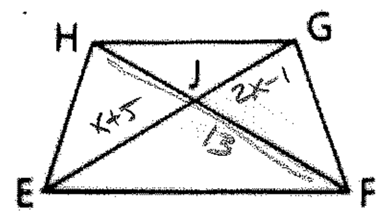
Geometry, 5.5 day 1 Notes –Properties of Quadrilaterals

Given: $\square ABCD$
Prove: $\Delta ABC \cong \Delta CDA$



S	R
1. $\square ABCD$	1. Given
2. $\overline{DC} \cong \overline{BA}$	2. $\square$ opp sides $\cong$
3. $\overline{AD} \cong \overline{CB}$	3. $\square$ opp sides $\cong$
4. $\overline{AC} \cong \overline{CA}$	4. Reflexive prop.
5. $\Delta ABC \cong \Delta CDA$	5. SSS

Given: EFGH is an isosceles trapezoid with legs $\overline{HE}$ and $\overline{GF}$
$EJ = x+5$
$JG = 2x-1$
$HF = 13$
Find: EJ, JG, and HJ.



isos. trap diags $\cong$

$x+5+2x-1=13$

$3x+4=13$

$3x=9$

$x=3$

$EJ = x+5 = 3+5 = \boxed{8}$

$JG = 2x-1 = 2(3)-1 = \boxed{5}$

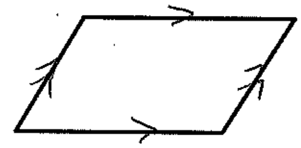
$HJ = GJ = \boxed{5}$

Geometry, 5.6 Notes –Proving a quadrilateral is a parallelogram

There are 5 ways to prove a quadrilateral is a parallelogram:

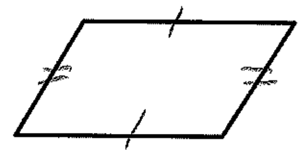
1. If both pairs of opposite sides are parallel.

   both pairs opp. sides ∥ ⇒ ▱



2. If both pairs of opposite sides are congruent.

   both pairs opp. sides ≅ ⇒ ▱

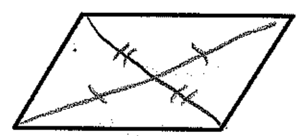


3. If one pair of opposite sides are parallel and congruent.

   one pair opp sides ∥ & ≅ ⇒ ▱

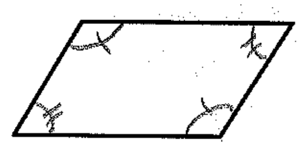
4. If the diagonals bisect each other.

   diagonals bisect ⇒ ▱



5. If both pairs of opposite angles are congruent.

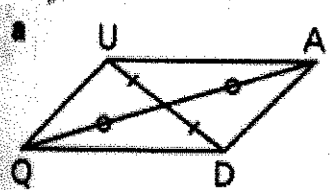
   both pairs opp. ∠'s ≅ ⇒ ▱



For each quadrilateral, state how you know it is a parallelogram:

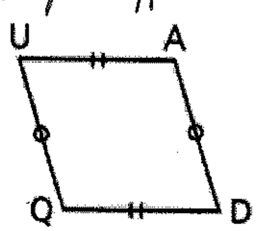
diagonals bisect ⇒ ▱

a



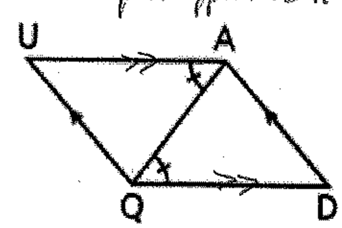
both pairs opp sides ≅ ⇒ ▱

c

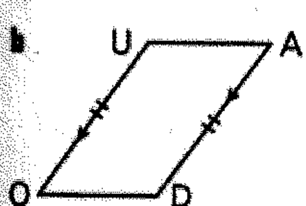


both pairs opp. sides ∥ ⇒ ▱

e

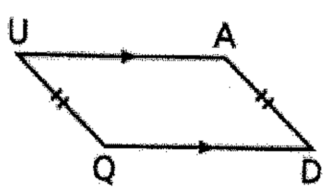


b



one pair opp sides ∥ & ≅ ⇒ ▱

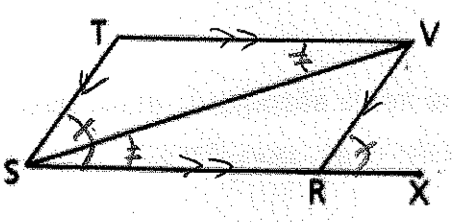
d



Can't say ▱

Given: $\angle XRV \cong \angle RST$
$\angle RSV \cong \angle TVS$

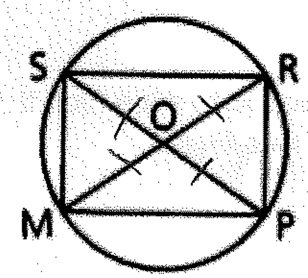
Prove: *RSTV is a* ▱



S	R
1. $\angle XRV \cong \angle RST$	1. Given
2. $\angle RSV \cong \angle TVS$	2. Given
3. $\overline{TS} \parallel \overline{VR}$	3. corr. ∠s ≅ ⇒ ∥ lines
4. $\overline{TV} \parallel \overline{SR}$	4. alt int ∠s ≅ ⇒ ∥ lines
5. RSTV ▱	5. both pairs opp. sides ∥ ⇒ ▱

Given: $\odot O$

Prove: *MPRS is a* ▱



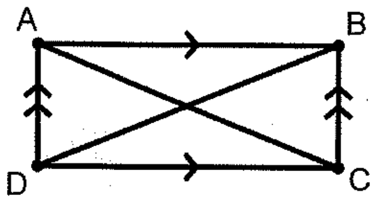
S	R
1. $\odot O$	1. Given
2. $\overline{OS} \cong \overline{OR} \cong \overline{OP} \cong \overline{OM}$	2. radii ≅
3. $\overline{SP}$ bis. $\overline{MR}$	3. def. of bisect
4. $\overline{RM}$ bis. $\overline{PS}$	4. def of bisect
5. MPRS is ▱	5. diagonals bisect ⇒ ▱

Geometry, 5.7 Notes –Proving figures are special quadrilaterals

Proving a quadrilateral is a rectangle:

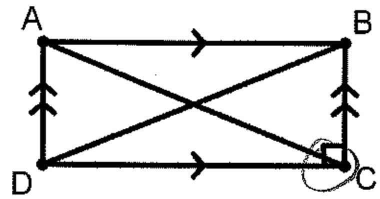
- show is a ▱ (parallelogram), then:
  - show at least one right angle, or
  - show diagonals are ≅
- or
- show all 4 angles are right angles

Are these rectangles? How do you know?

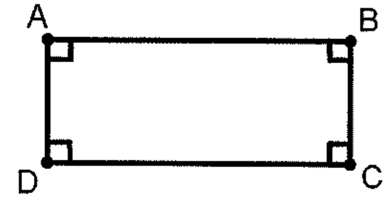


$\overline{AC} \cong \overline{BD}$

▱ w/ ≅ diagonals



▱ w/ right angle

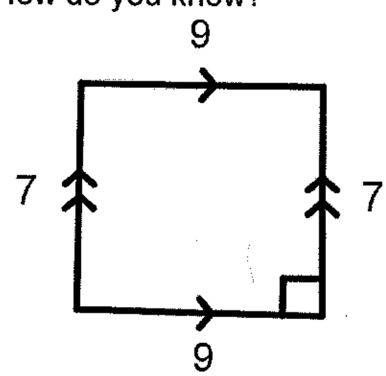


all right angles

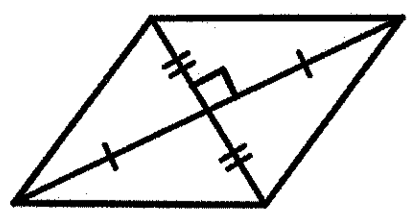
Use the reference sheet for proving a quadrilateral is a parallelogram, rectangle, kite, rhombus, square or isosceles trapezoid.

Examples: What is the most descriptive term for the shape? How do you know?

- parallelogram? yes (2 pairs opp sides ∥ → ▱)
- $\boxed{\text{rectangle?}}$ yes (▱ w/ rt angle)
- rhombus? no (sides not all ≅)
- square? no (not a rhombus, sides not equal)



- parallelogram? yes (diagonals bisect each other)
- rectangle? no (diagonals ≇, no right angle)
- $\boxed{\text{rhombus?}}$ yes (diagonals ⊥ bis of each other)
- square? no (not a rectangle)



If $\overline{AB} \cong \overline{DC}$ show ABCD is not a rhombus:

$AB = DC$

$2x+5 = 4x-17$

$-2x \quad -2x$

$5 = 2x-17$

$+17 \quad +17$

$22 = 2x$

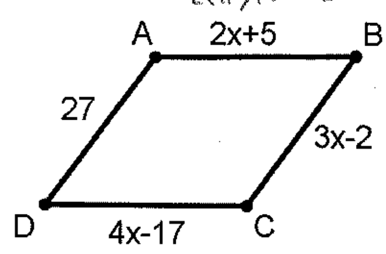
$\frac{22}{2} = \frac{2x}{2}$

$11 = x$

$\overline{BC} \cong \overline{AD}$

(not all sides are equal)

$2(11)+5=27$



$3(11)-2=31$

27