Geometry, 3-1 Notes – Congruent Figures

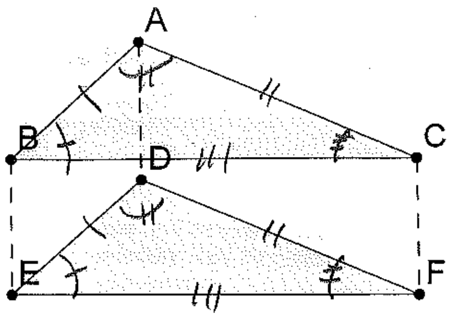
Extend our idea of congruent to figures (shapes):

Congruent angles ⇔ same angle measure

Congruent segments ⇔ same length

Congruent triangles ⇔ all pairs of corresponding parts (angles & segments) are ≅

Congruent polygons ⇔ all pairs of corresponding parts are ≅



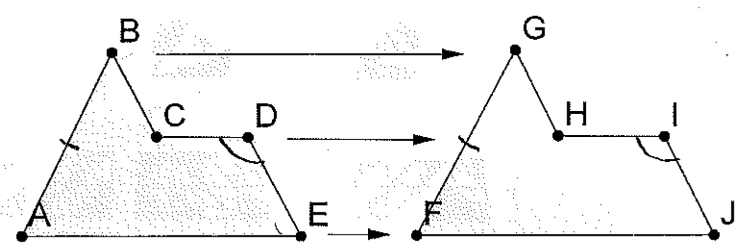
Writing a statement, all parts must 'match up':

$\triangle ABC \cong \triangle DEF$ ✓

$\triangle ABC \cong \triangle FED$ ✗ no, parts don't correspond.

To test if two shapes are congruent, try moving them in ways so that the parts match up:

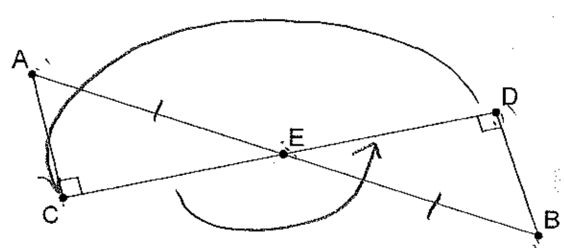
Slide



In figure above: $\overline{AB}$ shifts onto $\overline{FG}$

$\angle CDE$ shifts onto $\angle HIJ$

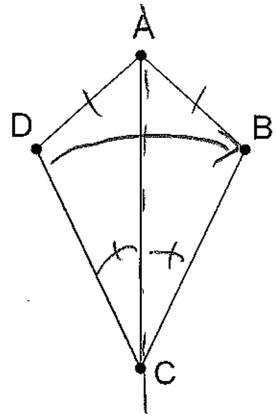
Rotate



In figure above: $\overline{AE}$ rotates onto $\overline{BE}$

$\angle CAE$ rotates onto $\angle DBE$

Reflection



In figure above: $\overline{AD}$ reflects onto $\overline{AB}$
$\angle DCA$ reflects onto $\angle BCA$
$\overline{AC}$ reflects onto $\overline{AC}$

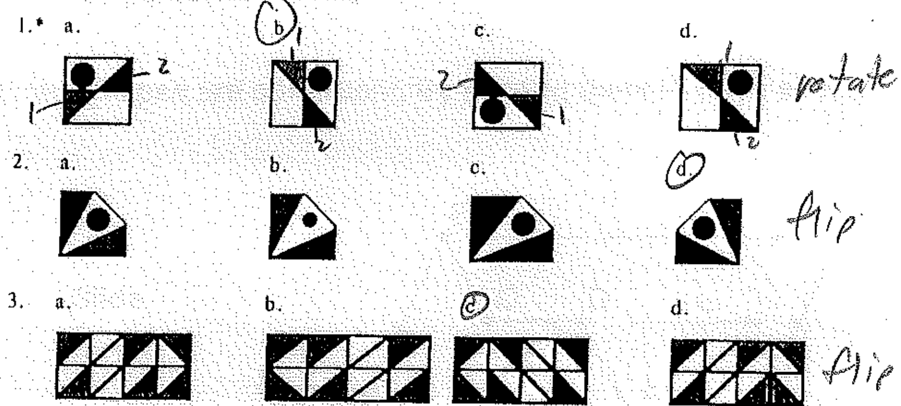
Reflexive Property Any segment of angle is congruent to itself.

Practice problems:

#1.

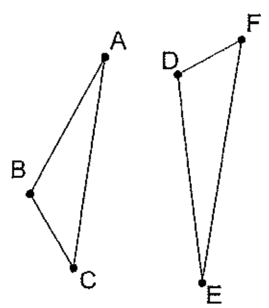
3

In each of the puzzles below, determine which two figures are identical. If necessary, trace the figures and flip, turn, or slide the tracings over the others to make your decision.



1

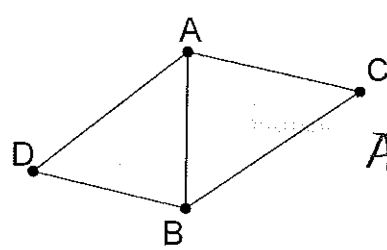
#2. Below, $\Delta ABC \cong \Delta EDF$
List all congruent pairs of angles and segments.



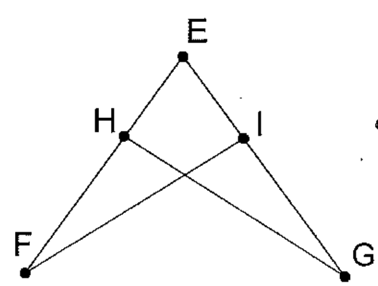
$\angle A \cong \angle E$
$\angle B \cong \angle D$
$\angle C \cong \angle F$
$\overline{AB} \cong \overline{ED}$
$\overline{AC} \cong \overline{EF}$
$\overline{BC} \cong \overline{DF}$

2

#3. In each figure, list which part is $\cong$ to itself by Reflexive Property.



$\overline{AB}$



$\angle HEI$

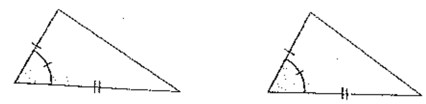
Geometry, 3-2 day1 Notes – Congruent Triangles

How do you know if 2 triangles are congruent?
Could compare every pair of corresponding angles and sides.

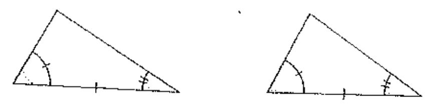
3 shortcuts – can compare just 3 pairs of sides or angles, but have to compare certain things.

Side-Side-Side (SSS)
If 3 sides are congruent, then the triangles are congruent.

Side-Angle-Side (SAS)
If 2 sides and the angle in between are congruent, then the triangles are congruent.



Angle-Side-Angle (ASA)
If 2 angles and the side in between are congruent, then the triangles are congruent.

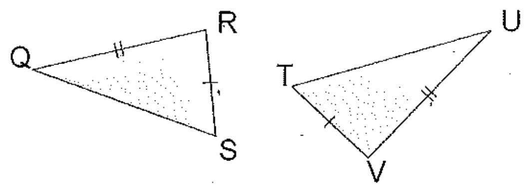


Examples:

Given the diagram, what additional sides or angles would need to be marked congruent for you to know the two triangles are congruent by:

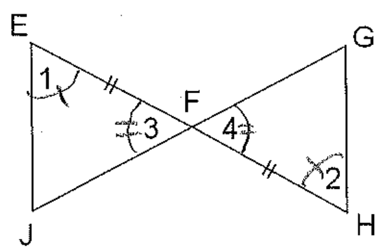
(a) SSS $\overline{QS} \cong \overline{TU}$

(b) SAS $\angle QRS \cong \angle TVU$



Given: $\angle 1 \cong \angle 2$
$\overline{EF} \cong \overline{HF}$

Prove: $\Delta EFJ \cong \Delta HFG$

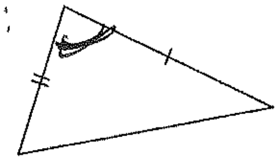


Statement	Reason
1. $\angle 1 \cong \angle 2$, $\overline{EF} \cong \overline{HF}$	1. Given
2. $\angle 3 \cong \angle 4$	2. vertical angles are $\cong$
3. $\Delta EFJ \cong \Delta HFG$	3. ASA

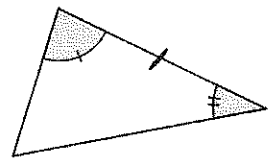
Geometry, 3-2 day2 Notes – Congruent Triangles

New term: 'included' = 'between'

Included angle = angle between 2 sides:

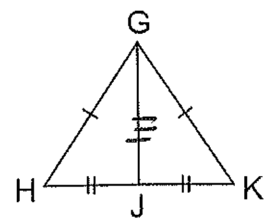


Included side = side between 2 angles

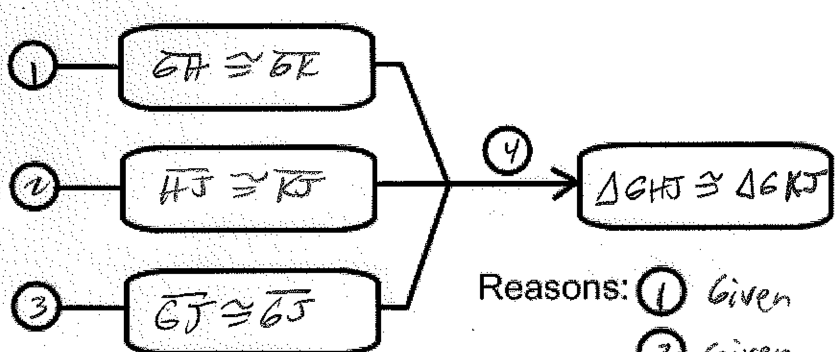


Flowchart proofs:

Example: Given the diagram, what can you conclude?

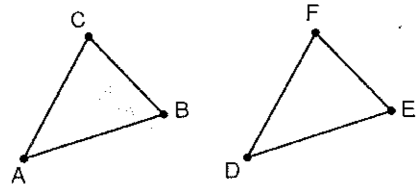


Flowchart proof...



Reasons:
1. Given
2. Given
3. Reflexive Property
4. SSS

Geometry, 3-3 day1 Notes – CPCTC and Circles



If $\Delta ABC \cong \Delta DEF$

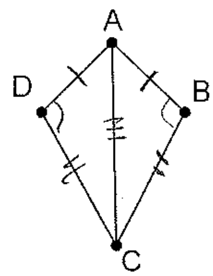
Is $\angle A \cong \angle D$? Yes
Is $\overline{BC} \cong \overline{EF}$? Yes

Every corresponding pair of parts (angles or sides) of congruent triangles are congruent.

When we use this as a reason in a proof, we write CPCTC:

C – Corresponding
P – parts of
C – Congruent
T – Triangles are
C – Congruent

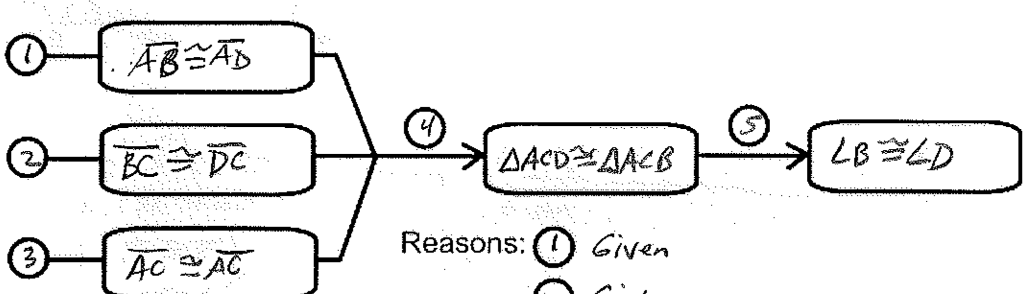
Example:



Given: $\overline{AB} \cong \overline{AD}$
$\overline{BC} \cong \overline{DC}$

Prove: $\angle B \cong \angle D$

Flowchart Proof:

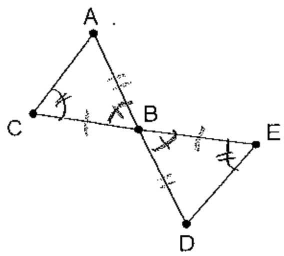


Reasons: 1 Given
2 Given
3 Reflexive Property
4 SSS
5 CPCTC

Same proof, in two-column form…

Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{BC} \cong \overline{DC}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property
4. $\Delta ACD \cong \Delta ACB$	4. SSS
5. $\angle B \cong \angle D$	5. CPCTC

Example 2:



Given: $\overline{CB} \cong \overline{EB}$

$\overline{AB} \cong \overline{DB}$

Prove: $\angle C \cong \angle E$

(1) $\overline{CB} \cong \overline{EB}$

(2) $\overline{AB} \cong \overline{DB}$

(3) $\angle ABC \cong \angle DBE$

(4) $\rightarrow \triangle ABC \cong \triangle DBE$ (5) $\rightarrow \angle C \cong \angle E$

(1) Given

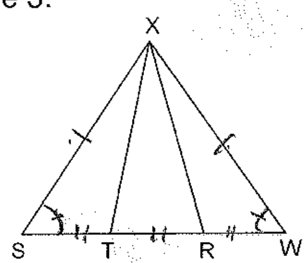
(2) Given

(3) vert. ∠s are ≅

(4) SAS

(5) CPCTC

Example 3:



Given: T and R trisect $\overline{SW}$

$\overline{XS} \cong \overline{XW}$

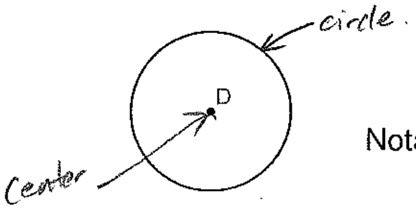
$\angle S \cong \angle W$

Prove: $\overline{XT} \cong \overline{XR}$

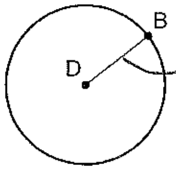
Statement	Reason
1. T & R trisect $\overline{SW}$	1. Given
2. $\overline{XS} \cong \overline{XW}$	2. Given
3. $\angle S \cong \angle W$	3. Given
4. $\overline{ST} \cong \overline{TR} \cong \overline{RW}$	4. def. of trisect
5. $\overline{SR} \cong \overline{WT}$	5. Addition prop.
6. $\triangle XTS \cong \triangle XRW$	6. SAS
7. $\overline{XT} \cong \overline{XR}$	7. CPCTC

Geometry, 3-3 day2 Notes – CPCTC and Circles

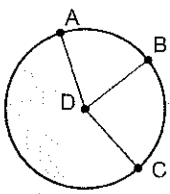
Definition of a circle: every point is the same distance from the center



Notation: $\odot D$



radius = line segment from center to a point on the circle.



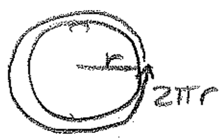
$\overline{DA} \cong \overline{DB} \cong \overline{DC}$

all radii (radiuses) are congruent to each other

For a circle:

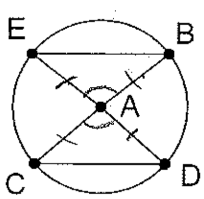
Area = πr^2

Perimeter = circumference = $2\pi r$



"Pi" $= \pi = 3.14159...$ (a little more than 3)

Example:



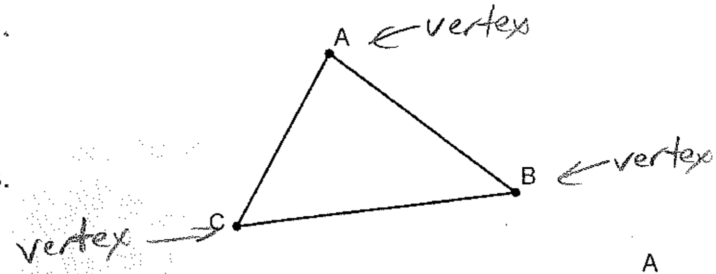
Given: $\odot A$

Prove: $\overline{EB} \cong \overline{CD}$

Statements	Reasons
1. $\odot A$	1. Given
2. $\overline{AC} \cong \overline{AD} \cong \overline{AB} \cong \overline{AE}$	2. all radii are $\cong$
3. $\angle EAB \cong \angle DAC$	3. vertical ∠'s are $\cong$
4. $\Delta EAB \cong \Delta DAC$	4. SAS
5. $\overline{EB} \cong \overline{CD}$	5. CPCTC

Geometry, 3-4 Notes – Triangle Medians, Altitudes and Auxiliary Lines

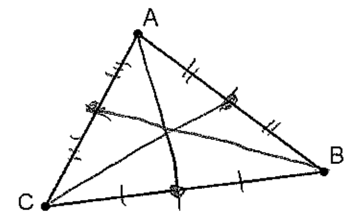
Triangle terms.....



Vertex: A corner of a triangle.

A triangle has 3 vertices.

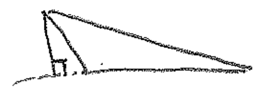
Median: A line from a vertex to the midpoint of the opposite side.

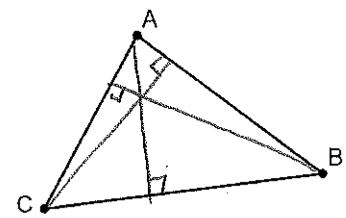


A triangle has 3 medians.

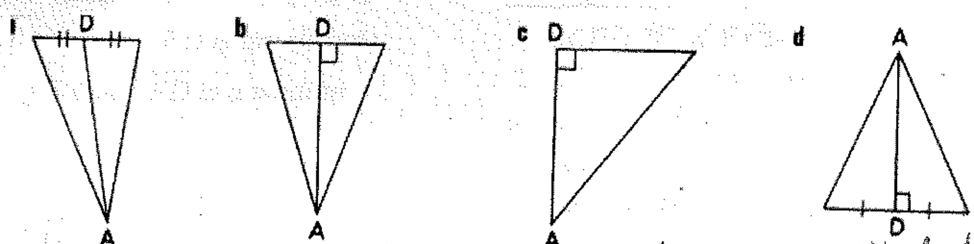
Altitude: A line from a vertex to the opposite side (extended if needed) perpendicular to the side.

A triangle has 3 altitudes.





Practice: #1. In each figure below, say whether $\overline{AD}$ is a median, an altitude, neither or both.



median | altitude | altitude | altitude & median

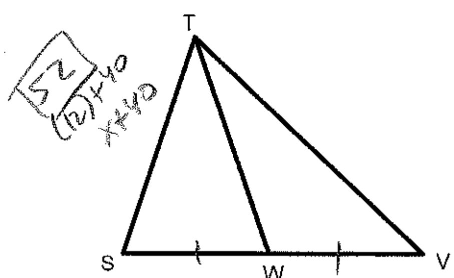
#6. Given: $\overline{TW}$ *is a median*

$ST = x + 40$

$SW = 2x + 30$

$WV = 5x - 6$

Find: SW, WV, and ST



$2x+30$ $5x-6$

$2(12)+30$ $5(12)-6$

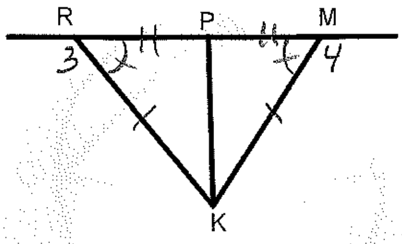
$\boxed{54}$ $\boxed{54}$

$$2x+30=5x-6$$
$$-2x \quad -2x$$
$$30=3x-6$$
$$+6 \quad +6$$
$$\frac{36}{3}=\frac{3x}{3}$$
$$12=x$$

#7. Given: $\overline{KP}$ *is a median*

$\overline{MK} \cong \overline{RK}$

Prove: $\angle 3 \cong \angle 4$

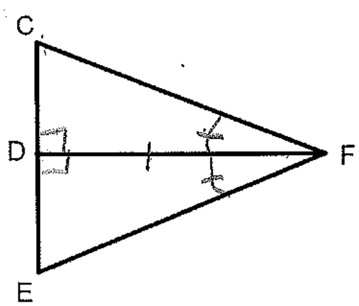


S	R
1. $\overline{KP}$ is a median	1. Given
2. $\overline{MK} \cong \overline{RK}$	2. Given
3. $\overline{RP} \cong \overline{MP}$	3. Definition of median
4. $\overline{PK} \cong \overline{PK}$	4. Reflexive property
5. $\triangle RPK \cong \triangle MPK$	5. SSS
6. $\angle PRK \cong \angle PMK$	6. CPCTC
7. $\angle 3 \cong \angle 4$	7. $\angle$'s supp. to $\cong$ $\angle$'s are $\cong$

#4. Given: $\angle CFD \cong \angle EFD$

$\overline{FD}$ *is an altitude*

Prove: $\overline{FD}$ *is a median*

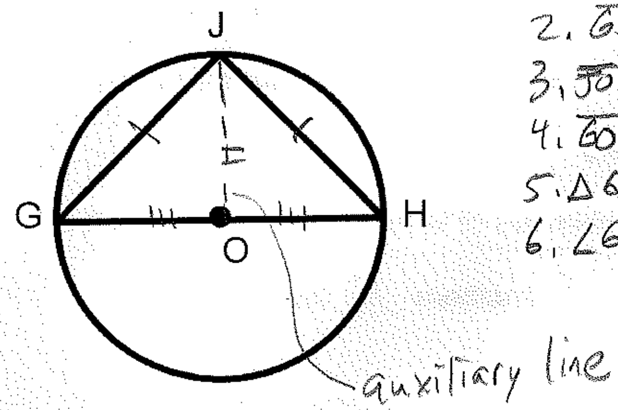


S	R
1. $\angle CFD \cong \angle EFD$	1. Given
2. $\overline{FD}$ is an altitude	2. Given
3. $\angle FDC$, $\angle FDE$ are rt $\angle$'s	3. Definition of altitude
4. $\angle FDC \cong \angle FDE$	4. all rt $\angle$'s $\cong$
5. $\overline{FD} \cong \overline{FD}$	5. Reflexive prop.
6. $\triangle CDF \cong \triangle EDF$	6. ASA
7. $\overline{CD} \cong \overline{ED}$	7. CPCTC
8. $\overline{FD}$ is a median	8. Def. of median

Given: $\odot O$

$\overline{GJ} \cong \overline{HJ}$

Prove: $\angle G \cong \angle H$



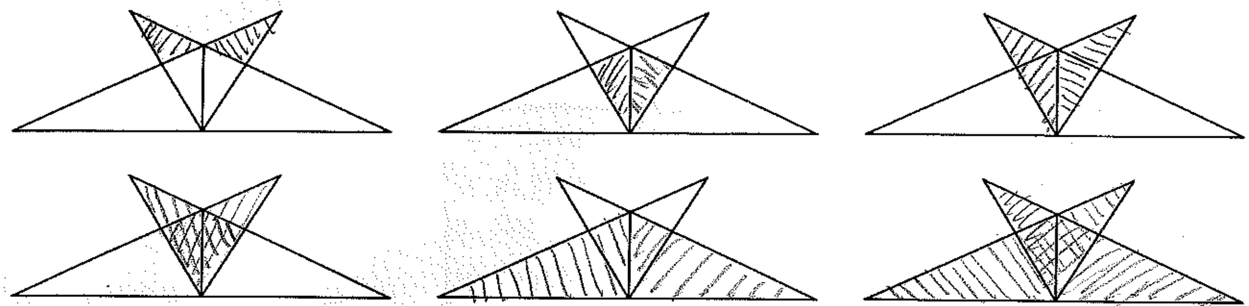
S	R
1. $\odot O$	1. Given
2. $\overline{GJ} \cong \overline{HJ}$	2. Given
3. $\overline{JO} \cong \overline{JO}$	3. Reflexive Prop.
4. $\overline{GO} \cong \overline{HO}$	4. radii $\cong$
5. $\triangle GJO \cong \triangle HJO$	5. SSS
6. $\angle G \cong \angle H$	6. CPCTC

Can add a line segment between two points because: **two points make a line, ray or segment**

Auxiliary Lines: Lines added to a diagram to help prove something.

Geometry, 3-5 Notes – Overlapping Triangles

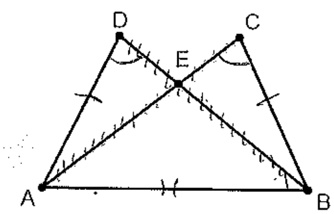
There are 6 pairs of congruent triangles in the figure below. In each copy, shade a pair of matching triangles:



We can often prove things using CPCTC by first proving 2 triangles congruent, but it is important to choose the right pair of triangles to use.

Example: Given: $\overline{BD} \cong \overline{AC}$
$\overline{AD} \cong \overline{BC}$

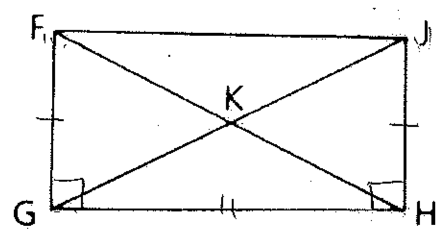
Prove: $\angle D \cong \angle C$



1. $\overline{BD} \cong \overline{AC}$, $\overline{AD} \cong \overline{BC}$	1. Givens
2. $\overline{AB} \cong \overline{BA}$	2. Reflexive Prop.
3. $\triangle DAB \cong \triangle CBA$	3. SSS
4. $\angle D \cong \angle C$	4. CPCTC

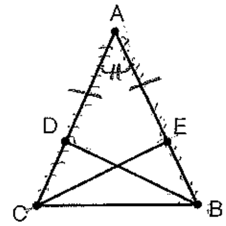
Example: Given: $\angle FGH$ is a right angle
$\angle JHG$ is a right angle
$\overline{FG} \cong \overline{JH}$

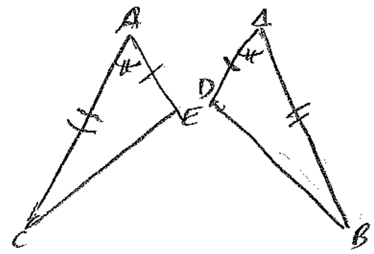
Prove: $\Delta FGH \cong \Delta JHG$



1. $\angle FGH$ is rt $\angle$, $\angle JHG$ rt $\angle$, $\overline{FG} \cong \overline{JH}$	1. Givens
2. $\overline{GH} \cong \overline{HG}$	2. Reflexive Prop.
3. $\angle FGH \cong \angle JHG$	3. all rt $\angle$'s are $\cong$
4. $\Delta FGH \cong \Delta JHG$	4. SAS

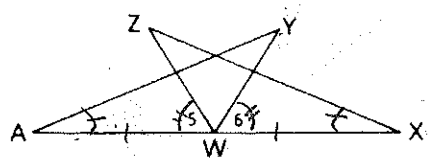
Practice: Given: $\overline{AC} \cong \overline{AB}$
$\overline{AE} \cong \overline{AD}$
Prove: $\overline{CE} \cong \overline{BD}$

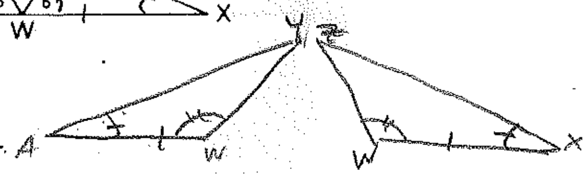




1. $\overline{AC} \cong \overline{AB}, \overline{AE} \cong \overline{AD}$	1. Givens
2. $\angle A \cong \angle A$	2. Reflexive Prop.
3. $\triangle AEC \cong \triangle ADB$	3. SAS
4. $\overline{CE} \cong \overline{BD}$	4. CPCTC

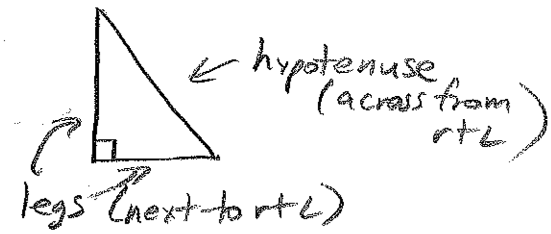
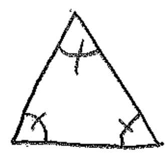
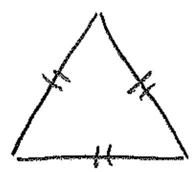
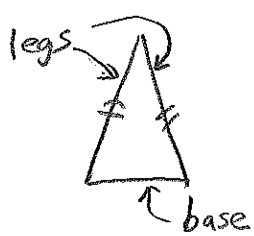
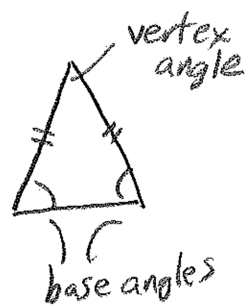
Practice: Given: $\overline{YW}$ bisects $\overline{AX}$
$\angle A \cong \angle X$
$\angle 5 \cong \angle 6$
Prove: $\overline{ZW} \cong \overline{YW}$



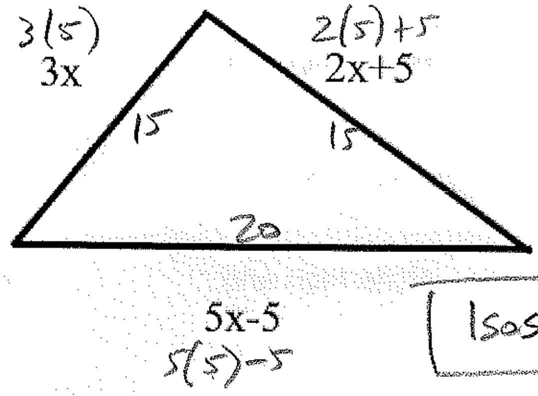


1. $\overline{YW}$ bis. $\overline{AX}$, $\angle A \cong \angle X$, $\angle 5 \cong \angle 6$	1. Givens
2. $\angle AWY \cong \angle XWZ$	2. Addition prop.
3. $\triangle AWY \cong \triangle XWZ$	3. ASA
4. $\overline{ZW} \cong \overline{YW}$	4. CPCTC

Geometry, 3-6 Notes – Types of Triangles

Type of Triangle	Definition	Examples - Special Naming
Acute	all angles acute	
Right	one right angle	
Obtuse	one obtuse angle	
Equiangular	3 angles congruent	
Equilateral	3 sides congruent	
Isosceles	2 sides congruent	 
Scalene	no sides congruent	

If the perimeter of the triangle is 50, determine what kind of triangle it is.



$P = 3x + 2x + 5 + 5x - 5$

$\frac{50}{10} = \frac{10x}{10}$

$5 = x$

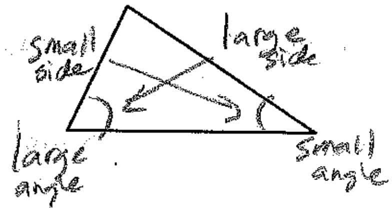
$\boxed{\text{Isosceles triangle}}$

Geometry, 3-7 Notes – Angle-Side Theorems

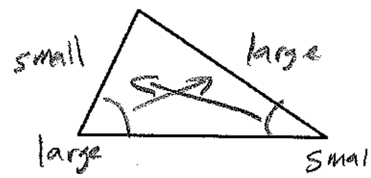
The angle-side theorems describe the relationship of angles and their opposite sides.

Unequal angles/sides:

If two sides of a triangle are not congruent,
then the angles opposite them are not congruent,
and the larger angle is opposite the longer side.

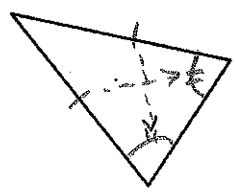


If two angles of a triangle are not congruent,
then the sides opposite them are not congruent,
and the longer side is opposite the larger angle.

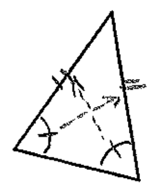


Equal angles/sides:

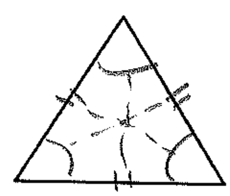
If two sides of a triangle are congruent,
then the angles opposite the sides are congruent.
Short version: If △ then △



If two angles of a triangle are congruent,
then the sides opposite the angles are congruent.
Short version: If △ then △

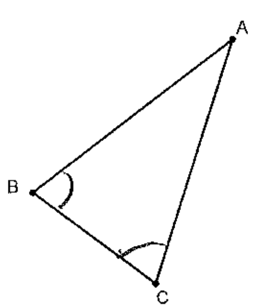


Equilateral = Equiangular (only for triangles) Why?



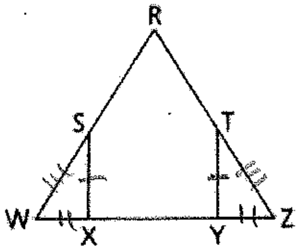
Two ways to prove triangles are isosceles:

1. If at least 2 sides of a triangle are congruent, then the triangle is isosceles.
2. If at least 2 angles of a triangle are congruent, then the triangle is isosceles.



Given: $\angle B \cong \angle C$
Prove: ΔABC is isosceles.

S	R
1. $\angle B \cong \angle C$	1. Given
2. ΔABC is isosceles	2. base angles of isosceles Δ are $\cong$



Given: $\overline{SX} \cong \overline{TY}$

$\overline{WX} \cong \overline{YZ}$

$\overline{SW} \cong \overline{TZ}$

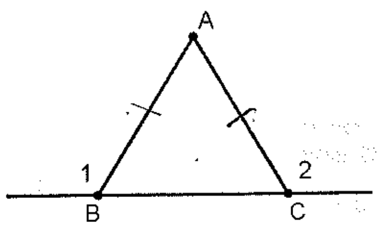
Prove: $\overline{RW} \cong \overline{RZ}$

S	R
1. $\overline{SX} \cong \overline{TY}$	1. Given
2. $\overline{WX} \cong \overline{YZ}$	2. Given
3. $\overline{SW} \cong \overline{TZ}$	3. Given
4. $\Delta WSX \cong \Delta ZTY$	4. SSS
5. $\angle W \cong \angle Z$	5. CPCTC
6. ΔRWZ is isosceles	6. base ∠'s of isosceles Δ are ≅
7. $\overline{RW} \cong \overline{RZ}$	7. legs of isosceles Δ are ≅

Practice:

#1. Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle 1 \cong \angle 2$



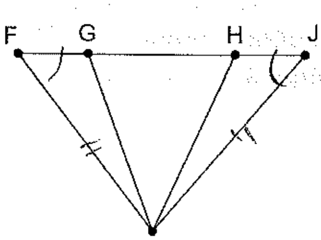
S	R
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\angle 1 \cong \angle 2$	2. If Δ then Δ

#2. Given: $\overline{FH} \cong \overline{GJ}$

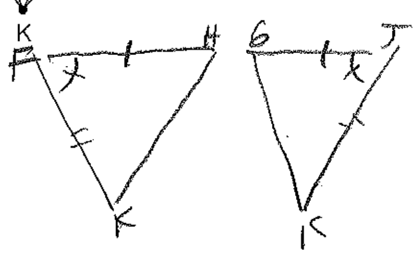
ΔFKJ is isosceles,

($\overline{FK} \cong \overline{JK}$)

Prove: $\Delta FKH \cong \Delta JKG$

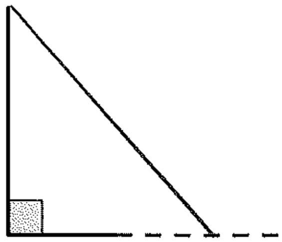


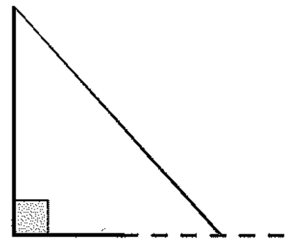
S	R
1. $\overline{FH} \cong \overline{GJ}$	1. Given
2. ΔFKJ is isosceles ($\overline{FK} \cong \overline{JK}$)	2. Given
3. $\angle F \cong \angle J$	3. base ∠s of iso. Δ ≅
4. $\Delta FKH \cong \Delta JKG$	4. SAS



Geometry, 3-8 Notes – HL Shortcut

Activity: (need a ruler)





A 4th triangle congruency shortcut: HL (hypotenuse-leg)

If the Hypotenuse and one Leg are congruent between two right triangles, then the triangles are congruent.

Note: only true for right triangles.

Still need 3 things for this shortcut:

1. One leg congruent.
2. Hypotenuse congruent.
3. Right triangles.

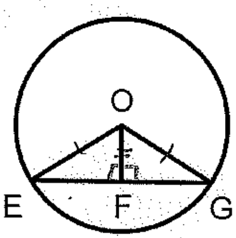
Example:

Given: $\odot O$

$\overline{OF}$ is an altitude.

Prove: $\overline{EF} \cong \overline{FG}$

$\triangle OFE \cong \triangle OFG$
$\overline{OF} \cong \overline{OF}$
$\overline{OE} \cong \overline{OG}$

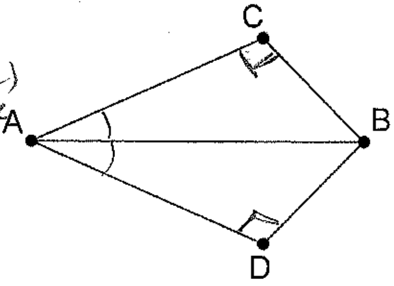


S	R
1. $\odot O$	1. Given
2. $\overline{OF}$ is altitude	2. Given
3. $\angle OFG = \angle OFE =$ right $\angle$	3. def. of altitude
4. $\overline{OE} \cong \overline{OG}$	4. radii $\cong$
5. $\overline{OF} \cong \overline{OF}$	5. Reflexive prop.
6. $\triangle OFE \cong \triangle OFG$	6. HL
7. $\overline{EF} \cong \overline{FG}$	7. CPCTC

Example: Given: $\overline{BC} \perp \overline{AC}$, $\overline{BD} \perp \overline{AD}$, $\overline{AC} \cong \overline{AD}$

Prove: $\overline{AB}$ bisects $\angle CAD$

$\angle CAB \cong \angle DAB$
$\triangle CAB \cong \triangle DAB$ (HL)
$\angle ACB = \angle ADB =$ rt $\angle$s
$\overline{AB} \cong \overline{AB}$



S	R
1. $\overline{BC} \perp \overline{AC}$, $\overline{BD} \perp \overline{AD}$	1. Given
2. $\overline{AC} \cong \overline{AD}$	2. Given
3. $\overline{AB} \cong \overline{AB}$	3. Reflexive prop.
4. $\angle ACB \cong \angle ADB =$ right $\angle$	4. $\perp$ lines meet at right $\angle$s
5. $\triangle CAB \cong \triangle DAB$	5. HL
6. $\angle CAB \cong \angle DAB$	6. CPCTC
7. $\overline{AB}$ bis. $\angle CAD$	7. bis divides $\angle$ into $\cong$ parts.