DiffEq - Ch 8 - Required Practice **Name: ______________________________**

8.1

#1. Write the linear system in matrix form:

$$\frac{dx}{dt} = 3x - 5y$$

$$\frac{dy}{dt} = 4x + 8y$$

#2. Write the linear system in matrix form:

$$\frac{dx}{dt} = -3x + 4y - 9z$$

$$\frac{dy}{dt} = 6x - y$$

$$\frac{dz}{dt} = 10x + 4y + 3z$$

#3. Write the linear system in matrix form:

$$\frac{dx}{dt} = x - y + z + t - 1$$

$$\frac{dy}{dt} = 2x + y - z - 3t^2$$

$$\frac{dz}{dt} = x + y + z + t^2 - t + 2$$

#4. Write the given system without the use of matrices:

$$\vec{X}' = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \vec{X} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t$$

#5. Write the given system without the use of matrices:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 1 \\ -2 & 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} e^{-t} - \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} t$$

#6. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\frac{dx}{dt} = 3x - 4y \qquad \vec{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-5t}$$

$$\frac{dy}{dt} = 4x - 7y$$

#7. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\vec{X}' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} \vec{X}; \qquad \vec{X} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-\frac{3}{2}t}$$

#8. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\vec{X}' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} \vec{X}; \qquad \vec{X} = \begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix}$$

#9. The given vectors are solutions of a system $\vec{X}' = \vec{A}\,\vec{X}$. Determine whether the vectors form a fundamental set on the interval $(-\infty, \infty)$:

$$\vec{X}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}, \quad \vec{X}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t}$$

#10. Verify that the vector $\vec{X}_P$ is a particular solution of the given system:

$$\vec{X}' = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \vec{X} + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix}; \qquad \vec{X}_P = \begin{bmatrix} 2 \\ -1 \end{bmatrix} t + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

8.2 day 1

#1. Find the general solution of the system:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = 4x + 3y$$

#2. Find the general solution of the system:

$$\frac{dx}{dt} = -4x + 2y$$

$$\frac{dy}{dt} = -\frac{5}{2}x + 2y$$

#3. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} \vec{X}$$

#4. Find the general solution of the system:

$$\frac{dx}{dt} = x + y - z$$

$$\frac{dy}{dt} = 2y$$

$$\frac{dz}{dt} = y - z$$

#5. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix} \vec{X}$$

#6. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 1/2 & 0 \\ 1 & -1/2 \end{bmatrix} \vec{X}, \quad \vec{X}(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

8.2 day 2

#1. Find the general solution of the system:

$$\frac{dx}{dt} = 3x - y$$

$$\frac{dy}{dt} = 9x - 3y$$

#2. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \vec{X}$$

#3. Find the general solution of the system:

$$\frac{dx}{dt} = 3x - y - z$$

$$\frac{dy}{dt} = x + y - z$$

$$\frac{dz}{dt} = x - y + z$$

#4. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} \vec{X}$$

#5. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \vec{X}$$

#6. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

8.2 day 3

#1. Find the general solution of the system:

$$\frac{dx}{dt} = 6x - y$$

$$\frac{dy}{dt} = 5x + 2y$$

#2. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 4 & -5 \\ 5 & -4 \end{bmatrix} \vec{X}$$

#3. Find the general solution of the system:

$$\frac{dx}{dt} = z$$

$$\frac{dy}{dt} = -z$$

$$\frac{dz}{dt} = y$$

#4. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 2 & 5 & 1 \\ -5 & -6 & 4 \\ 0 & 0 & 2 \end{bmatrix} \vec{X}$$

#5. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

8.3 day 1

#1. Use the method of undermined coefficients to solve the system:

$$\frac{dx}{dt} = 2x + 3y - 7$$

$$\frac{dy}{dt} = -x - 2y + 5$$

#2. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} -2t^2 \\ t+5 \end{bmatrix}$$

#3. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} 4 & \frac{1}{3} \\ 9 & 6 \end{bmatrix} \vec{X} + \begin{bmatrix} -3 \\ 10 \end{bmatrix} e^t$$

#4. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{bmatrix} \vec{X} + \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} e^{4t}$$

#5. Use the method of undermined coefficients to solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \vec{X} + \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \qquad \vec{X}(0) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

8.3 day 2

#1. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \vec{X} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t$$

#2. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & 8 \\ 1 & -1 \end{bmatrix} \vec{X} + \begin{bmatrix} 12 \\ 12 \end{bmatrix} t$$

#3. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{X} + \begin{bmatrix} \sec t \\ 0 \end{bmatrix}$$

#4. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^t$$

8.3 day 3

#1. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}\vec{X} + \begin{bmatrix} 4e^{2t} \\ 4e^{4t} \end{bmatrix}, \quad \vec{X}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

8.3 day 4

#1. Rewrite the following 3rd-order differential equation as a system of 3 1st-order differential equations (and also write the initial conditions in matrix form):

$$y''' - 2y'' + 3y' = 9 + e^x$$

$$y(0) = 3, \qquad y'(0) = 0, \qquad y''(0) = -2$$

(do not solve the system)

Ch8 Test Review

#1. Find the general solution of the system:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = 4x + 3y$$

#2. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} \vec{X}$$

#3. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} -6 & 2 \\ -3 & 1 \end{bmatrix} \vec{X}$$

#4. Find the general solution of the system:

$$\frac{dx}{dt} = 3x - y$$

$$\frac{dy}{dt} = 9x - 3y$$

#5. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \vec{X}$$

#6. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

#7. Find the general solution of the system:

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = -2x - y$$

#8. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 4 & -5 \\ 5 & -4 \end{bmatrix} \vec{X}$$

#9. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

#10. Solve both ways (method of undermined coefficients and variation of parameters):

$$\vec{X}' = \begin{bmatrix} 4 & 1/3 \\ 9 & 6 \end{bmatrix} \vec{X} + \begin{bmatrix} -3 \\ 10 \end{bmatrix} e^t$$

#11. Solve both ways (method of undermined coefficients and variation of parameters):

$$\vec{X}' = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} \vec{X} + \begin{bmatrix} 2e^{-t} \\ e^{-t} \end{bmatrix}$$

#12. Rewrite the following 3rd-order differential equation as a system of 3 1st-order differential equations (and also write the initial conditions in matrix form):

$$y''' - y'' + 2y' - 3y = t^3 - 4t$$

$$y(0) = 3, \qquad y'(0) = 7, \qquad y''(0) = 5$$

(do not solve the system)