DiffEq - Ch 7 - Required Practice **Name: ______________________________**

7.1

#1. Use the integral definition of the Laplace Transform to find $\mathscr{L}\left\{f\left(t\right)\right\}$

$$f\left(t\right)=\begin{cases}-1, & 0\le t<1\\ 1, & t\ge 1\end{cases}$$

#2. Use the integral definition of the Laplace Transform to find $\mathscr{L}\left\{f\left(t\right)\right\}$

$$f\left(t\right)=\begin{cases}t, & 0\le t<1\\ 1, & t\ge 1\end{cases}$$

#3. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$. Then use the table to verify your answer.

$f(t)=e^{t+7}$

#4. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$. Then use the table to verify your answer.

$f(t)=te^{4t}$

#5. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=2t^4$

#6. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^2+6t-3$

#7. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=1+e^{4t}$

#8. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=\left(1+e^{2t}\right)^2$

#9. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=4t^2-5\sin(3t)$

7.2 day 1

#1. Use theorems and the table to find $\mathscr{L}^{-1}\left\{\frac{1}{s^3}\right\}$

#2. Use theorems and the table to find $\mathscr{L}^{-1}\left\{\frac{1}{s^2}-\frac{48}{s^5}\right\}$

#3. Use theorems and the table to find $\mathscr{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\}$

#4. Use theorems and the table to find $\mathscr{L}^{-1}\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s-2}\right\}$

#5. Use theorems and the table to find $\mathscr{L}^{-1}\left\{\frac{1}{4s+1}\right\}$

#6. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{5}{s^2+49}\right\}$$

#7. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\}$$

#8. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\}$$

#9. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2+3s}\right\}$$

#10. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\}$$

#11. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\}$$

#12. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$$

#13. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{s^3+5s}\right\}$$

#14. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{2s-4}{\left(s^2+s\right)\left(s^2+1\right)}\right\}$$

#15. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{\left(s^2+1\right)\left(s^2+4\right)}\right\}$$

7.2 day 2

#1. Use the Laplace transform to solve the initial-value problem:

$$\frac{dy}{dt} - y = 1, \quad y(0) = 0$$

#2. Use the Laplace transform to solve the initial-value problem:

$$y' + 6y = e^{4t}, \quad y(0) = 2$$

#3. Use the Laplace transform to solve the initial-value problem:

$$y' - y = 2\cos(5t), \quad y(0) = 0$$

#4. Use the Laplace transform to solve the initial-value problem:

$y'' + 5y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$

#5. Use the Laplace transform to solve the initial-value problem:

$y'' + y = \sqrt{2}\sin\left(\sqrt{2}\,t\right), \quad y(0) = 10, \quad y'(0) = 0$

#6. Use the Laplace transform to solve the initial-value problem:

$$2y''' + 3y'' - 3y' - 2y = e^{-t}$$

$$y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1$$

7.3 day 1

#1. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{te^{10t}\right\}$

#2. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{t^3e^{-2t}\right\}$

#3. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{t\left(e^t+e^{2t}\right)^2\right\}$

#4. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{e^t\sin(3t)\right\}$

#5. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{\left(1-e^t+3e^{-4t}\right)\cos(5t)\right\}$

#6. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\}$

#7. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

#8. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

#9. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\}$

#10. Use the Laplace transform to solve the initial-value problem

$y' + 4y = e^{-4t}, \quad y(0) = 2$

#11. Use the Laplace transform to solve the initial-value problem

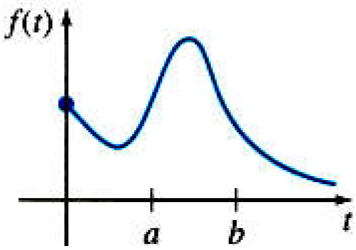
$y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1$

#12. Use the Laplace transform to solve the initial-value problem

$y'' - 6y' + 9y = t, \quad y(0) = 0, \quad y'(0) = 1$

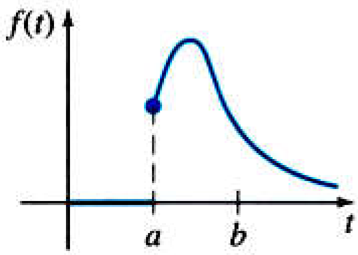
7.3 day 2

For #1-6, the function $f(t)$ is given by the graph:

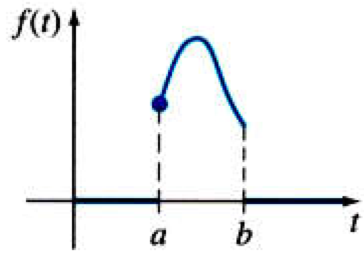


Given the graph of a modified version of this function, use a combination of terms with the original function, time shifted original function and the unit step function to write a new function that produces the function in the problem's graph.

#1.



#2.



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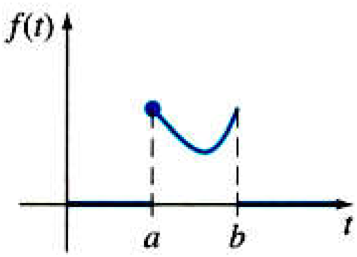

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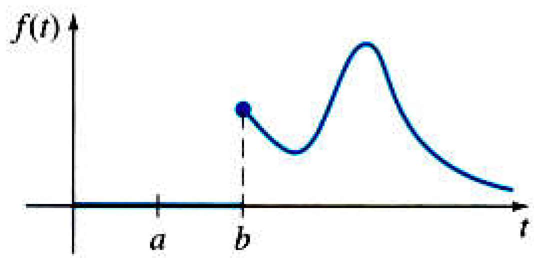
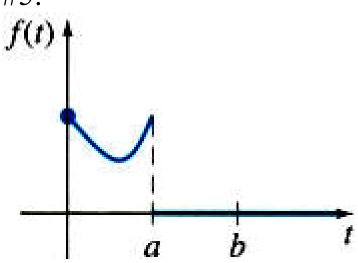
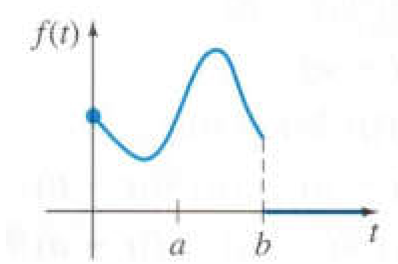


#5.



#6.



7.3 day 3

#1. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{(t-1)\,\mathcal{U}(t-1)\}$

#2. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t\,\mathcal{U}(t-2)\}$

#3. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{\cos(2t)\,\mathcal{U}(t-\pi)\}$

#4. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}}{s^3}\right\}$

#5. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$

#6. Evaluate the Inverse Laplace transform to find

$f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\}$

#7. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t) = \begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$$

#8. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t^2, & t \ge 1 \end{cases}$$

#9. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

#10. Use the Laplace transform to solve the initial-value problem:

$$y' + y = f(t), \quad y(0) = 0$$

$$\textit{where}\ \ f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 5, & t \ge 1 \end{cases}$$

#11. Use the Laplace transform to solve the initial-value problem:

$$y' + 2y = f(t), \quad y(0) = 0$$

$$\text{where } f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

#12. Use the Laplace transform to solve the initial-value problem:

$$y'' + 4y = \sin t \, \mathcal{U}(t - 2\pi),$$

$$y(0) = 1, \quad y'(0) = 0$$

#13. Use the Laplace transform to solve the initial-value problem:

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1$$

$$\text{where } f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 1, & \pi \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$$

7.4

#1. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{te^{-10t}\right\}$

#2. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{t\cos(2t)\right\}$

#3. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{te^{2t}\sin(6t)\right\}$

#4. Use the Laplace transform to solve the initial-value problem:

$y' + y = t\sin(t), \quad y(0) = 0$

#5. Use the Laplace transform to solve the initial-value problem:

$$y'' + 9y = \cos(3t), \quad y(0) = 2, \quad y'(0) = 5$$

#6. Use the Laplace transform to solve the initial-value problem:

$$y'' + 16y = f(t), \quad y(0) = 0, \quad y'(0) = 1$$

$$\textit{where} \;\; f(t) = \begin{cases} \cos(4t), & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$

7.5

#1. Use the Laplace transform to solve the initial-value problem:
$y' - 3y = \delta(t-2), \quad y(0) = 0$

#2. Use the Laplace transform to solve the initial-value problem:
$y'' + y = \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 1$

#3. Use the Laplace transform to solve the initial-value problem:
$y'' + y = \delta\left(t - \frac{1}{2}\pi\right) + \delta\left(t - \frac{3}{2}\pi\right),$
$y(0) = 0, \quad y'(0) = 0$

#4. Use the Laplace transform to solve the initial-value problem:
$y'' + 2y' = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 1$

Ch7 Test Review

#1. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t) = 2t^4$

#2. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t) = t^2 + 6t - 3$

#3. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t) = t^2 - e^{-9t} + 5$

#4. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$

#5. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$

#6. Use the Laplace transform to solve the initial-value problem:

$y' + 6y = e^{4t}, \quad y(0) = 2$

#7. Use the Laplace transform to solve the initial-value problem:

$y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1, \quad y'(0) = -1$

#8. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

#9. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$

#10. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

#11. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}$

#12. Use the Laplace transform to solve the initial-value problem

$y'' - 6y' + 9y = t, \quad y(0) = 0, \quad y'(0) = 1$

#13. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}}{s^3}\right\}$

#14. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$

#15. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$$

#16. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

#17. Evaluate the Laplace transform to find $F(s)$

$$\mathscr{L}\left\{te^{-10t}\right\}$$

#18. Evaluate the Laplace transform to find $F(s)$

$$\mathscr{L}\left\{te^{2t}\sin(6t)\right\}$$

#19. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{te^{-3t}\cos(3t)\right\}$

#20. Use the Laplace transform to solve the initial-value problem

$y'' - 2y' + y = e^t, \quad y(0) = 0, \quad y'(0) = 5$