DiffEq - Ch 4 - Required Practice **Name: ____________________________**

4.1

#1. The given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

$y = C_1e^x + C_2e^{-x}, \quad (-\infty, \infty);$

$y'' - y = 0, \quad y(0) = 0, \quad y'(0) = 1$

#2. The given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

$y = C_1x + C_2x\ln x, \quad (0, \infty);$

$x^2y'' - xy' + y = 0, \quad y(1) = 3, \quad y'(1) = -1$

#3. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = x, \quad f_2(x) = x^2, \quad f_3(x) = 4x - 3x^2$

#4. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = 0, \quad f_2(x) = x, \quad f_3(x) = e^x$

#5. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = 1 + x, \quad f_2(x) = x, \quad f_3(x) = x^2$

#6. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$y'' - y' - 12y = 0; \quad e^{-3x}, e^{4x}, \ (-\infty, \infty)$

#7. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$y'' - 2y' + 5y = 0; \quad e^x \cos 2x, e^x \sin 2x, \ (-\infty, \infty)$

#8. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$x^2 y'' - 6xy' + 12y = 0; \quad x^3, x^4, \ (0, \infty)$

4.2

#1. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$y'' - 4y' + 4y = 0; \quad y_1 = e^{2x}$

#2. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$y'' + 16y = 0; \quad y_1 = \cos(4x)$

#3. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$9y'' - 12y' + 4y = 0; \quad y_1 = e^{\left(\frac{2}{3}x\right)}$

#4. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$x^2 y'' - 7xy' + 16y = 0; \quad y_1 = x^4$

#5. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$xy'' + y' = 0; \quad y_1 = \ln x$

4.3

#1. Find the general solution of the differential equation.
$4y'' + y' = 0$

#2. Find the general solution of the differential equation.
$y'' + 8y' + 16y = 0$

#3. Find the general solution of the differential equation.
$12y'' - 5y' - 2y = 0$

#4. Find the general solution of the differential equation.
$y'' + 9y = 0$

#5. Find the general solution of the differential equation.
$y'' - 4y' + 5y = 0$

#6. Find the general solution of the differential equation.
$3y'' + 2y' + y = 0$

#7. Find the general solution of the differential equation.
$y''' - 5y'' + 3y' + 9y = 0$

#8. Find the general solution of the differential equation.

$y''' + 3y'' + 3y' + y = 0$

#9. Solve the initial-value problem.

$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = 0, \quad y(1) = 0, \; y'(1) = 2$

#10. Solve the initial-value problem.

$y'' + y' + 2y = 0, \quad y(0) = 0, \;\; y'(0) = 0$

#11. Solve the initial-value problem.

$y''' + 12y'' + 36y' = 0, \quad y(0) = 0, \; y'(0) = 1, \; y''(0) = -7$

4.4

#1. Solve the differential equation using the method of undermined coefficients.
$y'' + 3y' + 2y = 6$

#2. Solve the differential equation using the method of undermined coefficients.
$y'' - 10y' + 25y = 30x + 3$

#3. Solve the differential equation using the method of undermined coefficients.

$$\frac{1}{4}y'' + y' + y = x^2 - 2x$$

#4. Solve the differential equation using the method of undermined coefficients.

$$y'' + 3y = -48x^2e^{3x}$$

#5. Solve the differential equation using the method of undermined coefficients.

$y'' - 16y = 2e^{4x}$

#6. Solve the differential equation using the method of undermined coefficients.
$y'' + y = 2x \sin x$

#7. Solve the initial-value problem.

$5y'' + y' = -6x, \quad y(0) = 0, \quad y'(0) = -10$

4.6 (we skip 4.5)

#1. Solve the differential equation using the method of variation of parameters.
$y'' + y = \sec x$

#2. Solve the differential equation using the method of variation of parameters.
$y'' + y = \sin x$

#3. Solve the differential equation using the method of variation of parameters.

$$y'' - 9y = \frac{9x}{e^{3x}}$$

#4. Solve the differential equation using the method of variation of parameters.

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

#5. Solve the initial value problem using variation of parameters.

$$4y'' - y = xe^{\left(\frac{1}{2}x\right)}$$

4.7

#1. Solve the differential equation.
$x^2 y'' - 2y = 0$

#2. Solve the differential equation.
$x^2 y'' + 5xy' + 3y = 0$

#3. Solve the differential equation.
$x^2 y'' - 3xy' - 2y = 0$

#4. Solve the differential equation by variation of parameters.

$xy'' - 4y' = x^4$

#5. Use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients, then solve.

$x^2 y'' + 9xy' - 20y = 0$

#6. Use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients, then solve.

$$x^2 y'' + 10xy' + 8y = x^2$$

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#1. Given one solution to the differential equation (no need to verify), use reduction of order to find a second solution.

$y'' - 25y = 0; \quad y_1 = e^{5x}$

#2. Given one solution to the differential equation (no need to verify), use reduction of order to find a second solution.

$9y'' - 12y' + 4y = 0; \quad y_1 = e^{\left(\frac{2}{3}x\right)}$

#3. Use a Wronskian to determine if the following set of functions is linearly independent on the interval $(0,\infty)$

$f_1(x) = e^{4x}, \quad f_2(x) = e^{2x}$

#4. Use a Wronskian to determine if the following set of functions is linearly independent on the interval $(0,\infty)$

$f_1(x) = x, \quad f_2(x) = x-1, \quad f_3(x) = x+3$

#5. Solve the differential equation.

$y'' - 2y' - 2y = 0$

#6. Solve the differential equation.

$2y'' + 2y' + 3y = 0$

#7. Solve the differential equation.
$y''' - 5y'' + 3y' + 9y = 0$

#8. Solve the differential equation by method of undetermined coefficients (table method).
$y'' + 4y = 3\sin(2x)$

#9. Solve the differential equation by method of undetermined coefficients (table method).

$$y'' + 8y' + 16y = 2x^2 - 3, \quad y(0) = \frac{247}{64}, \quad y'(0) = \frac{-153}{8}$$

#10. Solve the differential equation by method of undetermined coefficients (table method).

$$y'' - 16y = 2e^{4x}$$

#11. Solve the differential equation by variation of parameters (Wronskian method).

$$y'' - 9y = \frac{9x}{e^{3x}}$$

#12. Solve the differential equation by variation of parameters (Wronskian method).

$y'' + y = \sec^3 x$

#13. Solve this Cauchy-Euler form differential equation using the Cauchy-Euler procedure for the homogeneous solution. Then use the variation of parameters (Wronskian method) to find the particular solution. Finally, join the solutions to produce the general solution.

$$2x^2 y'' + 5xy' + y = x^2 - x$$

#14. Solve this Cauchy-Euler form differential equation using the Cauchy-Euler procedure for the homogeneous solution. Then use the variation of parameters (Wronskian method) to find the particular solution. Finally, join the solutions to produce the general solution.

$xy'' - 4y' = x^4$