DiffEq - Ch 1-2 - Required Practice **Name: ______________________________**

1.1

State the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear.

#1. $(1-x)y''-4xy'+5y=\cos x$

#2. $t^5y^{(4)}-t^3y''+6y=0$

#3. $\dfrac{d^2y}{dx^2}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

#4. $(\sin\theta)y'''-(\cos\theta)y'=2$

#5. Determine whether $\left(y^2-1\right)dx+xdy=0$

(i) is linear in y?

(ii) is linear is x?

#6. Verify that $y=e^{-\frac{1}{2}x}$ is an explicit solution of
$2y'+y=0$
(don't worry about interval of definition).

#7. Verify that $y = x + 4\sqrt{x+2}$ is an explicit solution of $(y-x)y' = y - x + 8$

Then give at least one interval *I* of definition for this solutions.

#8. Verify that $\ln\left(\frac{2x-1}{x-1}\right) = t$ is an implicit solution of $\frac{dx}{dt} = (x-1)(1-2x)$

Then find at least one explicit solution $x = \phi(t)$, and graph this solution. Finally, give an interval *I* of definition for your solution.

#9. Find values of m so that the function $y = x^m$ is a solution of $xy'' + 2y' = 0$

#10. Verify that the pair of functions…

$x = e^{-2t} + 3e^{6t}$

$y = -e^{-2t} + 5e^{6t}$

is a solution of the system of differential equations…

$$\begin{cases} \dfrac{dx}{dt} = x + 3y \\ \dfrac{dy}{dt} = 5x + 3y \end{cases}$$

1.2

#1. $y = \dfrac{1}{1+C_1e^{-x}}$ is a one-parameter family of solutions of the first-order DE $y' = y - y^2$. Find a solution of the first-order Initial Value Problem given initial condition $y(0) = -\dfrac{1}{3}$.

#2. $y = \dfrac{1}{x^2+c}$ is a one-parameter family of solutions of the first-order DE $y' + 2xy^2 = 0$. Find a solution of the first-order Initial Value Problem given initial condition $y(2) = \dfrac{1}{3}$. Also, give the largest interval *I* over which the solution is defined.

#3. $x = C_1 \cos t + C_2 \sin t$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$. Find a solution of the second-order Initial Value Problem given initial conditions $x(0) = -1,\ \ x'(0) = 8$.

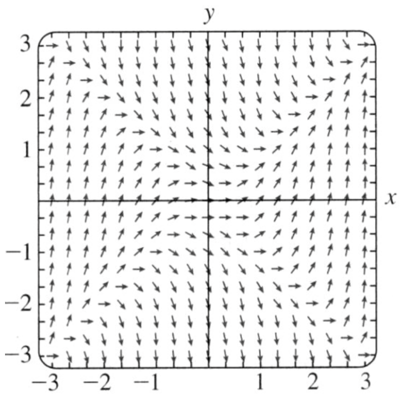
#4. $y = C_1 e^x + C_2 e^{-x}$ is a two-parameter family of solutions of the second-order DE $y'' - y = 0$. Find a solution of the second-order Initial Value Problem given initial conditions $y(0) = 1, \quad y'(0) = 2$.

#5. Determine a region of the *xy*-plane for which the given differential equation would have a unique solution whose graph passes through a point (x_0, y_0) in the region. $\frac{dy}{dx} = y^{2/3}$

#6. Determine whether the differential equation $y' = \sqrt{y^2 - 9}$ possesses a unique solutions through the point $(1, 4)$.

2.1

#1. A direction field is given for the differential equation $\dfrac{dy}{dx} = x^2 - y^2$:



Without using a calculator, sketch an approximate solution curve on the direction field that passes through each of the indicated points.

(a) $y(-2) = 1$ (b) $y(3) = 0$

(c) $y(0) = 2$ (d) $y(0) = 0$

#2. For the differential equation $\dfrac{dy}{dx} = x + y$, sketch a few isoclines $\left(f(x,y) = c\right)$ for $-5 \le c \le 5$. Then, construct a direction field by drawing lineal elements with the appropriate slope to match each isocline. Finally, use this rough direction field to sketch an approximate solution curve with the initial condition $y(0) = 1$.

#3. For the autonomous, first-order differential equation $\dfrac{dy}{dx} = y^2 - 3y$, find the critical points and phase portrait. Then, classify each critical point as asymptotically stable, unstable, or semi-stable. Finally, sketch typical solution curves in the regions in the *xy*-plan determined by the graphs of the equilibrium solutions.

2.2

#1. Solve by separation of variables: $\frac{dy}{dx} = \sin 5x$

#2. Solve by separation of variables:
$dx + e^{3x}dy = 0$

#3. Solve by separation of variables: $x\frac{dy}{dx} = 4y$

#4. Solve by separation of variables: $\frac{dy}{dx} = e^{3x+2y}$
(leave solution in implicit form)

#5. Solve by separation of variables:
$y\ln x\frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$
(leave solution in implicit form)

#6. Solve by separation of variables:
$\csc y\, dx + \sec^2 x\, dy = 0$

#7. Find an explicit solution of the given initial-value problem: $\dfrac{dx}{dt} = 4\left(x^2+1\right), \quad x\left(\dfrac{\pi}{4}\right) = 1$

2.3

#1. Find the general solutions of the differential equation $\frac{dy}{dx} = 5y$. Then, give the largest interval I over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#2. Find the general solutions of the differential equation $x^2 y' + xy = 1$. Then, give the largest interval I over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#3. Find the general solutions of the differential equation $\frac{dr}{d\theta} + r\sec\theta = \cos\theta$. Then, give the largest interval I over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#4. Solve the initial-value problem:

$xy' + y = e^x, \quad y(1) = 2$

#5. Solve the initial-value problem:

$L\frac{di}{dt} + Ri = E, \quad i(0) = i_0$

$\left[L, R, E, \text{ and } i_0 \text{ are constants}\right]$.

2.4

#1. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(2x-1)\,dx+(3y+7)\,dy=0$$

#2. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(2xy^2-3)\,dx+(2x^2y+4)\,dy=0$$

#3. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(x^2-y^2)\,dx+(x^2-2xy)\,dy=0$$

#4. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$\frac{dy}{dx}=\frac{x-y^3+y^2\sin x}{3xy^2+2y\cos x}$$

#5. Solve the initial-value problem:
$$(x+y)^2\,dx+(2xy+x^2-1)\,dy=0,\quad y(1)=1$$

#6. Solve the given differential equation by first finding an integrating factor, then using it to convert the differential equation to exact form:
$$(2y^2+3x)\,dx+(2xy)\,dy=0$$

2.5

#1. Solve the Bernoulli form differential equation by using an appropriate substitution:

$$x\frac{dy}{dx}+y=\frac{1}{y^2}$$

#2. Solve the Bernoulli form differential equation by using an appropriate substitution:

$$\frac{dy}{dx}=y\left(xy^3-1\right)$$

#3. Solve the differential equation by using an appropriate substitution: $\dfrac{dy}{dx} = (x + y + 1)^2$

#4. Solve the differential equation by using an appropriate substitution: $\dfrac{dy}{dx} = \tan^2(x + y)$

2.6

#1. Use Euler's method to obtain a four-decimal approximation of $y(0.5)$ if $y(0)=0$ and $y' = e^{-y}$. Use x-increments of 0.1.

#2. Use Euler's method to obtain a four-decimal approximation of $y(1.5)$ if $y(1)=1$ and $y' = xy^2 - \frac{y}{x}$. Use x-increments of 0.1.

Diff Eq Ch1-2 Test Review

#1. Classify each differential equation as 'separable', 'exact', 'linear' or 'Bernoulli', or 'composite' form. Some equations may be more than one kind. <u>Do not solve the differential equations, just classify them so you can identify which methods could use used to solve them</u>.

(a) $\dfrac{dy}{dx} = \dfrac{x-y}{x}$

(b) $\dfrac{dy}{dx} = \dfrac{1}{y-x}$

#1 continued...

(c) $\left(x+1\right)\dfrac{dy}{dx}=-y+10$

(d) $\dfrac{dy}{dx}=\dfrac{1}{x\left(x-y\right)}$

#1 continued...

(e) $\dfrac{dy}{dx} = \dfrac{y^2 + y}{x^2 + x}$

(f) $\dfrac{dy}{dx} = 5y + y^2$

#1 continued...

(g) $y\,dx = \left(y - xy^2\right) dy$

(h) $xyy' + y^2 = 2x$

#1 continued...

(i) $y\,dx + x\,dy = 0$

(j) $\left(x^2 + \dfrac{2y}{x}\right) dx = \left(3 - \ln\left(x^2\right)\right) dy$

#1 continued...

(k) $\dfrac{y}{x^2}\dfrac{dy}{dx}+e^{\left(2x^3+y^2\right)}=0$

For #2 and #3, Given the autonomous, first-order differential equation, find the critical points and phase portrait. Then, classify each critical point as asymptotically stable, unstable, or semi-stable. Finally, sketch typical solution curves in the regions in the *xy*-plan determined by the graphs of the equilibrium solutions.

#2. $\dfrac{dy}{dx}=y^2-3y$

#3. $\dfrac{dy}{dx}=y^2-y^3$

#4. Solve and write the solution in explicit form:

$dy-(y-1)^2\,dx=0$

#5. Solve and write the solution in explicit form:

$\csc y\,dx+\sec^2 x\,dy=0$

#6. Solve and write the solution in explicit form:

$3\dfrac{dy}{dx}+12y=4$

#7. Solve and write the solution in explicit form:

$x\dfrac{dy}{dx}-y=x^2\sin x$

#8. Solve and write the solution in <u>implicit</u> form:

$$\left(3x^2y+e^y\right)dx+\left(x^3+xe^y-2y\right)dy=0$$

#9. Solve (leave answer in implicit form):

$$\frac{dy}{dx}=\frac{1-x-y}{x+y}$$

#10. Find the particular solution of the initial value problem (you can leave your answer in implicit form):

$$\left(\frac{3y^2 - t^2}{y^5}\right)\frac{dy}{dt} + \frac{t}{2y^4} = 0; \qquad y(1) = 1$$

#11. Solve and write the solution in explicit form:

$$\frac{dy}{dx} = y\left(xy^3 - 1\right)$$

#12. Solve and write the solution in explicit form:

$$t^2 \frac{dy}{dt} + y^2 = ty$$