PROJECT 7

MURDER AT THE MAYFAIR

Dawn at the Mayfair (see Figure 1). The amber glow of streetlights mixed with the violent red flash of police cruisers begins to fade with the rising furnace-orange sun. Detective Diff E. Quasion exits the Mayfair Diner* holding a cup of hot joe in one hand and a summary of the crime scene evidence in the other. Taking a seat on the bumper of his tan LTD, Detective Quasion begins to review the evidence.

At 5:30 A.M. the body of the diner's owner, Joe D. Wood, was found in the walk-in refrigerator in the diner's basement. At 6:00 A.M. the coroner arrived and determined that the core body temperature of the corpse was 85° Fahrenheit. Thirty minutes later the coroner again measured the core body temperature; this time the reading was 84° Fahrenheit. A thermostat inside the refrigerator reads 50° Fahrenheit.

FIGURE 1 The Mayfair Diner

Diff takes a faded yellow legal pad and a ketchup-spattered calculator from the front seat of his cruiser and begins to compute. He knows that Newton's Law of Cooling says that the rate at which an object cools is proportional to the difference between the temperature T of the body at time t and the temperature T_m of the environment or medium surrounding the body. He jots down the differential equation

$$\frac{dT}{dt} = k(T - T_m), \quad t > 0, \tag{1}$$

where k is a constant of proportionality, T and T_m are measured in degrees Fahrenheit, and time t is measured in hours. Because Diff wants to investigate the past using positive values of time, he decides to correspond $t = 0$ with 6:00 A.M., and so, for example, $t = 4$ is 2:00 A.M. After a few scratches on his yellow pad, Diff realizes that with this time convention the constant k in (1) will turn out to be *positive*. Diff jots a reminder to himself that 6:30 A.M. is now $t = -\frac{1}{2}$.

PROBLEM 1. After much deep thought Diff decides to begin by assuming that Mr. Wood was murdered inside the refrigerator. What is the estimated time of death?

As the cool and quiet dawn gives way to a steamy midsummer morning, Diff begins to sweat and wonders aloud, "But what if he was killed in the diner and then the body was moved into the fridge in a feeble attempt to hide it? How does this change my estimate of the time of death?" He reenters the diner and finds a grease-streaked thermostat above the empty cash register. It reads 70° Fahrenheit.

"But *when* was the body moved?" Diff asks. He decides to leave this question unanswered for now and simply lets h denote the number of hours the body has been in the refrigerator prior to 6:00 A.M. For example, if $h = 6$, then the body was moved into the refrigerator at midnight.

Diff flips a page on his legal pad and begins jotting. As the rapidly cooling coffee begins to do its work, he realizes that a way to model the environmental temperature change caused by the move is with the unit step function $\mathscr{U}(t)$. He writes

$$T_m(t) = 50 + 20\,\mathscr{U}(t - h) \tag{2}$$

*The historic Mayfair Diner opened in 1932 in Philadelphia, Pennsylvania. Photo used with permission.

(…here is where we will go a slightly different direction from the classic published version of this problem ☺)

…and below that, Diff write Newton's Law of Cooling: $\frac{dT}{dt} = k\left(T - T_m\left(t\right)\right)$

But the ambient temperature is a function of time (it changes abruptly at *t = h* when the body was moved. So the differential equation that describes the warming/cooling of the body is actually:

$$\frac{dT}{dt} = k\left(T - \left(50 + 20\,\mathcal{U}\left(t-h\right)\right)\right)$$

The problem is, Diff doesn't know the cooling constant, *k*, nor the time, *h*, when the body was moved.

Let's help Diff solve the case! We can establish the *k* constant with the two temperature readings taken:

$$T\left(0\right) = 85, \qquad T\left(-0.5\right) = 84$$

We can use OCTAVE to solve the differential equation. If we set the initial condition to be *T = 84* when time is *t = -0.5*, then we can use the differential equation solution curve to model how the body 'warms up' as we go backward in time.

We will start by making a guess at the *k* constant, and see if the body temp, *T* warms up to 85 at *t = 0*. If it doesn't, we can adjust *k* and keep trying until we find the *k* that matches.

But first, we will need some way to model the Unit Step Function. In OCTAVE you can import libraries that have a number of additional useful functions, but this is cumbersome to do for a quick project, so we will just write our own function. In the OCTAVE library, this function is called 'heaviside' (named for Oliver Heaviside – the person who is actually responsible for taking Maxwell's 16 equations describing electromagnetism and expressing them in the form we have today using divergence and curl).

Step 1) Create a Unit Step Function called 'heaviside'

<u>Step 1a</u>) Copy any of the function.m file to a new file called heavisidefunction.m.

<u>Step 1b</u>) Change the code to the following…

```
function y = heavisidefunction (x,h)
  y=((x-h)>0)+0.5*((x-h)==0);
endfunction
```

Step 2) Create the files to model the body warming differential equation and find *k*

Step 2a) Copy any of the function.m and script.m files to create new files called DVmayfairfunction.m and DVmayfairscript.m.

Step 2b) Change the coding to the following:

DVmayfairfunction.m

```
function returnValue = DVmayfairfunction (t, T)
  k=0.2;
  returnValue=k*(T-50);
endfunction
```

This first version of the function file doesn't include the Unit Step Function, because we are only going to use it from time *t = -0.5* to *t = 0* when the ambient temperature is a constant 50. We're starting with our first guess at *k* to be 0.2.

DVmayfairscript.m

```
clear all
clc
trange=[-0.5,1];
initialT=84;
[t,T]=ode45(@DVmayfairfunction,trange,initialT);
plot(t,T)
ylabel('Temperature, *F')
xlabel('time (hrs)')
```

This version of the script file just models from *t = -0.5* to *t = 0* hours, and starts the body at the last temperature measured of 84 (remember, time is positive backwards).

Step 2c) Run the program and see if *k* is accurate. When you run the simulation, the solution curve shows the body temperature warming up as time is running backwards. If our value of *k* is correct, the body temperature will be 85 at *t = 0*. (Our first guess is not accurate…*k* is something different from 0.2).

Step 2d) Edit the function file to try different values of *k* until the body temp curve is as close as possible to 85 when *t=0*. Record your value of *k* here, and leave that value in your function file for the rest of this activity.

k = ________

Step 3) Update the files to include the Heaviside function to model the entire night

Step 3a) Change the coding to the following:

DVmayfairfunction.m

```
function returnValue = DVmayfairfunction (t, T)
  k=(the value you determined to be correct)
  h=2;
  returnValue=k*(T-(50+20*heavisidefunction(t,h)));
endfunction
```

This new version of the function file includes the Unit Step Function to change the temperature at the time the body was moved). We've introduced a new constant, *h*, which is the number of hours before 6:00AM when the body was moved into the freezer. We will try adjusting this constant like we did the *k* constant.

DVmayfairscript.m

```
clear all
clc
trange=[0,10];
initialT=85;
[t,T]=ode45(@DVmayfairfunction,trange,initialT);
plot(t,T)
ylabel('Temperature, *F')
xlabel('time (hrs)')
```

The only change here is we make the time range from 0 to 10 hours and change the initialT to 85 (so this is modeling from 6:00AM backwards in time).

Step 3b) Now we can try different values for *h* and see how the body temperature changes. But we need something to use to find time of death. We can assume that the body temp was normal 98.6 *F at the time of death. But unlike when we determined *k*, we don't know the time when this temp of 98.6 occurred. So the best we can do is to try some different values of *h* and record what time the body would have been at 98.6 to see if that provides more insight into who might have committed the murder.

To collect our info, try each integer value of *h* from 2 to 12, run the model each time and on the plot, determine the time *t* when the body would have been 98.6 *F. Fill in the table on the next page with your findings.

h	Time body was moved	*t* when T = 98.6	Time of death
7			
6			
5			
4			
3			
2			

Step 3c) Only some of these trials are even possible. For example, if you calculated a time of death that was after the body was moved, you should cross that off – the victim wouldn't have allowed himself to be put in the freezer if he were still alive ☺

Step 4) Solve the Murder!

Okay, how does all this help us solve the murder? Well, the classic problem also includes some additional information about possible suspects….

Shoving away the empty platter, Diff picks up his cell phone to check with his partner Marie. "Any suspects?" Diff asks.

"Yeah," she replies, "we've got three of 'em. The first is the late Mr. Wood's ex-wife, a dancer by the name of Twinkles. She was seen in the Mayfair between 5:00 and 6:00 P.M. engaged in a shouting match with Wood. We haven't found any witnesses that saw Joe after this fight."

"When did she leave?"

"A witness says she left in a hurry a little after 6:00 P.M. The second suspect is an East End bookie who goes by the name of Slim. Slim was in last night asking for Joe around 10:00 P.M. Witnesses say that there was a lot of hand gesturing, like Slim was upset about something. One witness says that Slim stormed into the back room."

"Did anyone see him leave?"

"Yeah. He left quietly around 11:00 P.M. The third suspect is the cook."

"The cook?"

"Yep, the cook. Goes by the name of Shorty. The cashier says she heard Joe and Shorty arguing over the proper way to present a plate of veal scaloppine. She said that Shorty took an unusually long break at 10:30 P.M. He then took off in a huff when the diner closed at 2:00 A.M. Guess that explains why the place was in such a mess."

"Great work, partner. I think I know who to bring in for questioning."

Can you solve it? Which suspect does Diff want to question?