DiffEq - Ch 8 - Extra Practice

8.1

#1b. Write the linear system in matrix form:

$$\frac{dx}{dt} = 4x - 7y$$

$$\frac{dy}{dt} = 5x$$

#3b. Write the linear system in matrix form:

$$\frac{dx}{dt} = -3x + 4y + e^{-t}\sin 2t$$

$$\frac{dy}{dt} = 5x + 9z + 4e^{-t}\cos 2t$$

$$\frac{dz}{dt} = y + 6z - e^{-t}$$

#2b. Write the linear system in matrix form:

$$\frac{dx}{dt} = x - y$$

$$\frac{dy}{dt} = x + 2z$$

$$\frac{dz}{dt} = -x + z$$

#4b. Write the given system without the use of matrices:

$$\vec{X}' = \begin{bmatrix} 7 & 5 & -9 \\ 4 & 1 & 1 \\ 0 & -2 & 3 \end{bmatrix}\vec{X} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}e^{5t} + \begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}e^{-2t}$$

#5b. Write the given system without the use of matrices:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 8 \end{bmatrix}\sin t + \begin{bmatrix} t-4 \\ 2t+1 \end{bmatrix}e^{4t}$$

#7b. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\vec{X}' = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\vec{X}; \qquad \vec{X} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}e^{t} + \begin{bmatrix} 4 \\ -4 \end{bmatrix}te^{t}$$

#6b. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\frac{dx}{dt} = -2x + 5y \qquad \vec{X} = \begin{bmatrix} 5\cos t \\ 3\cos t - \sin t \end{bmatrix}e^{t}$$

$$\frac{dy}{dt} = -2x + 4y$$

#8b. Verify that the vector $\vec{X}$ is a solution of the given system:

$$\vec{X}' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \vec{X}; \qquad \vec{X} = \begin{bmatrix} \sin t \\ -\frac{1}{2}\sin t - \frac{1}{2}\cos t \\ -\sin t + \cos t \end{bmatrix}$$

#9b. The given vectors are solutions of a system $\vec{X}' = \vec{A}\,\vec{X}$. Determine whether the vectors form a fundamental set on the interval $(-\infty, \infty)$:

$$\vec{X}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t, \quad \vec{X}_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix} e^t + \begin{bmatrix} 8 \\ -8 \end{bmatrix} te^t$$

#10b. Verify that the vector $\vec{X}_P$ is a particular solution of the given system:

$$\vec{X}' = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & 0 \\ -6 & 1 & 0 \end{bmatrix} \vec{X} + \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} \sin 3t; \qquad \vec{X}_P = \begin{bmatrix} \sin 3t \\ 0 \\ \cos 3t \end{bmatrix}$$

8.2 day 1

#1b. Find the general solution of the system:

$$\frac{dx}{dt} = 2x + 2y$$

$$\frac{dy}{dt} = x + 3y$$

#2b. Find the general solution of the system:

$$\frac{dx}{dt} = -\frac{5}{2}x + 2y$$

$$\frac{dy}{dt} = \frac{3}{4}x - 2y$$

#3b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} -6 & 2 \\ -3 & 1 \end{bmatrix} \vec{X}$$

#4b. Find the general solution of the system:

$$\frac{dx}{dt} = 2x - 7y$$

$$\frac{dy}{dt} = 5x + 10y + 4z$$

$$\frac{dz}{dt} = 5y + 2z$$

#5b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \vec{X}$$

#6b. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \vec{X}, \quad \vec{X}(0) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

8.2 day 2

#1b. Find the general solution of the system:

$$\frac{dx}{dt} = -6x + 5y$$

$$\frac{dy}{dt} = -5x + 4y$$

#2b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 12 & -9 \\ 4 & 0 \end{bmatrix} \vec{X}$$

#3b. Find the general solution of the system:

$$\frac{dx}{dt} = 3x + 2y + 4z$$

$$\frac{dy}{dt} = 2x + 2z$$

$$\frac{dz}{dt} = 4x + 2y + 3z$$

#4b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} \vec{X}$$

#5b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix} \vec{X}$$

#6b. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

8.2 day 3

#1b. Find the general solution of the system:

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = -2x - y$$

#2b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 1 & -8 \\ 1 & -3 \end{bmatrix} \vec{X}$$

#3b. Find the general solution of the system:

$$\frac{dx}{dt} = 2x + y + 2z$$

$$\frac{dy}{dt} = 3x + 6z$$

$$\frac{dz}{dt} = -4x - 3z$$

#4b. Find the general solution of the system:

$$\vec{X}' = \begin{bmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix} \vec{X}$$

#5b. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 1 & -12 & -14 \\ 1 & 2 & -3 \\ 1 & 1 & -2 \end{bmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{bmatrix} 4 \\ 6 \\ -7 \end{bmatrix}$$

8.3 day 1

#1b. Use the method of undermined coefficients to solve the system:

$$\frac{dx}{dt} = 5x + 9y + 2$$

$$\frac{dy}{dt} = -x + 11y + 6$$

#2b. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} 4t + 9e^{6t} \\ -t + e^{6t} \end{bmatrix}$$

#3b. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} -1 & 5 \\ -1 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} \sin t \\ -2\cos t \end{bmatrix}$$

#4b. Use the method of undermined coefficients to solve the system:

$$\vec{X}' = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix} \vec{X} + \begin{bmatrix} 5 \\ -10 \\ 40 \end{bmatrix}$$

#5b. Use the method of undermined coefficients to solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} -1 & 5 \\ -1 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} \sin t \\ -2\cos t \end{bmatrix}, \qquad \vec{X}(0) = \begin{bmatrix} 7 \\ -14/3 \end{bmatrix}$$

8.3 day 2

#1b. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \vec{X} + \begin{bmatrix} 2 \\ e^{-3t} \end{bmatrix}$$

#2b. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} \vec{X} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}$$

#3b. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{X} + \begin{bmatrix} 0 \\ \sec t \tan t \end{bmatrix}$$

#4b. Use variation of parameters to solve the system:

$$\vec{X}' = \begin{bmatrix} 1 & 2 \\ -\frac{1}{2} & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} \csc t \\ \sec t \end{bmatrix} e^{t}$$

8.3 day 3

#1b. Solve the initial-value problem:

$$\vec{X}' = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \vec{X} + \begin{bmatrix} 1/t \\ 1/t \end{bmatrix}, \quad \vec{X}(1) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

8.3 day 4

#1b. Rewrite the following 3rd-order differential equation as a system of 3 1st-order differential equations (and also write the initial conditions in matrix form):

$$2y''' + 4y'' - 6y' = 12 - 8\sin x$$

$$y(0) = 4, \quad y'(0) = 0, \quad y''(0) = -5$$

(do not solve the system)