DiffEq - Ch 7 - Extra Practice

7.1

#1b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$

$$f(t)=\begin{cases}4, & 0\le t<2\\ 0, & t\ge 2\end{cases}$$

#2b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$

$$f(t)=\begin{cases}2t+1, & 0\le t<1\\ 0, & t\ge 1\end{cases}$$

#3b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\left\{f\left(t\right)\right\}$. Then use the table to verify your answer.
$f\left(t\right)=e^{-2t-5}$

#4b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\left\{f\left(t\right)\right\}$. Then use the table to verify your answer.
$f\left(t\right)=t^{2}e^{-2t}$

#5b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^5$

#6b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=-4t^2+16t+9$

#7b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^2-e^{-9t}+5$

#8b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=\left(e^t-e^{-t}\right)^2$

#9b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=\cos(5t)+\sin(2t)$

7.2 day 1

#1b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{s^4}\right\}$$

#2b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\left(\frac{2}{s}-\frac{1}{s^3}\right)^2\right\}$$

#3b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$$

#4b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{4}{s}+\frac{6}{s^5}-\frac{1}{s+8}\right\}$$

#5b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{5s-2}\right\}$$

#6b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{10s}{s^2+16}\right\}$$

#7b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{4s^2+1}\right\}$$

#8b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\}$$

#9b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-4s}\right\}$$

#10b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2+s-20}\right\}$$

#11b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{0.7s}{(s-0.3)(s+0.4)}\right\}$$

#12b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$$

#13b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$$

#14b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{3s-9}{\left(s^2+s\right)\left(s^2+4\right)}\right\}$$

#15b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{6s+3}{s^4+5s^2+4}\right\}$$

7.2 day 2

#1b. Use the Laplace transform to solve the initial-value problem:

$$2\frac{dy}{dt} + y = 0, \quad y(0) = -3$$

#2b. Use the Laplace transform to solve the initial-value problem:

$$y' + 3y = e^{2t}, \quad y(0) = 5$$

#3b. Use the Laplace transform to solve the initial-value problem:

$$y' - y = 4\sin(3t), \quad y(0) = 5$$

#4b. Use the Laplace transform to solve the initial-value problem:

$y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1, \quad y'(0) = -1$

#5b. Use the Laplace transform to solve the initial-value problem:

$y'' + 9y = e^{t}, \quad y(0) = 0, \quad y'(0) = 0$

#6b. Use the Laplace transform to solve the initial-value problem:

$$y''' + 2y'' - y' - 2y = \sin(3t)$$

$$y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1$$

7.3 day 1

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{te^{-6t}\}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^{10}e^{-7t}\}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{e^{2t}(t-1)^2\}$

#4b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{e^{-2t}\cos(4t)\}$

#5b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{e^{3t}\left(9-4t+10\sin\left(\frac{t}{2}\right)\right)\right\}$

#6b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{(s-1)^4}\right\}$

#7b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$

#8b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}$

#9b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\}$

#10b. Use the Laplace transform to solve the initial-value problem

$y' - y = 1 + te^t, \quad y(0) = 0$

#11b. Use the Laplace transform to solve the initial-value problem

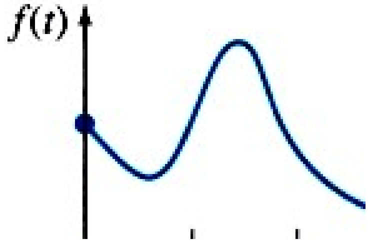
$y'' - 4y' + 4y = t^3 e^{2t}, \qquad y(0) = 0, \qquad y'(0) = 0$

#12b. Use the Laplace transform to solve the initial-value problem

$y'' - 4y' + 4y = t^3, \qquad y(0) = 1, \qquad y'(0) = 0$

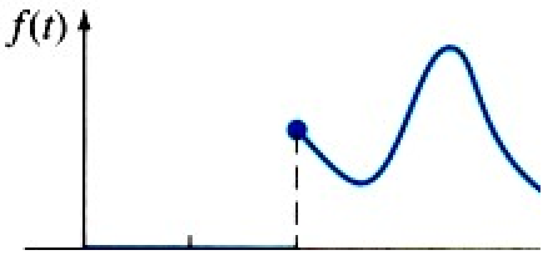
7.3 day 2

For #1-6, the function $f(t)$ is given by the graph:



Given the graph of a modified version of this function, use a combination of terms with the original function, time shifted original function and the unit step function to write a new function that produces the function in the problem's graph.

#4.



7.3 day 3

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{e^{2-t}\,\mathcal{U}(t-2)\right\}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{(3t+1)\,\mathcal{U}(t-1)\right\}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\left\{\sin t\,\mathcal{U}\left(t-\frac{\pi}{2}\right)\right\}$

#4b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{\left(1+e^{-2s}\right)^2}{s+2}\right\}$

#5b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{se^{-\frac{\pi}{2}s}}{s^2+4}\right\}$

#6b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\}$

#7b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}1, & 0\le t<4\\ 0, & 4\le t<5\\ 1, & t\ge 5\end{cases}$$

#8b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}0, & 0\le t<3\\ t+2, & t\ge 3\end{cases}$$

#9b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}\sin t, & 0\le t<2\pi\\ 0, & t\ge 2\pi\end{cases}$$

#10b. Use the Laplace transform to solve the initial-value problem:

$$y'+y=f(t), \quad y(0)=0$$

$$\textit{where } f(t)=\begin{cases}1, & 0\le t<1\\ -1, & t\ge 1\end{cases}$$

#11b. Use the Laplace transform to solve the initial-value problem:

$$y'' - 5y' + 6y = \mathcal{U}(t-1),$$

$$y(0) = 0, \quad y'(0) = 1$$

#12b. Use the Laplace transform to solve the initial-value problem:

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1$$

$$where \;\; f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

#13b. Use the Laplace transform to solve the initial-value problem:

$$y'' + 4y' + 3y = 1 - \mathcal{U}(t-2) - \mathcal{U}(t-4) + \mathcal{U}(t-6),$$

$$y(0) = 0, \quad y'(0) = 0$$

7.4

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^3 e^t\}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^2 \cos t\}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{te^{-3t} \cos(3t)\}$

#4b. Use the Laplace transform to solve the initial-value problem:

$y' - y = te^t \sin(t), \quad y(0) = 0$

#5b. Use the Laplace transform to solve the initial-value problem:

$$y'' + y = \sin t, \quad y(0) = 1, \quad y'(0) = -1$$

#6b. Use the Laplace transform to solve the initial-value problem:

$$y'' + y = f(t), \quad y(0) = 1, \quad y'(0) = 0$$

$$where \;\; f(t) = \begin{cases} 1, & 0 \le t < \dfrac{\pi}{2} \\ \sin t, & t \ge \dfrac{\pi}{2} \end{cases}$$

7.5

#1b. Use the Laplace transform to solve the initial-value problem:
$y' + y = \delta(t-1), \quad y(0) = 2$

#2b. Use the Laplace transform to solve the initial-value problem:
$y'' + 16y = \delta(t-2\pi), \quad y(0) = 0, \quad y'(0) = 0$

#3b. Use the Laplace transform to solve the initial-value problem:
$y'' + y = \delta(t-2\pi) + \delta(t-4\pi),$
$y(0) = 1, \quad y'(0) = 0$

#4b. Use the Laplace transform to solve the initial-value problem:
$y'' - 2y' = 1 + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1$