DiffEq - Ch 7 - Extra Practice

7.1

#1b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$

$$f(t)=\begin{cases}4, & 0\le t<2\\ 0, & t\ge 2\end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^{-st}(4)\,dt + \int_2^\infty e^{-st}(0)\,dt$$

$$= 4\left(\frac{1}{-s}\right)e^{-st}\Big]_0^2 + 0$$

$$= -\frac{4}{s}\left[e^{-2s} - e^{0}\right]$$

$$= \boxed{-\frac{4}{s}\left(e^{-2s} - 1\right)}$$

#2b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$

$$f(t)=\begin{cases}2t+1, & 0\le t<1\\ 0, & t\ge 1\end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(2t+1)\,dt + \int_1^\infty e^{-st}(0)\,dt$$

$$= \underbrace{2\int_0^1 te^{-st}\,dt}_{\text{by parts}} + \int_0^1 e^{-st}\,dt + \int_1^\infty 0\,dt$$

$$+\left[\frac{1}{-s}e^{-st}\right]_0^1 + 0$$

$u=t \qquad dv=e^{-st}\,dt$

$\frac{du}{dt}=1, \qquad \int dv=\int e^{-st}\,dt$

$du=dt \qquad v=\frac{1}{-s}e^{-st}$

$2\left[uv - \int v\,du\right]$

$$2\left[t\left(\frac{1}{-s}e^{-st}\right) - \int\left(\frac{1}{-s}\right)e^{-st}\,dt\right]_0^1$$

$$+\frac{1}{s}\int e^{-st}\,dt$$

$$\left[\frac{2}{-s}te^{-st} - \frac{2}{s^2}e^{-st}\right]_0^1$$

$$\left[-\frac{2}{s}te^{-st} - \frac{2}{s^2}e^{-st} - \frac{1}{s}e^{-st}\right]_0^1$$

$$\left[-\frac{2}{s}(1)e^{-s(1)} - \frac{2}{s^2}e^{-s(1)} - \frac{1}{s}e^{-s(1)}\right]$$

$$-\left[-\frac{2}{s}(0)e^{-s(0)} - \frac{2}{s^2}e^{-s(0)} - \frac{1}{s}e^{-s(0)}\right]$$

$$= -\frac{2}{s}e^{-s} - \frac{2}{s^2}e^{-s} - \frac{1}{s}e^{-s} + \frac{2}{s^2} + \frac{1}{s}$$

$$= \boxed{\frac{1}{s}\left(1-3e^{-s}\right) + \frac{2}{s^2}\left(1-e^{-s}\right)}$$

#3b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$. Then use the table to verify your answer.

$f(t)=e^{-2t-5}$

$$\mathcal{L}\{e^{-2t-5}\} = \int_0^\infty e^{-st} e^{(-2t-5)}\,dt = \int_0^\infty e^{-(s+2)t} e^{-5}\,dt$$

$$= e^{-5}\int_0^\infty e^{-(s+2)t}\,dt = e^{-5}\left[\frac{e^{-(s+2)t}}{-(s+2)}\right]_0^\infty$$

$$= \frac{-e^{-5}}{(s+2)}\left[\lim_{b\to\infty} e^{-(s+2)b} - e^{-(s+2)(0)}\right]$$

for $(s>-2)$

$$= \frac{-e^{-5}}{(s+2)}\left[(0)-1\right]$$

$$= \boxed{\frac{e^{-5}}{s+2}}$$

table: $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$\mathcal{L}\{e^{-2t-5}\}$

$\mathcal{L}\{e^{-2t}e^{-5}\}$

$e^{-5}\mathcal{L}\{e^{-2t}\}$ $a=-2$

$e^{-5}\frac{1}{s-(-2)}$

$\boxed{\frac{e^{-5}}{s+2}}$

#4b. Use the integral definition of the Laplace Transform to find $\mathscr{L}\{f(t)\}$. Then use the table to verify your answer.

$f(t)=t^2e^{-2t}$

$$\mathcal{L}\{t^2e^{-2t}\} = \int_0^\infty e^{-st}t^2e^{-2t}\,dt = \int_0^\infty t^2e^{-(s+2)t}\,dt$$

by parts: $u=t^2$ $dv=e^{-(s+2)t}dt$

$\frac{du}{dt}=2t$ $\int dv=\int e^{-(s+2)t}dt$

$du=2t\,dt$ $v=\frac{e^{-(s+2)t}}{-(s+2)}$

table: $\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$

$\mathcal{L}\{t^2e^{-2t}\}$ $n=2$, $a=-2$

$\frac{2!}{(s-(-2))^3} = \boxed{\frac{2}{(s+2)^3}}$

$uv-\int v\,du$

$$-\frac{t^2}{(s+2)}e^{-(s+2)t} + \frac{2}{(s+2)}\left[\int_0^\infty te^{-(s+2)t}dt\right]$$

by parts again

$u=t$ $dv=e^{-(s+2)t}dt$

$\frac{du}{dt}=1$ $\int dv=\int e^{-(s+2)t}dt$

$du=dt$ $v=\frac{e^{-(s+2)t}}{-(s+2)}$

$$-\frac{t^2}{(s+2)}e^{-(s+2)t} + \frac{2}{(s+2)}\left[uv-\int v\,du\right]$$

$$-\frac{t^2}{(s+2)}e^{-(s+2)t} + \frac{2}{(s+2)}\left\{-\frac{t}{(s+2)}e^{-(s+2)t} + \frac{1}{(s+2)}\int_0^\infty e^{-(s+2)t}dt\right\}$$

$$\left[\frac{-t^2}{(s+2)}e^{-(s+2)t} - \frac{2t}{(s+2)^2}e^{-(s+2)t} + \frac{2}{(s+2)^2}\cdot\frac{e^{-(s+2)t}}{-(s+2)}\right]_0^\infty$$

$$\lim_{b\to\infty}\left[-\frac{t^2}{(s+2)e^{(s+2)t}} - \frac{2t}{(s+2)^2e^{(s+2)t}} - \frac{2}{(s+3)^3e^{(s+2)t}}\right] - \left[0-0-\frac{2}{(s+2)^3}\right]$$

L'Ho x2 L'Ho x1

$$\lim_{b\to\infty}\left[\frac{-2}{(s+2)^2e^{(s+2)t}} - \frac{2}{(s+2)^3e^{(s+2)t}} - \frac{2}{(s+3)^3e^{(s+2)t}}\right] - \left(\frac{-2}{(s+2)^3}\right) = \boxed{\frac{2}{(s+2)^3}}$$

0

#5b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^5$ table: $\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}$

$\mathcal{L}\{t^5\}$

$\frac{5!}{s^6}=\boxed{\frac{120}{s^6}}$

#6b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=-4t^2+16t+9$

$-4\mathcal{L}\{t^2\}+16\mathcal{L}\{t\}+9\mathcal{L}\{1\}$

table: $\left(\frac{n!}{s^{n+1}}\right)$ $\left(\frac{1}{s^2}\right)$ $\left(\frac{1}{s}\right)$

$(n=2)$

$-4\frac{2!}{s^3}+16\frac{1}{s^2}+9\frac{1}{s}$

$\boxed{-8\frac{1}{s^3}+16\frac{1}{s^2}+9\frac{1}{s}}$

#7b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^2-e^{-9t}+5$

$\mathcal{L}\{t^2\}-\mathcal{L}\{e^{-9t}\}+5\mathcal{L}\{1\}$

table: $\left(\frac{n!}{s^{n+1}}\right)$ $\left(\frac{1}{s-a}\right)$ $\left(\frac{1}{s}\right)$

$n=2$ $a=-9$

$\frac{2!}{s^3}-\frac{1}{s+9}+5\frac{1}{s}$

$\boxed{2\frac{1}{s^3}-\frac{1}{s+9}+5\frac{1}{s}}$

#8b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=\left(e^t-e^{-t}\right)^2=(e^t-e^{-t})(e^t-e^{-t})$

$=e^te^t-2e^te^{-t}+e^{-t}e^{-t}$

$=e^{2t}-2+e^{-2t}$

$\mathcal{L}\{e^{2t}\}-2\mathcal{L}\{1\}+\mathcal{L}\{e^{-2t}\}$

table: $\left(\frac{1}{s-a}\right)$ $\left(\frac{1}{s}\right)$ $\left(\frac{1}{s-a}\right)$

$a=2$ $a=-2$

$\boxed{\frac{1}{s-2}-2\frac{1}{s}+\frac{1}{s+2}}$

#9b. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=\cos(5t)+\sin(2t)$

$\mathcal{L}\{\cos(5t)\}+\mathcal{L}\{\sin(2t)\}$

table: $\left(\frac{s}{s^2+k^2}\right)$ $\left(\frac{k}{s^2+k^2}\right)$

$(k=5)$ $(k=2)$

$\boxed{\frac{s}{s^2+25}+\frac{2}{s^2+4}}$

7.2 day 1

#1b. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{1}{s^4}\right\}$ table; $\left\{\frac{n!}{s^{n+1}}\right\} \to t^n$

$n=3$ $\mathscr{L}^{-1}\left\{\frac{1}{s^4}\right\}$

$\frac{1}{3!}\mathscr{L}^{-1}\left\{\frac{3!}{s^4}\right\} = \frac{1}{3!}t^3 = \boxed{\frac{1}{6}t^3}$

#2b. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\left(\frac{2}{s}-\frac{1}{s^3}\right)^2\right\} = \mathscr{L}^{-1}\left\{\frac{4}{s^2} - \frac{4}{s^4} + \frac{1}{s^6}\right\}$

$= 4\mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{2}{3}\mathscr{L}^{-1}\left\{\frac{3!}{s^4}\right\} + \frac{1}{5!}\mathscr{L}^{-1}\left\{\frac{5!}{s^6}\right\}$

$\boxed{4t - \frac{2}{3}t^3 + \frac{1}{120}t^5}$

#3b. Use theorems and the table to find

(Binomial theorem)!

$\mathscr{L}^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$ $(s+2)^2 = 1\,(s)^2(2)^0 + 2(s)^1(2)^1 + 1\,(s)^0(2)^2$

1
1 1
1 2 1

$(s+2)^2 = s^2+4s+4$

$\mathscr{L}^{-1}\left\{\frac{s^2}{s^3} + \frac{4s}{s^3} + \frac{4}{s^3}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\} + 4\mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} + 4\frac{1}{2!}\mathscr{L}^{-1}\left\{\frac{2!}{s^3}\right\}$

$\boxed{1 + 4t + 2t^2}$

#4b. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{4}{s}+\frac{6}{s^5}-\frac{1}{s+8}\right\} = 4\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} + 6\left(\frac{1}{4!}\right)\mathscr{L}^{-1}\left\{\frac{4!}{s^5}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{s+8}\right\}$

$\boxed{4(1) + \frac{1}{4}t^4 - e^{-8t}}$

#5b. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{1}{5s-2}\right\} = \mathscr{L}^{-1}\left\{\frac{(1/5)}{s-(2/5)}\right\} = \frac{1}{5}\mathscr{L}^{-1}\left\{\frac{1}{s-2/5}\right\}$

$\boxed{\frac{1}{5}e^{2/5t}}$

#6b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{10s}{s^2+16}\right\} = 10\mathscr{L}^{-1}\left\{\frac{s}{s^2+16}\right\}$$

$$\boxed{10\cos(4t)}$$

#7b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{4s^2+1}\right\} = \mathscr{L}^{-1}\left\{\frac{(1/4)}{s^2+(1/4)}\right\}$$

$$= \frac{1}{4}\mathscr{L}^{-1}\left\{\frac{1}{s^2+(\frac{1}{2})^2}\right\}$$

$$\boxed{\frac{1}{4}\sin(\tfrac{1}{2}t)}$$

#8b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\} = \mathscr{L}^{-1}\left\{\frac{s}{s^2+2}\right\} + \mathscr{L}^{-1}\left\{\frac{1}{s^2+2}\right\}$$

$$= \mathscr{L}^{-1}\left\{\frac{s}{s^2+2}\right\} + \frac{1}{\sqrt{2}}\mathscr{L}^{-1}\left\{\frac{\sqrt{2}}{s^2+(\sqrt{2})^2}\right\}$$

$$\boxed{\cos(\sqrt{2}t) + \frac{1}{\sqrt{2}}\sin(\sqrt{2}t)}$$

<u>Note!</u> you are not allowed to factor and cancel ... instead, separate using a common denominator like this

#9b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-4s}\right\} \quad \frac{s+1}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$= -\frac{1}{4}\frac{1}{s} + \frac{5}{4}\frac{1}{s-4}$$

$A(s-4)+B(s) = s+1$, $As-4A+Bs = s+1$

$(A+B)s + (-4A) = (1)s + (1)$

$\begin{cases} A+B=1 \\ -4A=1 \end{cases}$ $\quad A = -1/4 \quad B = 1+\frac{1}{4} = \frac{5}{4}$

$$-\frac{1}{4}\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{5}{4}\mathscr{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= -\frac{1}{4}(1) + \frac{5}{4}e^{4t} = \boxed{-\frac{1}{4}+\frac{5}{4}e^{4t}}$$

#10b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2+s-20}\right\} \quad \frac{1}{(s+5)(s-4)} = \frac{A}{s+5} + \frac{B}{s-4}$$

$$= -\frac{1}{9}\frac{1}{s+5} + \frac{1}{9}\frac{1}{s-4}$$

$A(s-4)+B(s+5) = 1$, $As-4A+Bs+5B = 1$

$(A+B)s + (-4A+5B) = (0)s + (1)$

$\begin{cases} A+B=0 \\ -4A+5B=1 \end{cases}$, $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -4 & 5 & 1 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & -1/9 \\ 0 & 1 & 1/9 \end{array}\right]$

$$-\frac{1}{9}\mathscr{L}^{-1}\left\{\frac{1}{s+5}\right\} + \frac{1}{9}\mathscr{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$\boxed{-\frac{1}{9}e^{-5t} + \frac{1}{9}e^{4t}}$$

#11b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{0.7s}{(s-0.3)(s+0.4)}\right\} \quad \frac{0.7s}{(s-.3)(s+.4)} = \frac{A}{s-.3} + \frac{B}{s+.4}$$

$A(s+.4)+B(s-.3) = .7s$, $As+.4A+Bs-.3B=.7s$

$(A+B)s+(.4A-.3B) = (.7)s+(0)$

$$\begin{cases} A+B=.7 \\ .4A-.3B=0 \end{cases} \quad \left[\begin{array}{cc|c} 1 & 1 & .7 \\ .4 & -.3 & 0 \end{array}\right] \text{rref}\left(\begin{array}{cc|c} 1 & 0 & 0.3 \\ 0 & 1 & 0.4 \end{array}\right)$$

$$0.3\mathscr{L}^{-1}\left\{\frac{1}{s-0.3}\right\} + 0.4\mathscr{L}^{-1}\left\{\frac{1}{s+0.4}\right\}$$

$$\boxed{0.3e^{0.3t} + 0.4e^{-0.4t}}$$

#12b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$$

online PFE: $= \frac{1}{2}\frac{1}{s} - \frac{1}{s-1} - \frac{1}{3}\frac{1}{s+1} + \frac{5}{6}\frac{1}{s-2}$

$$\frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{3}\mathscr{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{5}{6}\left\{\frac{1}{s-2}\right\}$$

$$\frac{1}{2}(1) - e^{t} - \frac{1}{3}e^{-t} + \frac{5}{6}e^{2t}$$

$$\boxed{\frac{1}{2} - e^{t} - \frac{1}{3}e^{-t} + \frac{5}{6}e^{2t}}$$

#13b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$$

online PFE: $-\frac{1}{4}\frac{1}{s+2} + \frac{s+2}{4(s^2+4)}$

$$-\frac{1}{4}\mathscr{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{4}\mathscr{L}^{-1}\left\{\frac{s}{s^2+(2)^2}\right\} + \frac{1}{4}\mathscr{L}^{-1}\left\{\frac{2}{s^2+(2)^2}\right\}$$

$$\boxed{-\frac{1}{4}e^{-2t} + \frac{1}{4}\cos(2t) + \frac{1}{4}\sin(2t)}$$

#14b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{3s-9}{(s^2+s)(s^2+4)}\right\}$$ online PFE: $-\frac{9}{4}\frac{1}{s}+\frac{12}{5}\frac{1}{s+1}+\frac{-3s+48}{20(s^2+4)}$

$$-\frac{9}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}+\frac{12}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}-\frac{3}{20}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}+\frac{16}{5}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$-\frac{9}{4}(1)+\frac{12}{5}e^{-t}-\frac{3}{20}\cos(2t)+\frac{6}{5}\sin(2t)$$

$$\boxed{-\frac{9}{4}+\frac{12}{5}e^{-t}-\frac{3}{20}\cos(2t)+\frac{6}{5}\sin(2t)}$$

#15b. Use theorems and the table to find

$$\mathscr{L}^{-1}\left\{\frac{6s+3}{s^4+5s^2+4}\right\}$$ $\frac{2s+1}{s^2+1}+\frac{-2s-1}{s^2+4}$

$$2\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}-2\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}-\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$\boxed{2\cos(t)+\sin(t)-2\cos(2t)-\frac{1}{2}\sin(2t)}$$

7.2 day 2

#1b. Use the Laplace transform to solve the initial-value problem:

$$2\frac{dy}{dt}+y=0, \quad y(0)=-3$$

$2[sY(s)-y(0)]+Y(s)=0$

$2[sY(s)+3]+Y(s)=0$

$(2s+1)Y(s)+6=0$

$Y(s)=\frac{-6}{2s+1}=$

$Y(s)=\frac{-6}{2s+1}=\frac{2(-3)}{2(s+1/2)}=-3\frac{1}{s+1/2}$ (no PFE needed, just algebra)

$\mathcal{L}^{-1}$:

$y(t)=-3\mathcal{L}^{-1}\{\frac{1}{s+1/2}\}$

$\boxed{y(t)=-3e^{-1/2t}}$

#2b. Use the Laplace transform to solve the initial-value problem:

$$y'+3y=e^{2t}, \quad y(0)=5$$

$[sY(s)-y(0)]+3Y(s)=\mathcal{L}\{e^{2t}\}$

$(s+3)Y(s)-5=\frac{1}{s-2}$

$(s+3)Y(s)=5+\frac{1}{s-2}$

$Y(s)=\frac{5}{s+3}+\frac{1}{(s-2)(s+3)}$

online PFE: $\frac{1}{(s-2)(s+3)}=\frac{1}{5}\frac{1}{s-2}-\frac{1}{5}\frac{1}{s+3}$

$Y(s)=\frac{5}{s+3}+\frac{1}{5}\frac{1}{s-2}-\frac{1}{5}\frac{1}{s+3}$

$Y(s)=\frac{24}{5}\frac{1}{s+3}+\frac{1}{5}\frac{1}{s-2}$

$\mathcal{L}^{-1}$:

$\boxed{y(t)=\frac{24}{5}e^{-3t}+\frac{1}{5}e^{2t}}$

#3b. Use the Laplace transform to solve the initial-value problem:

$$y'-y=4\sin(3t), \quad y(0)=5$$

$[sY(s)-y(0)]-Y(s)=\mathcal{L}\{4\sin(3t)\}$

$sY(s)-5-Y(s)=4\frac{3}{s^2+9}$

$(s-1)Y(s)=5+\frac{12}{s^2+9}$

$Y(s)=\frac{5}{s-1}+\frac{12}{(s-1)(s^2+9)}$

online PFE: $\frac{12}{(s-1)(s^2+9)}=\frac{6}{5}\frac{1}{s-1}+\frac{-6s-6}{5(s^2+9)}$

$Y(s)=\frac{5}{s-1}+\frac{6}{5}\frac{1}{s-1}-\frac{6}{5}\frac{s}{s^2+9}-\frac{6}{5}\frac{1}{s^2+9}$

$Y(s)=\frac{31}{5}\frac{1}{s-1}-\frac{6}{5}\frac{s}{s^2+9}-\frac{2}{5}\frac{3}{s^2+9}$

$\mathcal{L}^{-1}$:

$\boxed{y(t)=\frac{31}{5}e^{t}-\frac{6}{5}\sin(3t)-\frac{2}{5}\cos(3t)}$

#4b. Use the Laplace transform to solve the initial-value problem:

$y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0)=1, \quad y'(0)=-1$

online PFE:

$\frac{s-5}{s^2-4s} = \frac{5}{4}\frac{1}{s} - \frac{1}{4}\frac{1}{s-4}$

$\frac{6}{(s-3)(s^2-4s)} = -2\frac{1}{s-3} + \frac{1}{2}\frac{1}{s} + \frac{3}{2}\frac{1}{s-4}$

$\frac{-3}{(s+1)(s^2-4s)} = -\frac{3}{5}\frac{1}{s+1} + \frac{3}{4}\frac{1}{s} - \frac{3}{20}\frac{1}{s-4}$

$[s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] = \mathcal{L}\{6e^{3t} - 3e^{-t}\}$

$s^2Y(s) - s(1) - (-1) - 4sY(s) + 4(1) = 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$(s^2-4s)Y(s) - s + 1 + 4 = 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$(s^2-4s)Y(s) = s - 5 + 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$Y(s) = \frac{s-5}{s^2-4s} + 6\frac{1}{(s-3)(s^2-4s)} - 3\frac{1}{(s+1)(s^2-4s)}$

$Y(s) = \frac{5}{4}\frac{1}{s} - \frac{1}{4}\frac{1}{s-4} - 2\frac{1}{s-3} + \frac{1}{2}\frac{1}{s} + \frac{3}{2}\frac{1}{s-4} - \frac{3}{5}\frac{1}{s+1} + \frac{3}{4}\frac{1}{s} - \frac{3}{20}\frac{1}{s-4}$

$Y(s) = \frac{5}{2}\frac{1}{s} + \frac{11}{10}\frac{1}{s-4} - 2\frac{1}{s-3} - \frac{3}{5}\frac{1}{s+1}$

$\mathcal{L}^{-1}$: $y(t) = \frac{5}{2}(1) + \frac{11}{10}e^{4t} - 2e^{3t} - \frac{3}{5}e^{-t}$

$\boxed{y(t) = \frac{5}{2} + \frac{11}{10}e^{4t} - 2e^{3t} - \frac{3}{5}e^{-t}}$

#5b. Use the Laplace transform to solve the initial-value problem:

$y'' + 9y = e^{t}, \quad y(0)=0, \quad y'(0)=0$

online PFE:

$\frac{1}{(s-1)(s^2+9)} = \frac{1}{10}\frac{1}{s-1} + \frac{-s-1}{10(s^2+9)}$

$[s^2Y(s) - sy(0) - y'(0)] + 9Y(s) = \mathcal{L}\{e^{t}\}$

$s^2Y(s) - s(0) - 0 + 9Y(s) = \frac{1}{s-1}$

$(s^2+9)Y(s) = \frac{1}{s-1}$

$Y(s) = \frac{1}{(s-1)(s^2+9)}$

$Y(s) = \frac{1}{10}\frac{1}{s-1} - \frac{1}{10}\frac{s}{s^2+9} - \frac{1}{10}\frac{1}{s^2+9}$

$Y(s) = \frac{1}{10}\frac{1}{s-1} - \frac{1}{10}\frac{s}{s^2+9} - \frac{1}{30}\frac{3}{s^2+9}$

$\mathcal{L}^{-1}$:

$\boxed{y(t) = \frac{1}{10}e^{t} - \frac{1}{10}\cos(3t) - \frac{1}{30}\sin(3t)}$

#6b. Use the Laplace transform to solve the initial-value problem:

$$y''' + 2y'' - y' - 2y = \sin(3t)$$

$$y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1$$

$$\left[s^3Y(s) - s^2y(0) - sy'(0) - y''(0)\right] + 2\left[s^2Y(s) - sy(0) - y'(0)\right] - \left[sY(s) - y(0)\right] - 2Y(s) = \mathcal{L}\{\sin(3t)\}$$

$$s^3Y(s) - s^2(0) - s(0) - 1 + 2s^2Y(s) - 2s(0) + 2(0) - sY(s) + 0 - 2Y(s) = \frac{3}{s^2+9}$$

$$(s^3 + 2s^2 - s - 2)Y(s) - 1 = \frac{3}{s^2+9}$$

$$(s^3 + 2s^2 - s - 2)Y(s) = 1 + \frac{3}{s^2+9}$$

$$Y(s) = \frac{1}{s^3+2s^2-s-2} + \frac{3}{(s^2+9)(s^3+2s^2-s-2)}$$

online PFE:

$$Y(s) = \left(\frac{1}{3}\frac{1}{s+2} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s-1}\right) + \left(\frac{3s-6}{130(s^2+9)} + \frac{1}{13(s+2)} - \frac{3}{20(s+1)} + \frac{1}{20(s-1)}\right)$$

$$Y(s) = \frac{1}{3}\frac{1}{s+2} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s-1} + \frac{3}{130}\frac{s}{s^2+9} - \frac{6}{130}\frac{1}{s^2+9} - \frac{3}{20}\frac{1}{s+1} + \frac{1}{20}\frac{1}{s-1}$$

$$Y(s) = \frac{1}{3}\frac{1}{s+2} - \frac{13}{20}\frac{1}{s+1} + \frac{13}{60}\frac{1}{s-1} + \frac{3}{130}\frac{s}{s^2+9} - \frac{6}{130}\frac{1}{s^2+9}$$

$$Y(s) = \frac{1}{3}\frac{1}{s+2} - \frac{13}{20}\frac{1}{s+1} + \frac{13}{60}\frac{1}{s-1} + \frac{3}{130}\frac{s}{s^2+9} - \frac{2}{130}\frac{3}{s^2+9}$$

$$\left(\frac{1}{65}\right)$$

$\mathcal{L}^{-1}$:

$$\boxed{y(t) = \frac{1}{3}e^{-2t} - \frac{13}{20}e^{-t} + \frac{13}{60}e^{t} + \frac{3}{130}\cos(3t) - \frac{1}{65}\sin(3t)}$$

7.3 day 1

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{te^{-6t}\}$ e^{-6t}: shift $s \Rightarrow s+6$

$\mathcal{L}\{t\} = \frac{1}{s^2}$

$\boxed{\mathcal{L}\{te^{-6t}\} = \frac{1}{(s+6)^2}}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^{10}e^{-7t}\}$ e^{-7t}: shift, $s \to s+7$

$\mathcal{L}\{t^{10}\} = \frac{10!}{s^{11}}$

$\boxed{\mathcal{L}\{t^{10}e^{-7t}\} = \frac{10!}{(s+7)^{11}}}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{e^{2t}(t-1)^2\} = \mathcal{L}\{e^{2t}(t^2-2t+1)\}$

$= \mathcal{L}\{t^2e^{2t}\} - 2\mathcal{L}\{te^{2t}\} + \mathcal{L}\{1e^{2t}\}$

e^{2t}: shift $s \Rightarrow s-2$

$\mathcal{L}\{t^2\} = \frac{2!}{s^3}$ $\mathcal{L}\{t\} = \frac{1}{s^2}$ $\mathcal{L}\{1\} = \frac{1}{s}$

$\boxed{\frac{2!}{(s-2)^3} - 2\frac{1}{(s-2)^2} + \frac{1}{(s-2)}}$

#4b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{e^{-2t}\cos(4t)\}$ e^{-2t}: shift, $s \Rightarrow s+2$

$\mathcal{L}\{\cos(4t)\} = \frac{s}{s^2+16}$

$\boxed{\mathcal{L}\{e^{-2t}\cos(4t)\} = \frac{(s+2)}{(s+2)^2+16}}$

#5b. Evaluate the Laplace transform to find $F(s)$

$$\mathscr{L}\left\{e^{3t}\left(9-4t+10\sin\left(\frac{t}{2}\right)\right)\right\}$$

$9\mathcal{L}\{1 \cdot e^{3t}\} - 4\{te^{3t}\} + 10\{e^{3t}\sin(\frac{t}{2})\}$

shift: e^{3t}, $s \Rightarrow s-3$

$\mathcal{L}\{1\}$ $\frac{1}{s}$ $\mathcal{L}\{t\}$ $\frac{1}{s^2}$ $\mathcal{L}\{\sin(\frac{1}{2}t)\}$ $\frac{(1/2)}{s^2+(1/4)}$

$9\frac{1}{(s-3)} - 4\frac{1}{(s-3)^2} + 10\frac{(1/2)}{(s-3)^2+(1/4)}$

$\boxed{9\frac{1}{s-3} - 4\frac{1}{(s-3)^2} + 5\frac{1}{(s-3)^2+1/4}}$

#6b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{1}{(s-1)^4}\right\}$

$s-1$: shift, e^t

$\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = \frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^4}\right\} = \frac{1}{6}t^3$

$$\boxed{\frac{1}{6}e^t t^3}$$

#7b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\}$

$s^2+2s+\underline{1}+5-\underline{1}$

$(s+1)^2+4$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$

$s+1$: shift e^{-t}

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$

$= \frac{1}{2}\sin(2t)$

$$\boxed{\frac{1}{2}e^{-t}\sin(2t)}$$

#8b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$

$s^2+6s+\underline{9}+34-\underline{9}$

$(s+3)^2+25$

$\mathcal{L}^{-1}\left\{\frac{2s+5}{(s+3)^2+25}\right\}$

$s+3$: shift e^{-3t}

$2\mathcal{L}^{-1}\left\{\frac{s+5/2+3-3}{(s+3)^2+25}\right\}$

$2\mathcal{L}^{-1}\left\{\frac{(s+3)}{(s+3)^2+25}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{5}{(s+3)^2+25}\right\}$

$2\cos(5t) - \frac{1}{5}\sin(5t)$

$$\boxed{e^{-3t}\left(2\cos(5t) - \frac{1}{5}\sin(5t)\right)}$$

$$\boxed{2e^{-3t}\cos(5t) - \frac{1}{5}e^{-3t}\sin(5t)}$$

#9b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\}$

$5\mathcal{L}^{-1}\left\{\frac{s-2+2}{(s-2)^2}\right\}$

$= 5\mathcal{L}^{-1}\left\{\frac{(s-2)}{(s-2)^2}\right\} + 10\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}$

$s-2$: shift e^{2t}

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$ $\quad$ $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$

1 $\quad$ t

$$\boxed{5e^{2t} + 10te^{2t}}$$

#10b. Use the Laplace transform to solve the initial-value problem

$y' - y = 1 + te^t, \quad y(0) = 0$

$[sY(s) - y(0)] - Y(s) = \mathcal{L}\{1\} + \mathcal{L}\{te^t\}$

e^t: shift $s \to s-1$

$sY(s) - 0 - Y(s) =$ $\quad$ $\mathcal{L}\{t\} = \frac{1}{s^2}$

$(s-1)Y(s) = \frac{1}{s} + \frac{1}{(s-1)^2}$

$Y(s) = \frac{1}{s(s-1)} + \frac{1}{(s-1)^3}$

$Y(s) = -\frac{1}{s} + \frac{1}{s-1} + \frac{1}{(s-1)^3}$

$y(t) = -\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3}\right\}$

$s-1$: shift $\to e^t$

$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

$\frac{1}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}$

$$\boxed{y(t) = -1 + e^t + \frac{1}{2}t^2e^t}$$

#11b. Use the Laplace transform to solve the initial-value problem

$y'' - 4y' + 4y = t^3 e^{2t}, \quad y(0) = 0, \quad y'(0) = 0$

$[s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] + 4Y(s) = \mathcal{L}\{t^3 e^{2t}\}$ $\quad e^{2t}$: shift $s \Rightarrow s-2$

$\mathcal{L}\{t^3\} = \frac{3!}{s^4}$

$s^2Y(s) - s(0) - 0 - 4sY(s) + 4(0) + 4Y(s) = \frac{3!}{(s-2)^4}$

$(s^2 - 4s + 4)Y(s) = \frac{6}{(s-2)^4}, \quad s^2 - 4s + \frac{4}{} + 4 - \frac{4}{}$
$\quad (s-2)^2 + 0$

$Y(s) = \frac{6}{(s-2)^4(s^2 - 4s + 4)}$

$Y(s) = \frac{6}{(s-2)^4(s-2)^2} = \frac{6}{(s-2)^6}$

$y(t) = 6\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^6}\right\}$ $\quad s-2$: shift e^{2t}

$\frac{1}{5!}\mathcal{L}\left\{\frac{5!}{s^6}\right\} = \frac{1}{5!}t^5$

$y(t) = 6\frac{1}{5!}t^5 e^{2t}, \quad \boxed{y(t) = \frac{1}{20}t^5 e^{2t}}$

#12b. Use the Laplace transform to solve the initial-value problem

$y'' - 4y' + 4y = t^3, \quad y(0) = 1, \quad y'(0) = 0$

$[s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] + 4Y(s) = \mathcal{L}\{t^3\}$

$s^2Y(s) - s(1) - 0 - 4sY(s) + 4(1) + 4Y(s) = \frac{3!}{s^4}$

$s^2 - 4s + \frac{4}{} + 4 - \frac{4}{}$
$(s-2)^2 + 0$

$(s^2 - 4s + 4)Y(s) - s + 4 = \frac{6}{s^4}$

$Y(s) = \frac{s-4}{(s^2-4s+4)} + \frac{6}{s^4(s^2-4s+4)} = \frac{s-4}{(s-2)^2} + \frac{6}{s^4(s-2)^2} = \frac{(s-2)-2}{(s-2)^2} + \frac{6}{s^4(s-2)^2}$ PFE

$Y(s) = \frac{1}{s-2} - \frac{2}{(s-2)^2} + \frac{3}{4}\frac{1}{s} + \frac{9}{8}\frac{1}{s^2} + \frac{3}{2}\frac{1}{s^3} + \frac{3}{2}\frac{1}{s^4} - \frac{3}{4}\frac{1}{s-2} + \frac{3}{8}\frac{1}{(s-2)^2}$

$Y(s) = \frac{1}{4}\frac{1}{s-2} - \frac{13}{8}\frac{1}{(s-2)^2} + \frac{3}{4}\frac{1}{s} + \frac{9}{8}\frac{1}{s^2} + \frac{3}{2}\frac{1}{s^3} + \frac{3}{2}\frac{1}{s^4}$

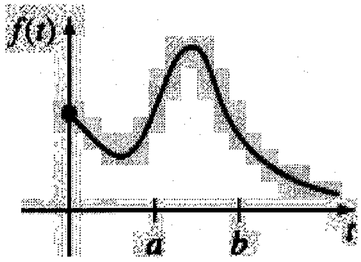
$y(t) = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{13}{8}\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} + \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{9}{8}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{3}{2\cdot 2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} + \frac{3}{2\cdot 6}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\}$

$s-2$: shift, e^{2t}

$\boxed{y(t) = \frac{1}{4}e^{2t} - \frac{13}{8}te^{2t} + \frac{3}{4} + \frac{9}{8}t + \frac{3}{4}t^2 + \frac{1}{4}t^3}$

7.3 day 2

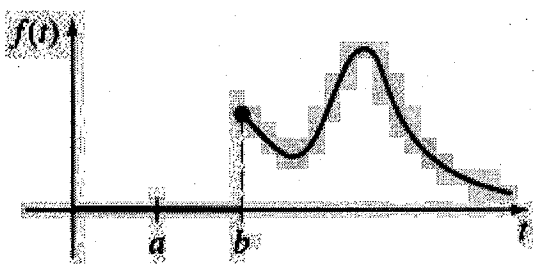
For #1-6, the function $f(t)$ is given by the graph:



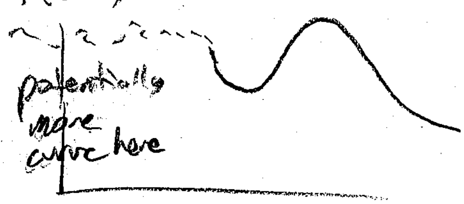
Given the graph of a modified version of this function, use a combination of terms with the original function, time shifted original function and the unit step function to write a new function that produces the function in the problem's graph.

(one example to show the idea)

#4.

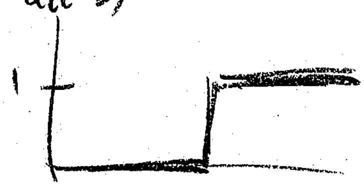


$f(t-b)$

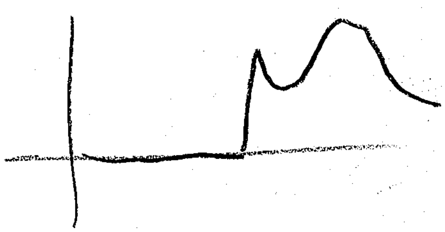


shifted version

$u(t-b)$



switch it on at $t=b$



$= \boxed{f(t-b)\,u(t-b)}$

7.3 day 3

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathcal{L}\{e^{2-t}\,u(t-2)\}$

$= \mathcal{L}\{e^{-(t-2)}u(t-2)\}$

$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ $\quad a=2$

$\mathcal{L}\{e^{-t}\} = \frac{1}{s+1}$

$\boxed{e^{-2s}\frac{1}{s+1}}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathcal{L}\{(3t+1)\,u(t-1)\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}, a=1$

$e^{-s}\mathcal{L}\{(3(t+1)+1\} = e^{-s}\mathcal{L}\{3t+4\}$

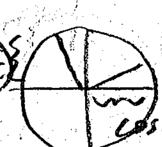
$\boxed{3e^{-s}\frac{1}{s^2} + 4e^{-s}\frac{1}{s}}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathcal{L}\left\{\sin t\, u\left(t-\frac{\pi}{2}\right)\right\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}, a=\pi/2$

$e^{-\pi/2\,t}\mathcal{L}\{\sin(t+\pi/2)\}$



$\sin(t+\frac{\pi}{2}) = \cos(t)$

$e^{-\pi/2\,t}\mathcal{L}\{\cos t\}$

$\boxed{e^{-\pi/2\,t}\frac{s}{s^2+1}}$

#4b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathcal{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{1+2e^{-2s}+e^{-4s}}{s+2}\right\}$

$\mathcal{L}^{-1}\{\frac{1}{s+2}\} + 2\mathcal{L}^{-1}\{\frac{e^{-2s}}{s+2}\} + \mathcal{L}^{-1}\{\frac{e^{-4s}}{s+2}\}$

e^{-2s}: shift $t \to t-2$ $\quad$ e^{-4s}: shift $t \to t-4$

$\boxed{e^{-2t} + 2e^{-2(t-2)}u(t-2) + e^{-2(t-4)}u(t-4)}$

#5b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathcal{L}^{-1}\left\{\frac{se^{-\frac{\pi}{2}s}}{s^2+4}\right\}$

$e^{-\pi/2\,s}$: shift $t \to t-\pi/2$

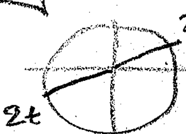
$\mathcal{L}\{\frac{s}{s^2+4}\} = \cos(2t)$

$\boxed{\cos(2(t-\pi/2))\,u(t-\pi/2)}$

or

$\cos(2t-\pi)\,u(t-\pi/2)$

$\boxed{-\cos(2t)\,u(t-\pi/2)}$



$\cos(2t-\pi) = -\cos(2t)$

#6b. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\}$

e^{-2s}: shift $t \to t-2$

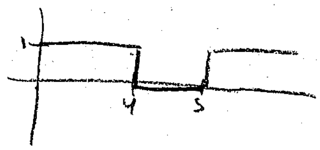
$\mathcal{L}^{-1}\{\frac{1}{s^2(s-1)}\} = \mathcal{L}^{-1}\{-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}\}$

(1) (t) (e^t)

$\boxed{-1u(t-2) - (t-2)u(t-2) + e^{(t-2)}u(t-2)}$

#7b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}1, & 0\le t<4\\ 0, & 4\le t<5\\ 1, & t\ge 5\end{cases}$$



$\boxed{f(t) = 1 - 1u(t-4) + 1u(t-5)}$

$\mathcal{L}\{1\} - \mathcal{L}\{1u(t-4)\} - \mathcal{L}\{1u(t-5)\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$\boxed{\frac{1}{s} - e^{-4s}\frac{1}{s} - e^{-5s}\frac{1}{s}}$

#8b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}0, & 0\le t<3\\ t+2, & t\ge 3\end{cases}$$

$\boxed{f(t) = (t+2)u(t-3)}$

$\mathcal{L}\{(t+2)u(t-3)\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$a=3$

$e^{-3s}\mathcal{L}\{((t+3)+2)\}$

$e^{-3s}\mathcal{L}\{t+5\}$

$\boxed{e^{-3s}\frac{1}{s^2} + 5e^{-3s}\frac{1}{s}}$

#9b. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases}\sin t, & 0\le t<2\pi\\ 0, & t\ge 2\pi\end{cases}$$

$\boxed{f(t) = \sin t - \sin t\, u(t-2\pi)}$

$\mathcal{L}\{\sin t\} - \mathcal{L}\{\sin t\, u(t-2\pi)\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$a=2\pi$

$\frac{1}{s^2+1} - e^{-2\pi s}\mathcal{L}\{\sin(t+2\pi)\}$

$\mathcal{L}\{\sin t\}$

$\boxed{\frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}}$

#10b. Use the Laplace transform to solve the initial-value problem:

$y'+y=f(t),\quad y(0)=0$

where $f(t)=\begin{cases}1, & 0\le t<1\\ -1, & t\ge 1\end{cases}$ $f(t)=1-2u(t-1)$

$[sY(s)-y(0)] + Y(s) = \mathcal{L}\{1-2u(t-1)\}$

$sY(s) - 0 + Y(s) = \mathcal{L}\{1\} - 2\mathcal{L}\{1\cdot u(t-1)\}$

$(\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+$

$a=1$

$(s+1)Y(s) = \frac{1}{s} - 2e^{-s}\frac{1}{s}$

$Y(s) = \frac{1}{s(s+1)} - 2e^{-s}\frac{1}{s(s+1)}$

$Y(s) = \frac{1}{s} - \frac{1}{s+1} - 2e^{-s}\frac{1}{s} + 2e^{-s}\frac{1}{s+1}$

$y(t) = \mathcal{L}^{-1}\{\frac{1}{s}\} - \mathcal{L}^{-1}\{\frac{1}{s+1}\} - 2\mathcal{L}^{-1}\{e^{-s}\frac{1}{s}\} + 2\mathcal{L}^{-1}\{e^{-s}\frac{1}{s}$

$(\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a))$

$a=1$

$y(t) = 1 - e^{-t} - 2(1)u(t-1) + 2e^{-(t-1)}u(t-1)$

$\boxed{y(t) = 1 - e^{-t} - 2u(t-1) + 2e^{-(t-1)}u(t-1)}$

#11b. Use the Laplace transform to solve the initial-value problem:

$y'' - 5y' + 6y = \mathcal{U}(t-1),$

$y(0) = 0, \quad y'(0) = 1$

$[s^2Y(s) - sy(0) - y'(0)] - 5[sY(s) - y(0)] + 6Y(s) = \mathcal{L}\{1\,\mathcal{U}(t-1)\}$

$(\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\})$

$s^2Y(s) - s(0) - 1 - 5sY(s) + 0 + 6Y(s) = e^{-s}\frac{1}{s}$

$(s^2 - 5s + 6)Y(s) = e^{-s}\frac{1}{s} + 1$

$Y(s) = e^{-s}\frac{1}{s(s^2-5s+6)} + \frac{1}{s^2-5s+6} = e^{-s}\left(\frac{1}{6}\frac{1}{s} - \frac{1}{2}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s-3}\right) + \left(-\frac{1}{s-2} + \frac{1}{s-3}\right)$

$y(t) = \underbrace{\frac{1}{6}\mathcal{L}^{-1}\{e^{-s}\frac{1}{s}\} - \frac{1}{2}\mathcal{L}^{-1}\{e^{-s}\frac{1}{s-2}\} + \frac{1}{3}\mathcal{L}^{-1}\{e^{-s}\frac{1}{s-3}\}}_{\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathcal{U}(t-a) \quad a=1} - \mathcal{L}^{-1}\{\frac{1}{s-2}\} + \mathcal{L}^{-1}\{\frac{1}{s-3}\}$

$$\boxed{y(t) = \frac{1}{6}(1)\mathcal{U}(t-1) - \frac{1}{2}e^{2(t-1)}\mathcal{U}(t-1) + \frac{1}{3}e^{3(t-1)}\mathcal{U}(t-1) - e^{2t} + e^{3t}}$$

#12b. Use the Laplace transform to solve the initial-value problem:

$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1$

where $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$ $\qquad f(t) = 1 - u(t-1)$

$$\left[s^2Y(s) - sy(0) - y'(0)\right] + 4Y(s) = \mathcal{L}\{1\} - \mathcal{L}\{1\,u(t-1)\}$$

$$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$$

$$a = 1$$

$$s^2Y(s) - s(0) - (-1) + 4Y(s) = \frac{1}{s} - e^{-s}\frac{1}{s}$$

$$(s^2+4)Y(s) + 1 = \frac{1}{s} - e^{-s}\frac{1}{s}$$

$$(s^2+4)Y(s) = \frac{1}{s} - e^{-s}\frac{1}{s} - 1$$

$$Y(s) = \frac{1}{s(s^2+4)} - e^{-s}\frac{1}{s(s^2+4)} - \frac{1}{s^2+4}$$

$$Y(s) = \left(\frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4}\right) - e^{-s}\left(\frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4}\right) - \frac{1}{s^2+4}$$

$$y(t) = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{e^{-s}\frac{1}{s}\right\} + \underbrace{\frac{1}{4}\mathcal{L}^{-1}\left\{e^{-s}\frac{s}{s^2+4}\right\}}_{\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$y(t) = \frac{1}{4}(1) - \frac{1}{4}\cos(2t) - \frac{1}{4}(1)u(t-1) + \frac{1}{4}\cos(2(t-1))u(t-1) - \frac{1}{2}\sin(2t)$$

$$\boxed{y(t) = \frac{1}{4} - \frac{1}{4}\cos(2t) - \frac{1}{4}u(t-1) + \frac{1}{4}\cos(2(t-1))u(t-1) - \frac{1}{2}\sin(2t)}$$

#13b. Use the Laplace transform to solve the initial-value problem:

$$y'' + 4y' + 3y = 1 - \mathcal{U}(t-2) - \mathcal{U}(t-4) + \mathcal{U}(t-6),$$

$$y(0) = 0, \quad y'(0) = 0$$

$$[s^2Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 3Y(s) = \mathcal{L}\{1\} - \underbrace{\mathcal{L}\{1\,\mathcal{U}(t-2)\} - \mathcal{L}\{1\,\mathcal{U}(t-4)\} - \mathcal{L}\{1\,\mathcal{U}(t-6)\}}_{\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}}$$

$$s^2Y(s) - s(0) - 0 + 4sY(s) - 4(0) + 3Y(s) = \frac{1}{s} - e^{-2s}\frac{1}{s} - e^{-4s}\frac{1}{s} - e^{-6s}\frac{1}{s}$$

$$(s^2+4s+3)Y(s) = \frac{1}{s} - e^{-2s}\frac{1}{s} - e^{-4s}\frac{1}{s} - e^{-6s}\frac{1}{s}$$

$$Y(s) = \frac{1}{s(s^2+4s+3)} - e^{-2s}\frac{1}{s(s^2+4s+3)} - e^{-4s}\frac{1}{s(s^2+4s+3)} - e^{-6s}\frac{1}{s(s^2+4s+3)}$$

$$Y(s) = \left(\frac{1}{3}\frac{1}{s} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s+3}\right) - e^{-2s}\left(\frac{1}{3}\frac{1}{s} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s+3}\right) - e^{-4s}\left(\frac{1}{3}\frac{1}{s} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s+3}\right) - e^{-6s}\left(\frac{1}{3}\frac{1}{s} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s+3}\right)$$

$$y(t) = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{e^{-2s}\frac{1}{s}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{e^{-2s}\frac{1}{s+1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{e^{-2s}\frac{1}{s+3}\right\}$$

$$- \frac{1}{3}\mathcal{L}^{-1}\left\{e^{-4s}\frac{1}{s}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{e^{-4s}\frac{1}{s+1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{e^{-4s}\frac{1}{s+3}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{e^{-6s}\frac{1}{s}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{e^{-6s}\frac{1}{s+1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{e^{-6s}\frac{1}{s+3}\right\}$$

$$\left(\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathcal{U}(t-a)\right)$$

$$\boxed{\begin{aligned} y(t) = {} & \frac{1}{3}(1) - \frac{1}{2}e^{-t} + \frac{1}{6}e^{-3t} - \frac{1}{3}(1)\mathcal{U}(t-2) + \frac{1}{2}e^{-(t-2)}\mathcal{U}(t-2) - \frac{1}{6}e^{-3(t-2)}\mathcal{U}(t-2) \\ & - \frac{1}{3}(1)\mathcal{U}(t-4) + \frac{1}{2}e^{-(t-4)}\mathcal{U}(t-4) - \frac{1}{6}e^{-3(t-4)}\mathcal{U}(t-4) - \frac{1}{3}(1)\mathcal{U}(t-6) + \frac{1}{2}e^{-(t-6)}\mathcal{U}(t-6) - \frac{1}{6}e^{-3(t-6)}\mathcal{U}(t-6) \end{aligned}}$$

7.4

#1b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^3 e^t\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)] \quad n=3$

$(-1)^3 \frac{d^3}{ds^3}\left[\frac{1}{s-1}\right] = (-1)^3 \frac{d^3}{ds^3}\left[(s-1)^{-1}\right]$

$= (-1)\frac{d^2}{ds^2}\left[-(s-1)^{-2}\right]$

$= -\frac{d}{ds}\left[-2(s-1)^{-3}\right]$

$= 6(s-1)^{-4} = \boxed{\frac{6}{(s-1)^4}}$

#3b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{te^{-3t}\cos(3t)\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)], \; n=1$

$(-1)\frac{d}{ds}\left[\mathcal{L}\{e^{-3t}\cos(3t)\}\right]$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a), \; a=-3$

$(-1)\frac{d}{ds}\left[\frac{(s+3)}{(s+3)^2+9}\right]$

$(-1)\left[\frac{[(s+3)^2+9](1) - (s+3)(2(s+3)'(1))}{[(s+3)^2+9]^2}\right] = \frac{-(s+3)^2 - 9 + 2(s+3)^2}{[(s+3)^2+9]^2} = \boxed{\frac{(s+3)^2-9}{[(s+3)^2+9]^2}}$

#2b. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{t^2 \cos t\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)] \quad n=2$

$(-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$

$\frac{d}{ds}\left[\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right] = \frac{d}{ds}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$

$\frac{d}{ds}\left[\frac{-s^2+1}{(s^2+1)^2}\right] = \frac{(s^2+1)^2(-2s) - (-s^2+1)2(s^2+1)(2s)}{(s^2+1)^4}$

$\frac{(s^2+1)[-2s(s^2+1) - 4s(-s^2+1)]}{(s^2+1)^4} = \frac{-2s^3-2s+4s^3-4s}{(s^2+1)^3}$

$= \frac{2s(-s^2-1+2s^2-2)}{(s^2+1)^3} = \boxed{\frac{2s(s^2-3)}{(s^2+1)^3}}$

#4b. Use the Laplace transform to solve the initial-value problem:

$y' - y = te^t \sin(t), \quad y(0) = 0$

$[sY(s) - y(0)] - Y(s) = \mathcal{L}\{te^t \sin(t)\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$

$\mathcal{L}\{e^t \sin t\}$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \quad a=1$

$F(s) = \frac{1}{(s-1)^2+1}$

$sY(s) - 0 - Y(s) = (-1)\frac{d}{ds}\left[\frac{1}{(s-1)^2+1}\right] = (-1)\left[\frac{((s-1)^2+1)(0) - (1)(2(s-1)'(1))}{((s-1)^2+1)^2}\right] = \frac{2(s-1)}{((s-1)^2+1)^2}$

$(s-1)Y(s)$

$Y(s) = \frac{2(s-1)}{(s-1)[(s-1)^2+1]^2} = \frac{2}{[(s-1)^2+1]^2}$

$\mathcal{L}^{-1}\{F(s-a)\} = e^{at} f(t)$

$s-1$: shift, e^t

extended table: $\mathcal{L}\{\sin kt - kt\cos kt\} = \frac{2k^3}{(s^2+k^2)^2} \quad (k=1)$

$y(t) = e^t \mathcal{L}^{-1}\left\{\frac{2}{(s^2+1)^2}\right\}$

$y(t) = e^t(\sin t - t\cos t)$

$\boxed{y(t) = e^t \sin t - te^t \cos t}$

#5b. Use the Laplace transform to solve the initial-value problem:

$y''+y=\sin t,\quad y(0)=1,\quad y'(0)=-1$

$[s^2Y(s)-sy(0)-y'(0)]+Y(s)=\mathcal{L}\{\sin t\}$

$s^2Y(s)-s(1)+1+Y(s)=\frac{1}{s^2+1}$

$(s^2+1)Y(s)-s+1=\frac{1}{s^2+1}$

extended table:

$\mathcal{L}\{\sin kt - kt\cos kt\}=\frac{2k^3}{(s^2+k^2)^2}\quad (k=1)$

$Y(s)=\frac{s}{s^2+1}-\frac{1}{s^2+1}+\frac{1}{2}\frac{\times 2}{(s^2+1)^2}$

$y(t)=\cos t-\sin t+\frac{1}{2}(\sin t-t\cos t)$

$\boxed{y(t)=\cos t-\frac{1}{2}\sin t-\frac{1}{2}t\cos t}$

#6b. Use the Laplace transform to solve the initial-value problem:

$y''+y=f(t),\quad y(0)=1,\quad y'(0)=0$

$$\text{where } f(t)=\begin{cases}1, & 0\le t<\frac{\pi}{2}\\ \sin t, & t\ge\frac{\pi}{2}\end{cases}$$

$f(t)=1-1u(t-\frac{\pi}{2})+\sin t\, u(t-\frac{\pi}{2})$

$[s^2Y(s)-sy(0)-y'(0)]+Y(s)=\mathcal{L}\{1\}-\mathcal{L}\{1u(t-\frac{\pi}{2})\}+\mathcal{L}\{\sin t\, u(t-\pi/2)\}$

$(\mathcal{L}\{g(t)u(t-a)\}=e^{-as}\mathcal{L}\{g(t+a)\})$

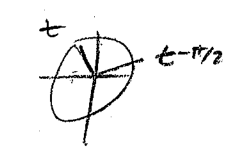
$s^2Y(s)-s(1)-0+Y(s)=$

$(s^2+1)Y(s)-s=\frac{1}{s}-e^{-\pi/2 s}\mathcal{L}\{1\}+e^{-\pi/2 s}\mathcal{L}\{\sin t\}$

extended table:

$\mathcal{L}\{\sin kt-kt\cos kt\}=\frac{2k^3}{(s^2+k^2)^2}$

$(s^2+1)Y(s)=s+\frac{1}{s}-e^{-\pi/2 s}\frac{1}{s}+e^{-\pi/2 s}\frac{1}{s^2+1}$



$Y(s)=\frac{s}{s^2+1}+\frac{1}{s(s^2+1)}-e^{-\pi/2 s}\frac{1}{s(s^2+1)}+e^{-\pi/2 s}\frac{1}{(s^2+1)^2}$

$Y(s)=\frac{s}{s^2+1}+\frac{1}{s}-\frac{s}{s^2+1}-e^{-\pi/2 s}\frac{1}{s}+e^{-\pi/2 s}\frac{s}{s^2+1}+\frac{1}{2}e^{-\pi/2 s}\frac{\times 2}{(s^2+1)^2}$

$(\mathcal{L}^{-1}\{e^{-as}F(s)\}=f(t-a)u(t-a)\ \ (a=\pi/2))$

$y(t)=\cancel{\cos t}+1-\cancel{\cos t}-(1)u(t-\pi/2)+\cos(t-\pi/2)u(t-\pi/2)+\frac{1}{2}(\sin(t-\pi/2)-t\cos(t-\pi/2))u(t-\pi/2)$

$\boxed{y(t)=1-u(t-\pi/2)+\cos(t-\pi/2)u(t-\pi/2)+\frac{1}{2}\sin(t-\pi/2)u(t-\pi/2)-\frac{1}{2}(t-\pi/2)\cos(t-\pi/2)u(t-\pi/2)}$

7.5

#1b. Use the Laplace transform to solve the initial-value problem:

$y' + y = \delta(t-1), \quad y(0) = 2$

$[sY(s) - y(0)] + Y(s) = \mathcal{L}\{\delta(t-1)\}$

$(s+1)Y(s) - 2 = e^{-t}$

$Y(s) = 2\frac{1}{s+1} + e^{-t}\frac{1}{s+1}$

$y(t) = 2\mathcal{L}^{-1}\{\frac{1}{s+1}\} + \mathcal{L}^{-1}\{e^{-t}\frac{1}{s+1}\}$

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$
$(a=1)$

$\boxed{y(t) = 2e^{-t} + e^{-(t-1)}u(t-1)}$

#2b. Use the Laplace transform to solve the initial-value problem:

$y'' + 16y = \delta(t-2\pi), \quad y(0) = 0, \quad y'(0) = 0$

$[s^2Y(s) - s\cancel{y(0)} - \cancel{y'(0)}] + 16Y(s) = \mathcal{L}\{\delta(t-2\pi)\}$
$0 \quad 0$

$(s^2+16)Y(s) = e^{-2\pi s}$

$Y(s) = \frac{1}{4}e^{-2\pi s}\frac{4}{s^2+16}$

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$
$(a=2\pi)$

$y(t) = \frac{1}{4}\sin(4(t-2\pi))u(t-2\pi)$

$\sin(4t - 8\pi)$

$\boxed{y(t) = \frac{1}{4}\sin(4t)u(t-2\pi)}$

#3b. Use the Laplace transform to solve the initial-value problem:

$y'' + y = \delta(t-2\pi) + \delta(t-4\pi),$

$y(0) = 1, \quad y'(0) = 0$

$[s^2Y(s) - sy(0) - y'(0)] + Y(s) = \mathcal{L}\{\delta(t-2\pi)\} + \mathcal{L}\{\delta(t-4\pi)\}$

$(s^2+1)Y(s) - s = e^{-2\pi s} + e^{-4\pi s}$

$Y(s) = \frac{s}{s^2+1} + e^{-2\pi s}\frac{1}{s^2+1} + e^{-4\pi s}\frac{1}{s^2+1} \qquad (\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a))$

$$\boxed{\begin{aligned} &y(t) = \cos(t) + \sin(t-2\pi)u(t-2\pi) + \sin(t-4\pi)u(t-4\pi) \\ &\text{or} \\ &y = \cos(t) + \sin t\, u(t-2\pi) + \sin t\, u(t-4\pi) \end{aligned}}$$

#4b. Use the Laplace transform to solve the initial-value problem:

$y'' - 2y' = 1 + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1$

$[s^2Y(s) - sy(0) - y'(0)] - 2[sY(s) - y(0)] = \mathcal{L}\{1\} + \mathcal{L}\{\delta(t-2)\}$

$s^2Y(s) - s(0) - 1 - 2sY(s) + 2(0) = \frac{1}{s} + e^{-2s}$

$(s^2 - 2s)Y(s) = 1 + \frac{1}{s} + e^{-2s}$

$Y(s) = \frac{1}{s(s-2)} + \frac{1}{s^2(s-2)} + e^{-2s}\frac{1}{s(s-2)} = -\frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{1}{s-2} - \frac{1}{4}\frac{1}{s} - \frac{1}{2}\frac{1}{s^2} + \frac{1}{4}\frac{1}{s-2} - \frac{1}{2}e^{-2s}\frac{1}{s} + \frac{1}{2}e^{-2s}\frac{1}{s-2}$

$(\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a))$
$(a=2)$

$Y(s) = -\frac{3}{4}\frac{1}{s} + \frac{3}{4}\frac{1}{s-2} - \frac{1}{2}\frac{1}{s^2} - \frac{1}{2}e^{-2s}\frac{1}{s} + \frac{1}{2}e^{-2s}\frac{1}{s-2}$

$y(t) = -\frac{3}{4}(1) + \frac{3}{4}e^{2t} - \frac{1}{2}t - \frac{1}{2}(1)u(t-2) + \frac{1}{2}e^{2(t-2)}u(t-2)$

$\boxed{y(t) = -\frac{3}{4} + \frac{3}{4}e^{2t} - \frac{1}{2}t - \frac{1}{2}u(t-2) + \frac{1}{2}e^{2(t-2)}u(t-2)}$

Ch7 Test Review

#1. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=2t^4$

$2\mathscr{L}\{t^4\}$

$2\frac{4!}{s^5}$

$$\boxed{\frac{48}{s^5}}$$

#2. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^2+6t-3$

$\mathscr{L}\{t^2\}+6\mathscr{L}\{t\}-3\mathscr{L}\{1\}$

$\frac{2!}{s^3}+6\frac{1}{s^2}-3\frac{1}{s}$

$$\boxed{\frac{2}{s^3}+\frac{6}{s^2}-\frac{3}{s}}$$

#3. Use theorems and the table to find $\mathscr{L}\{f(t)\}$

$f(t)=t^2-e^{-9t}+5$

$\mathscr{L}\{t^2\}-\mathscr{L}\{e^{-9t}\}+5\mathscr{L}\{1\}$

$\frac{2!}{s^3}-\frac{1}{s-(-9)}+5\frac{1}{s}$

$$\boxed{\frac{2}{s^3}-\frac{1}{s+9}+\frac{5}{s}}$$

#4. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$ (online PFE)

$\mathscr{L}^{-1}\left\{\frac{1}{2}\frac{1}{s-2}-\frac{1}{s-3}+\frac{1}{2}\frac{1}{s-6}\right\}$

$\frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s-2}\right\}-\mathscr{L}^{-1}\left\{\frac{1}{s-3}\right\}+\frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s-6}\right\}$

$$\boxed{\frac{1}{2}e^{2t}-e^{3t}+\frac{1}{2}e^{6t}}$$

#5. Use theorems and the table to find

$\mathscr{L}^{-1}\left\{\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$

$\mathscr{L}^{-1}\left\{\frac{1}{2}\frac{1}{s}-\frac{1}{s-1}-\frac{1}{3}\frac{1}{s+1}+\frac{5}{6}\frac{1}{s-2}\right\}$

$\frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s}\right\}-\mathscr{L}^{-1}\left\{\frac{1}{s-1}\right\}-\frac{1}{3}\mathscr{L}^{-1}\left\{\frac{1}{s+1}\right\}+\frac{5}{6}\mathscr{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$\frac{1}{2}(1)-e^{t}-\frac{1}{3}e^{-t}+\frac{5}{6}e^{2t}$

$$\boxed{\frac{1}{2}-e^{t}-\frac{1}{3}e^{-t}+\frac{5}{6}e^{2t}}$$

#6. Use the Laplace transform to solve the initial-value problem:

$y' + 6y = e^{4t}, \quad y(0) = 2$

$[sY(s) - y(0)] + 6Y(s) = \mathcal{L}\{e^{4t}\}$

$(s+6)Y(s) - 2 = \frac{1}{s-4}$

$Y(s) = \frac{2}{s+6} + \frac{1}{(s+6)(s-4)} = \frac{2}{s+6} - \frac{1}{10}\frac{1}{s+6} + \frac{1}{10}\frac{1}{s-4} = \frac{19}{10}\frac{1}{s+6} + \frac{1}{10}\frac{1}{s-4}$

$y(t) = \frac{19}{10}\mathcal{L}\{\frac{1}{s+6}\} + \frac{1}{10}\mathcal{L}\{\frac{1}{s-4}\}$

$\boxed{y(t) = \frac{19}{10}e^{-6t} + \frac{1}{10}e^{4t}}$

#7. Use the Laplace transform to solve the initial-value problem:

$y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1, \quad y'(0) = -1$

$[s^2Y(s) - sy(0) - y'(0)] - 4[sY(s) - y(0)] = 6\mathcal{L}\{e^{3t}\} - 3\mathcal{L}\{e^{-t}\}$

$s^2Y(s) - s(1) + 1 - 4sY(s) + 4(1) = 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$(s^2 - 4s)Y(s) = s - 5 + 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$Y(s) = \frac{s}{s(s-4)} - 5\frac{1}{s(s-4)} + 6\frac{1}{s(s-4)(s-3)} - 3\frac{1}{s(s-4)(s+1)}$

$Y(s) = \frac{1}{s-4} - 5\left(-\frac{1}{4}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4}\right) + 6\left(\frac{1}{12}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4} - \frac{1}{3}\frac{1}{s-3}\right) - 3\left(-\frac{1}{4s} + \frac{1}{20}\frac{1}{s-4} + \frac{1}{5}\frac{1}{s+1}\right)$

$Y(s) = \frac{11}{10}\frac{1}{s-4} + \frac{5}{2}\frac{1}{s} - 2\frac{1}{s-3} - \frac{3}{5}\frac{1}{s+1}$

$\boxed{y(t) = \frac{11}{10}e^{4t} + \frac{5}{2} - 2e^{3t} - \frac{3}{5}e^{-t}}$

#8. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

doesn't factor, so complete the square:

$s^2-6s+9+10-9$

$(s-3)^2+1$

$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1}\right\}$

$\mathcal{L}^{-1}\{F(s-a)\}=e^{at}f(t)$ $(a=3)$

$e^{3t}\mathcal{L}\left\{\frac{1}{s^2+1}\right\}$

$\boxed{e^{3t}\sin(t)}$

#9. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$

doesn't factor, so complete the square:

$s^2+2s+1+5-1$

$(s+1)^2+4$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$

$\mathcal{L}^{-1}\{F(s-a)\}=e^{at}f(t)$ $(a=-1)$

$e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$ fix up constant

$\frac{1}{2}e^{-t}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$

$\boxed{\frac{1}{2}e^{-t}\sin(2t)}$

#10. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

doesn't factor, so complete the square:

$s^2+4s+4+5-4$

$(s+2)^2+1$

$\mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\}$ shift? top must also be s+2

$\mathcal{L}^{-1}\left\{\frac{(s+2)-2}{(s+2)^2+1}\right\}$

$\mathcal{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+1}\right\}-2\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}$

$(\mathcal{L}^{-1}\{F(s-a)\}=e^{at}f(t)$, $(a=-2)$

$e^{-2t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}-2e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$

$\boxed{e^{-2t}\cos t-2e^{-2t}\sin t}$

#11. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathscr{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}$

doesn't factor, complete the square:

$s^2+6s+9+34-9$

$(s+3)^2+25$

(factor out)

$2\mathcal{L}^{-1}\left\{\frac{s+5/2}{(s+3)^2+25}\right\}$ shift? top must also be s+3

$2\mathcal{L}^{-1}\left\{\frac{(s+3)-3+5/2}{(s+3)^2+25}\right\}$

$2\mathcal{L}^{-1}\left\{\frac{(s+3)}{(s+3)^2+25}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+25}\right\}$ fix up constant

$2\mathcal{L}^{-1}\left\{\frac{(s+3)}{(s+3)^2+25}\right\}-\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{5}{(s+3)^2+25}\right\}$

shift! $\mathcal{L}^{-1}\{F(s-a)\}=e^{-at}f(t)$ $(a=-3)$

$2e^{-3t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\}-\frac{1}{5}e^{-3t}\mathcal{L}^{-1}\left\{\frac{5}{s^2+25}\right\}$

$\boxed{2e^{-3t}\cos(5t)-\frac{1}{5}e^{-3t}\sin(5t)}$

#12. Use the Laplace transform to solve the initial-value problem

$y'' - 6y' + 9y = t, \qquad y(0) = 0, \qquad y'(0) = 1$

$[s^2Y(s) - sy(0) - y'(0)] - 6[sY(s) - y(0)] + 9Y(s) = \mathcal{L}\{t\}$

$s^2Y(s) - s(0) - 1 - 6sY(s) + 6(0) + 9Y(s) = \frac{1}{s^2}$

$(s^2 - 6s + 9)Y(s) = 1 + \frac{1}{s^2}$

$Y(s) = \frac{1}{s^2 - 6s + 9} + \frac{1}{s^2(s^2 - 6s + 9)} = \frac{1}{(s-3)^2} + \frac{2}{27}\frac{1}{s} + \frac{1}{9}\frac{1}{s^2} - \frac{2}{27}\frac{1}{s-3} + \frac{1}{9}\frac{1}{(s-3)^2}$

$Y(s) = \frac{10}{9}\frac{1}{(s-3)^2} + \frac{2}{27}\frac{1}{s} + \frac{1}{9}\frac{1}{s^2} - \frac{2}{27}\frac{1}{s-3}$

$\boxed{y(t) = \frac{10}{9}te^{3t} + \frac{2}{27}(1) + \frac{1}{9}t - \frac{2}{27}e^{3t}}$

#13. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\} = \mathcal{L}^{-1}\{e^{-2s}\frac{1}{s^3}\}$

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$

$(a=2)$

$\mathcal{L}^{-1}\{\frac{1}{s^3}\}u(t-2)$

$\frac{1}{2}\mathcal{L}^{-1}\{\frac{2}{s^3}\}u(t-2)$

$\boxed{\frac{1}{2}(t-2)^2 u(t-2)}$

#14. Evaluate the Inverse Laplace transform to find $f(t)$: $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$

$\mathcal{L}^{-1}\{e^{-\pi s}\frac{1}{s^2+1}\}$

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a) \quad (a=\pi)$

$\mathcal{L}^{-1}\{\frac{1}{s^2+1}\}u(t-\pi)$

$\boxed{\sin(t-\pi)u(t-\pi)}$

or, because $\sin(t-\pi) = -\sin t$

$\boxed{-\sin t \, u(t-\pi)}$

#15. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$$

$\boxed{f(t) = 2 - 4\,u(t-3)}$

$2\mathcal{L}\{1\} - 4\mathcal{L}\{1\,u(t-3)\}$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$(a=3)$

$2\frac{1}{s} - 4e^{-3s}\mathcal{L}\{1\}$

$\boxed{2\frac{1}{s} - 4e^{-3s}\frac{1}{s}}$

#16. Write the function in terms of unit step functions, then find the Laplace transform of the given function:

$$f(t)=\begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

$\boxed{f(t) = t - t\,u(t-2)}$

$\mathcal{L}\{t\} - \mathcal{L}\{t\,u(t-2)\}$

$\mathcal{L}\{g(t)\,u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$(a=2)$

$\frac{1}{s^2} - e^{-2s}\mathcal{L}\{(t+2)\}$

$\boxed{\frac{1}{s^2} - e^{-2s}\frac{1}{s^2} - 2e^{-2s}\frac{1}{s}}$

#17. Evaluate the Laplace transform to find $F(s)$

$\mathcal{L}\{te^{-10t}\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)] \quad (n=1)$

$(-1)\frac{d}{ds}[\mathcal{L}\{e^{-10t}\}]$

$(-1)\frac{d}{ds}\left[\frac{1}{s+10}\right]$

$(-1)\left[\frac{(s+10)(0) - (1)(1)}{(s+10)^2}\right]$

$\boxed{\frac{1}{(s+10)^2}}$

#18. Evaluate the Laplace transform to find $F(s)$

$\mathcal{L}\{te^{2t}\sin(6t)\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)] \quad (n=1)$

$(-1)\frac{d}{ds}[\mathcal{L}\{e^{2t}\sin(6t)\}]$

$\mathcal{L}\{e^{at}f(t)\} = F(s-a) \quad (a=2)$

$(-1)\frac{d}{ds}\left[\frac{6}{(s-2)^2+36}\right]$

$(-1)\left[\frac{((s-2)^2+36)(0) - 6(2(s-2)(1))}{[(s-2)^2+36]^2}\right]$

$\boxed{\frac{12(s-2)}{[(s-2)^2+36]^2}}$

#19. Evaluate the Laplace transform to find $F(s)$

$\mathscr{L}\{te^{-3t}\cos(3t)\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$ $(n=1)$

$(-1)\frac{d}{ds}[\mathcal{L}\{e^{-3t}\cos(3t)\}]$

$\mathcal{L}\{e^{-as}f(t)\} = F(s-a)$ $(a=3)$

$(-1)\frac{d}{ds}\left[\frac{(s-3)}{(s-3)^2+9}\right]$

$(-1)\left(\frac{[(s-3)^2+9](1) - (s-3)(2(s-3)^1(1))}{[(s-3)^2+9]^2}\right)$

$(-1)\frac{(s-3)^2+9-2(s-3)^2}{[(s-3)^2+9]^2} = \boxed{\frac{(s-3)^2-9}{[(s-3)^2+9]^2}}$

#20. Use the Laplace transform to solve the initial-value problem

$y'' - 2y' + y = e^t, \quad y(0) = 0, \quad y'(0) = 5$

$[s^2Y(s) - s\,y(0) - y'(0)] - 2[sY(s) - y(0)] + Y(s) = \mathcal{L}\{e^t\}$

$s^2Y(s) - s(0) - 5 - 2sY(s) + 2(0) + Y(s) = \frac{1}{s-1}$

$(s^2-2s+1)Y(s) = 5 + \frac{1}{s-1}$

s^2-2s+1

$(s-1)^2$

$Y(s) = \frac{5}{s^2-2s+1} + \frac{1}{(s-1)(s^2-2s+1)}$

$Y(s) = \frac{5}{(s-1)^2} + \frac{1}{(s-1)^3}$

two possible approaches:

1) treat $s-1$ as a shift

$y(t) = 5\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3}\right\}$

$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$ $(a=1)$

$y(t) = 5e^t\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + e^t\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

fix up constant

$y(t) = 5e^t\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{1}{2}e^t\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$

$\boxed{y(t) = 5e^t t + \frac{1}{2}e^t t^2}$

or 2) use more advanced table entries

$y(t) = 5\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3}\right\}$

table: $\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2}\right\} = te^{at}$ $(a=1)$ $\quad \mathcal{L}^{-1}\left\{\frac{n!}{(s-a)^{n+1}}\right\} = t^n e^{at}$

fix up constant

$5te^t + \frac{1}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{(s-1)^3}\right\}$ $(a=1)$ $(n=2)$

$\boxed{5te^t + \frac{1}{2}t^2e^t}$