DiffEq - Ch 5 - Extra Practice

#1b. A mass weighing 6 pounds is attached to a spring whose spring constant is 18 lb/ft. What is the period of simple harmonic motion?

$T=\frac{1}{f}$, $w=2\pi f$, $w^2=\frac{k}{m}$, weight $=mg$

$mg = 6$ lbs

$m=\frac{6}{32}=\frac{3}{16}$ slug, $w^2=\frac{k}{m}=\frac{18}{(\frac{3}{16})}=96$

$w=\sqrt{96}=2\pi f$

$f=\frac{\sqrt{96}}{2\pi}$, $T=\frac{1}{f}$

$\boxed{T=\frac{2\pi}{\sqrt{96}}=0.6413 \text{ sec}}$

#2b. A 10-kilogram mass is attached to a spring. If the frequency of simple harmonic motion is $\frac{4}{\pi}$ cycles/s:

a) What is the spring constant k?

b) What is the frequency of simple harmonic motion if the original mass is replaced with an 80-kilogram mass?

(a) $f=\frac{4}{\pi}$, $w=2\pi f=2\pi\left(\frac{4}{\pi}\right)=8$

$w^2=\frac{k}{m}$

$(8)^2=\frac{k}{10}$, $K=10(8^2)=\boxed{640 \text{ N/m}}$

(b) (same k)

$w^2=\frac{k}{m}=\frac{640}{80}=8$

$w=\sqrt{8}=2\pi f$

$\boxed{f=\frac{\sqrt{8}}{2\pi}=0.450 \text{ cycles/s}}$

#3b. A mass weighing 10 pounds, attached to the end of a spring, stretches it 2 inches. Initially, the mass is released from rest from a point 4 inches below the equilibrium position. Find the equation of motion.

(positive x) $\quad \frac{2}{12}$ ft $\quad \frac{4}{12}$ ft

$mg=ks$

$10=k\left(\frac{2}{12}\right)=\frac{k}{6}$, $k=6(10)=60$

$mg=ks$

$m(32)=60\left(\frac{2}{12}\right)$, $m=\frac{5}{16}$ slug

$mx''+\beta x'+kx=0$ ($\beta=0$, no damping)

$\frac{5}{16}x''+60x=0$

$x''+192x=0$

$m^2+192=0$, $m^2=-192$, $m=0\pm\sqrt{192}\,i$

$x(t)=c_1e^{0t}\cos(\sqrt{192}t)+c_2e^{0t}\sin(\sqrt{192}t)$

$x=c_1\cos(\sqrt{192}t)+c_2\sin(\sqrt{192}t)$

$x'=-\sqrt{192}\,c_1\sin(\sqrt{192}t)+\sqrt{192}\,c_2\cos(\sqrt{192}t)$

now: $x(0)=\frac{4}{12}$ (must be in ft)

$x'(0)=0$ (from rest)

$\frac{4}{12}=c_1\cos(0)+c_2\sin(0) \rightarrow c_1=\frac{4}{12}=\frac{1}{3}$

$0=-\sqrt{192}\left(\frac{1}{3}\right)\sin(0)+\sqrt{192}\,c_2\cos(0)$

$0=\sqrt{192}\,c_2 \rightarrow c_2=0$

So $\boxed{x(t)=\frac{1}{3}\cos(\sqrt{192}t)}$

#4b. A mass weighing 5 pounds is attached to a spring. When set in motion, the spring/mass system exhibits simple harmonic motion. Determine the equation of motion if the spring constant is $2\ {}^{lb}/_{ft}$ and the mass is initially released from a point 8 inches below the equilibrium position with a downward velocity of $\frac{3}{4}\ {}^{ft}/_{s}$.

"positive"

$\frac{8}{12}$ ft

$mx'' + \beta x' + kx = 0$, $mg = 5$ lbs, $m = \frac{5}{32}$ slug, $k = 2$, $\beta = 0$ (no damping mentioned)

$\frac{5}{32}x'' + 2x = 0$

$x'' + \frac{64}{5}x = 0$

$m^2 + \frac{64}{5} = 0$, $m^2 = -\frac{64}{5}$ $\qquad$ $m = 0 \pm \sqrt{\frac{64}{5}}\, i = 0 \pm \frac{8}{\sqrt{5}} i$

$\alpha \pm \beta i$

$x(t) = C_1 e^{0t}\cos\left(\frac{8}{\sqrt{5}}t\right) + C_2 e^{0t}\sin\left(\frac{8}{\sqrt{5}}t\right)$

$x = C_1\cos\left(\frac{8}{\sqrt{5}}t\right) + C_2\sin\left(\frac{8}{\sqrt{5}}t\right)$

$x(0) = \frac{8}{12}$ (must be in ft, positive downward)

$\frac{8}{12} = C_1\cos(0) + C_2\sin(0) \rightarrow \underline{C_1 = \frac{8}{12}} = \frac{2}{3}$

$x' = -\frac{8}{\sqrt{5}}C_1\sin\left(\frac{8}{\sqrt{5}}t\right) + \frac{8}{\sqrt{5}}C_2\cos\left(\frac{8}{\sqrt{5}}t\right)$

$x'(0) = \frac{3}{4}$ (positive = downward)

$\frac{3}{4} = \frac{-8}{\sqrt{5}}\left(\frac{2}{3}\right)\sin(0) + \frac{8}{\sqrt{5}}C_2\cos(0) \rightarrow \frac{8}{\sqrt{5}}C_2 = \frac{3}{4} \rightarrow \underline{C_2 = \frac{3\sqrt{5}}{32}}$

so $\boxed{x(t) = \frac{2}{3}\cos\left(\frac{8}{\sqrt{5}}t\right) + \frac{3\sqrt{5}}{32}\sin\left(\frac{8}{\sqrt{5}}t\right)}$

#5b. A mass weighing 8 pounds is attached to a spring whose constant is 3 lb/ft. The medium offers a damping force that is numerically equal to the instantaneous velocity. The mass is initially released from a point 2 foot above the equilibrium position with a upward velocity of 8 ft/s.

a) Determine the time at which the mass passes through the equilibrium position.
b) Find the time at which the mass attains its extreme displacement from the equilibrium position.
c) What is the position of the mass at this instant?

(a) $mg = 8\,lbs \qquad k=3$

$m = \frac{8}{32} = \frac{1}{4}$ slug $\qquad \beta = 1$

$mx'' + \beta x' + kx = 0$

$\frac{1}{4}x'' + (1)x' + 3x = 0$

$x'' + 4x' + 12x = 0$

$m^2 + 4m + 12 = 0$

$m = \frac{-4 \pm \sqrt{16 - 4(1)(12)}}{2(1)} = \frac{-4 \pm \sqrt{32}\,i}{2} \qquad \sqrt{4}\sqrt{8}$

$m = -2 \pm \sqrt{8}\,i$

$x(t) = C_1 e^{-2t}\cos(\sqrt{8}t) + C_2 e^{-2t}\sin(\sqrt{8}t)$

$x(0) = -2$ ("above" = negative)

$-2 = C_1 e^0 \cos(0) + C_2 e^0 \sin(0) \rightarrow C_1 = -2$

$x'(t) = C_1 e^{-2t}(-\sqrt{8}\sin(\sqrt{8}t)) + \cos(\sqrt{8}t)(-2C_1 e^{-2t})$
$\quad + C_2 e^{-2t}(\sqrt{8}\cos(\sqrt{8}t)) + \sin(\sqrt{8}t)(-2C_2 e^{-2t})$

$x'(0) = -8$ ("upward" = negative)

$-8 = (-2)e^0(-\sqrt{8})\sin(0) + \cos(0)(-2)(-2)e^0$
$\quad + C_2 e^0(\sqrt{8})\cos(0) + \sin(0)(-2)C_2 e^0$

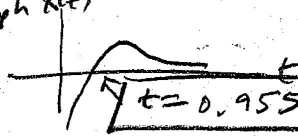
$-8 = 0 + 4 + \sqrt{8}\,C_2 + 0 \rightarrow C_2 = \frac{-12}{\sqrt{8}}$

$x(t) = -2e^{-2t}\cos(\sqrt{8}t) - \frac{12}{\sqrt{8}}e^{-2t}\sin(\sqrt{8}t)$

time when at equilibrium! $x=0$

$-2e^{-2t}\cos(\sqrt{8}t) - \frac{12}{\sqrt{8}}e^{-2t}\sin(\sqrt{8}t) = 0$

Calculator graph $x(t)$



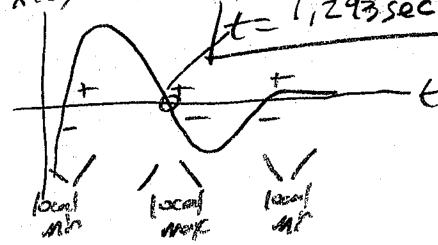
$\boxed{t = 0.955\text{ sec}}$

(b) extreme displacement at local max of $x(t)$ when $x'(t) = 0$:

$x'(t) = -2e^{-2t}(-\sqrt{8})\sin(\sqrt{8}t) - 2(-2)e^{-2t}\cos(\sqrt{8}t)$
$\quad - \frac{12}{\sqrt{8}}(\sqrt{8})e^{-2t}\cos(\sqrt{8}t) - 2\left(\frac{-12}{\sqrt{8}}\right)e^{-2t}\sin(\sqrt{8}t) =$

Solve with calculator graph zeroi

$x'(t)$



$\boxed{t = 1.293\text{ sec}}$

(c) $\boxed{x(1.293) = 0.2886\text{ ft}}$

(below equilibrium)

#6b. A 4-foot spring measures 12 feet long after a mass weighing 6 pounds is attached to it. The medium through which the mass moves offers a damping force numerically equal to $\sqrt{5}$ times the instantaneous velocity.

a) Find the equation of motion if the mass is initially released from the equilibrium position with a downward velocity of $4\ ^{ft}/_{s}$.

b) Find the time at which the mass attains its extreme displacement from the equilibrium position.

c) What is the position of the mass at this instant?

(a) $mg = ks$ $\qquad mg = 6$

$6 = k(8) \qquad m = \frac{6}{32} = \frac{3}{16}$ slug

$k = \frac{6}{8} = \frac{3}{4}$ lb/ft $\qquad \beta = \sqrt{5}$

$$mx'' + \beta x' + kx = 0$$

$$\frac{3}{16}x'' + \sqrt{5}x' + \frac{3}{4}x = 0$$

$$x'' + \frac{16\sqrt{5}}{3}x' + 4x = 0$$

$$m^2 + \frac{16\sqrt{5}}{3}m + 4 = 0$$

$$m = \frac{-\frac{16\sqrt{5}}{3} \pm \sqrt{\left(\frac{16\sqrt{5}}{3}\right)^2 - 4(1)(4)}}{2(1)}$$

$$m = \frac{-\frac{16\sqrt{5}}{3} \pm \sqrt{\frac{1280}{9} - 16}}{2} = \frac{-\frac{16\sqrt{5}}{3} \pm \frac{\sqrt{1136}}{3}}{2}$$

$$m = \frac{-16\sqrt{5} \pm \sqrt{1136}}{6} = -.3454,\ -11.580$$

$$x(t) = C_1 e^{-.3454t} + C_2 e^{-11.580t}$$

$x(0) = 0$

$0 = C_1 e^0 + C_2 e^0 \Rightarrow C_1 + C_2 = 0$

$x'(t) = -.3454 C_1 e^{-.3454t} - 11.580 C_2 e^{-11.58t}$

$x'(0) = 4$ ("downward" = positive)

$4 = -.3454 C_1 e^0 - 11.580 C_2 e^0$

$\Rightarrow -.3454 C_1 - 11.580 C_2 = 4$

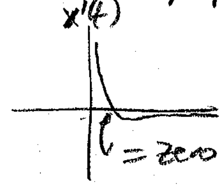
$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -.3454 & -11.58 & 4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & .356 \\ 0 & 1 & -.356 \end{array}\right] \begin{array}{l} = C_1 \\ = C_2 \end{array}$$

$$\boxed{x(t) = 0.356 e^{-.3454t} - 0.356 e^{-11.580t}}$$

(b) extreme displacement at max/min x, when $x' = 0$

$x'(t) = (-.3454)(.356)e^{-.3454t} - (.356)(-11.58)e^{-11.58t} =$

by calculator graph!



= zero at $\boxed{t = 0.3126 \text{ sec}}$

(c) $x(0.3126) = 0.356 e^{-.3454(.3126)} - .356 e^{-11.58(.3126)}$

$\boxed{= 0.310 \text{ ft}}$

#7b. A 3-kilogram mass is attached to a spring whose constant is $20\ {}^{N}/_{m}$, and the entire system is then submerged in a liquid that imparts a damping force numerically equal to 8 times the instantaneous velocity.
a) Find the equation of motion if the mass is initially released from rest from a point 2 meter above the equilibrium position.
b) Find the equation of motion if the mass is initially released from a point 2 meter below the equilibrium position with an downward velocity of $10\ {}^{m}/_{s}$.

$m=3,\quad k=20,\quad \beta=8$

$mx''+\beta x'+kx=0$

$3x''+8x'+20x=0$

$3m^2+8m+20=0$

$m=\dfrac{-8\pm\sqrt{64-4(3)(20)}}{2(3)}=\dfrac{-8\pm\sqrt{176}\,i}{6}$

$m=-\frac{4}{3}\pm\frac{\sqrt{176}}{6}i=-1.333\pm 6.1633i$

$x(t)=C_1e^{-\frac{4}{3}t}\cos\left(\frac{\sqrt{176}}{2}t\right)+C_2e^{-\frac{4}{3}t}\sin\left(\frac{\sqrt{176}}{2}t\right)$

$x'(t)=C_1e^{-4/3t}\left(-\frac{\sqrt{176}}{2}\right)\sin\left(\frac{\sqrt{176}}{2}t\right)+\cos\left(\frac{\sqrt{176}}{2}t\right)\left(-\frac{4}{3}C_1e^{-4/3t}\right)$
$\quad+C_2e^{-4/3t}\left(\frac{\sqrt{176}}{2}\right)\cos\left(\frac{\sqrt{176}}{2}t\right)+\sin\left(\frac{\sqrt{176}}{2}t\right)\left(-\frac{4}{3}C_2e^{-4/3t}\right)$

(a) $x(0)=-2\qquad x'(0)=0$

$-2=C_1$

$0=-\frac{4}{3}C_1+\frac{\sqrt{176}}{2}C_2$

$0=-\frac{4}{3}(-2)+\frac{\sqrt{176}}{2}C_2$

$-\frac{8}{3}=\frac{\sqrt{176}}{2}C_2\rightarrow C_2=\frac{-16}{3\sqrt{176}}$

$C_1=-2$

$$\boxed{x(t)=-2e^{-4/3t}\cos\left(\frac{\sqrt{176}}{2}t\right)-\frac{16}{3\sqrt{176}}e^{-4/3t}\sin\left(\frac{\sqrt{176}}{2}t\right)}$$

(b) $x(0)=2\qquad x'(0)=10$

$2=C_1$

$10=-\frac{4}{3}C_1+C_2\frac{\sqrt{176}}{2}$

$10=-\frac{4}{3}(2)+C_2\frac{\sqrt{176}}{2}$

$\frac{38}{3}=C_2\frac{\sqrt{176}}{2}\rightarrow C_2=\frac{76}{3\sqrt{176}}$

$$\boxed{x(t)=2e^{-4/3t}\cos\left(\frac{\sqrt{176}}{2}t\right)+\frac{76}{3\sqrt{176}}e^{-4/3t}\sin\left(\frac{\sqrt{176}}{2}t\right)}$$

#8b. A force of 6 pounds stretches a spring 1 foot. A mass weighing 4 pounds is attached to the spring, and the system is then immersed in a medium that offer a damping force that is numerically equal to 0.5 times the instantaneous velocity.

a) Find the equation of motion if the mass is initially released from rest from a point 2 foot above the equilibrium position.

b) Use the fact that…

$$A\sin(\omega t+\phi)=C_1\cos(\omega t)+C_2\sin(\omega t)$$

where $A=\sqrt{C_1^{\,2}+C_2^{\,2}}$ *and* $\tan\phi=\dfrac{C_1}{C_2}$

…to express the equation of motion as a sum of two terms with the same frequency without phase shift.

c) Find the first time at which the mass passes through the equilibrium position heading upward.

(a) $mg=ks$ $\quad mg=4$

$6=k(1)$ $\quad m=\frac{4}{32}=\frac{1}{8}$ slug

$k=6$ lb/ft $\quad \beta=0.5$

$mx''+\beta x'+kx=0$

$\frac{1}{8}x''+\frac{1}{2}x'+6x=0$

$x''+4x'+48x=0$

$m=\dfrac{-4\pm\sqrt{16-4(1)(48)}}{2(1)}=\dfrac{-4\pm 13.2665i}{2}$

$m=-2\pm 6.6332i$

$x(t)=C_1e^{-2t}\cos(6.6332t)+C_2e^{-2t}\sin(6.6332t)$

$x(0)=-2$ ("above" = negative)

$-2=C_1e^0\cos(0)+C_2e^0\sin(0)\rightarrow \underline{C_1=-2}$

$x'(t)=C_1e^{-2t}(-6.6332\sin(6.6332t))+\cos(6.6332t)(-2C_1e^{-2t})$
$\quad +C_2e^{-2t}(6.6332\cos(6.6332t))+\sin(6.6332t)(-2C_2e^{-2t})$

$x'(0)=0$ ("from rest")

$0=(-2)e^0(-6.6332)\sin(0)+\cos(0)(-2)(-2)e^0+C_2e^0(6.6332)\cos(0)+\sin(0)(-2)C_2e^0$

$0=0+4+6.6332C_2+0\rightarrow \quad C_2=\frac{-4}{6.6332}=-0.603$

$$\boxed{x(t)=-2e^{-2t}\cos(6.6332t)-0.603e^{-2t}\sin(6.6332t)}$$

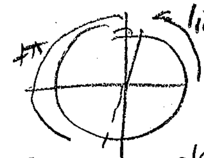
(b) $C_1\cos(\omega t)+C_2\sin(\omega t)=A\sin(\omega t+\phi)$

$x(t)=e^{-2t}[-2\cos(6.6332t)-0.603\sin(6.6332t)]$

$A=\sqrt{C_1^2+C_2^2}=\sqrt{(-2)^2+(-.603)^2}=2.0889$

$\tan\phi=\frac{C_1}{C_2}=\frac{-2}{-.603}=3.3167,$

$\phi=\tan^{-1}(3.3167)=1.278$

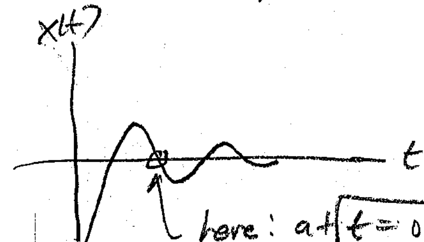


if C_1 & C_2 are both negative, we need to be in quad III

so $\phi=1.278+\pi=4.4196$

So $\boxed{x(t)=e^{-2t}[2.0889\sin(6.6332t+4.4196)]}$

(c) Calculator graph of $x(t)$:



here: at $\boxed{t=0.755 \text{ sec}}$

equilibrium position

$\uparrow -x$

heading upward is from $+x$ to $-x$

$\downarrow +x$

$+x$ direction

#9b. A mass weighing 12 pounds stretches a spring $\frac{7}{3}$ feet. The mass is initially released from rest from a point 1 feet above the equilibrium position, and the subsequent motion takes placed in a medium that offers a damping force that is numerically equal to $\frac{1}{3}$ the instantaneous velocity. Find the equation of motion if the mass is driven by an external force equal to $f(t) = 5\sin(5t)$.

① find x_c: $x'' + \frac{8}{9}x' + \frac{96}{7}x = 0$

$$m^2 + \frac{8}{9}m + \frac{96}{7} = 0$$

$$m = \frac{-\frac{8}{9} \pm \sqrt{\frac{64}{81} - 4(1)\left(\frac{96}{7}\right)}}{2(1)} = \frac{-\frac{8}{9} \pm \sqrt{\frac{30656}{567}}\,i}{2}$$

$$m = -.444 \pm 3.6765i$$

$$x_c = C_1 e^{-.444t}\cos(3.6765t) + C_2 e^{-.444t}\sin(3.6765t)$$

② find x_p:

$x_p = A\cos 5t + B\sin 5t$ (no absorption)

$x' = -5A\sin 5t + 5B\cos 5t$

$x'' = -25A\cos 5t - 25B\sin 5t$

$x'' + \frac{8}{9}x' + \frac{96}{7}x = \frac{40}{3}\sin 5t$

$$[-25A\cos 5t - 25B\sin 5t] + \frac{8}{9}[-5A\sin 5t + 5B\cos 5t] + \frac{96}{7}[A\cos 5t + B\sin 5t] = \frac{40}{3}\sin 5t$$

$$\left(-25A + \frac{40}{9}B + \frac{96}{7}A\right)\cos 5t + \left(-25B - \frac{40}{9}A + \frac{96}{7}B\right)\sin 5t = \frac{40}{3}\sin 5t$$

$$\left(-\frac{79}{7}A + \frac{40}{9}B\right)\cos 5t + \left(-\frac{40}{9}A - \frac{79}{7}B\right)\sin 5t = \left(\frac{40}{3}\right)\sin 5t$$

system: $\begin{cases} -\frac{79}{7}A + \frac{40}{9}B = 0 \\ -\frac{40}{9}A - \frac{79}{7}B = \frac{40}{3} \end{cases}$ $\left[\begin{array}{cc|c} -\frac{79}{7} & \frac{40}{9} & 0 \\ -\frac{40}{9} & -\frac{79}{7} & \frac{40}{3} \end{array}\right]$

rref $\rightarrow \left[\begin{array}{cc|c} 1 & 0 & -.40279 \\ 0 & 1 & -1.0228 \end{array}\right]$ $\begin{matrix} = A \\ = B \end{matrix}$

$x_p = -.40279\cos 5t - 1.0228\sin 5t$

So ... $x = x_c + x_p$

$$x = C_1 e^{-.444t}\cos(3.6765t) + C_2 e^{-.444t}\sin(3.6765t) - .40279\cos(5t) - 1.0228\sin(5t)$$

$mg = ks$

$12 = k\left(\frac{7}{3}\right)$

$k = \frac{12(3)}{7} = \frac{36}{7}$ lb/ft

$mg = 12$

$m = \frac{12}{32} = \frac{3}{8}$ slug

$\beta = \frac{1}{3}$

$mx'' + \beta x' + kx = f(t)$

$\frac{3}{8}x'' + \frac{1}{3}x' + \frac{36}{7}x = 5\sin(5t)$

$x'' + \frac{8}{9}x' + \frac{96}{7}x = \frac{40}{3}\sin(5t)$

③ use initial conditions:

$x(0) = -1$ ("above" = negative)

$-1 = C_1e^0\cos 0 + C_2e^0\sin 0 - .40279\cos 0 - 1.0228\sin 0$

$-1 = C_1 - .40279 \rightarrow C_1 = -.59721$

$x'(0) = 0$ ("from rest")

$$x'(t) = C_1 e^{-.444t}(-3.6765\sin(3.6765t)) + \cos(3.6765t)(-.444C_1e^{-.444t}) + C_2e^{-.444t}(3.6765\cos(3.6765t)) + \sin(3.6765t)(-.444C_2e^{-.444t}) + 5(.40279)\sin(5t) - 5(1.0228)\cos(5t)$$

$$0 = C_1e^0(-3.6765)\sin 0 - .444C_1e^0\cos 0 + 3.6765C_2e^0\cos 0 - .444C_2e^0\sin 0 + 5(.40279)\sin 0 - 5(1.0228)\cos 0$$

$0 = -.444(-.59721) + 3.6765C_2 - 5(1.0228)$

$0 = 3.6765C_2 - 4.84884 \rightarrow C_2 = 1.319$

④ final answer:

$$\boxed{x = -.59721e^{-.444t}\cos(3.6765t) + 1.319e^{-.444t}\sin(3.6765t) - .40279\cos(5t) - 1.0228\sin(5t)}$$

#10b. For the given LRC series circuit…

$L=\frac{5}{2}H, \quad R=8\,\Omega, \quad C=\frac{1}{20}F,$

$E(t)=250\,V, \quad q(0)=0\,Coulombs, \quad i(0)=0\,A$

a) Find the charge on the capacitor as a function of time.

b) Find the maximum charge on the capacitor.

a) ① q_c: $q''+\frac{16}{5}q'+8q=0$

$$m^2+\frac{16}{5}m+8=0$$

$$m=\frac{-\frac{16}{5}\pm\sqrt{(\frac{16}{5})^2-4(8)}}{2}=-1.6\pm 2.332i$$

$$\alpha\pm\beta i$$

$$q_c(t)=C_1e^{-1.6t}\cos(2.332t)+C_2e^{-1.6t}\sin(2.332t)$$

② q_p: (table)

$q_p=A$ (no absorption)

$q'=0$

$q''=0$

$q''+\frac{16}{5}q'+8q=100$

$[0]+\frac{16}{5}[0]+8A=100$

$A=\frac{100}{8}=12.5$

$q_p=12.5$

so $q(t)=q_c+q_p$

$$q(t)=C_1e^{-1.6t}\cos(2.332t)+C_2e^{-1.6t}\sin(2.332t)+12.5$$

$Lq''+Rq'+\frac{1}{C}q=E$

$\frac{5}{2}q''+8q'+20q=250$

$q''+\frac{16}{5}q'+8q=100$

③ use initial conditions:

$q(0)=0$

$0=C_1e^0\cos 0+C_2e^0\sin 0+12.5$

$0=C_1+12.5 \rightarrow C_1=-12.5$

$q'(0)=i(0)=0$

$q'(t)=C_1e^{-1.6t}(-2.332\sin(2.332t))$
$+\cos(2.332t)(-1.6C_1e^{-1.6t})$
$+C_2e^{-1.6t}(2.332\cos(2.332t))$
$+\sin(2.332t)(-1.6C_2e^{-1.6t})+0$

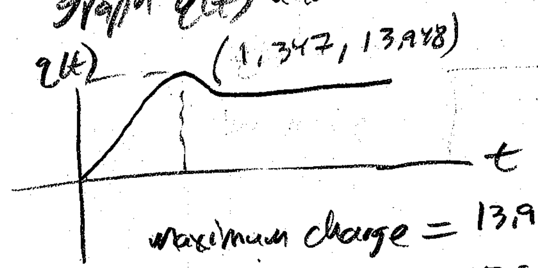
$0=(-12.5)e^0(-2.332)\sin 0-1.6(-12.5)e^0\cos 0$
$+2.332C_2e^0\cos 0-1.6C_2e^0\sin 0$

$0=0+20+2.332C_2$

$C_2=\frac{-20}{2.332}=-8.576$

$$\boxed{q(t)=-12.5e^{-1.6t}\cos(2.332t)-8.576e^{-1.6t}\sin(2.332t)+12.5}$$

(b) max charge: could now take derivative, $q'(t)=i(t)$ and set to zero, but we are going to need a calculator to find zeros anyway, so let's just graph $q(t)$ and use "maximum" feature



maximum charge = 13.948 Coulombs

(occurs at $t=1.347$ sec)

#11b. For the given LRC series circuit…
$L = 2\,H$, $R = 5\,\Omega$, $C = \cancel{.05}\ 0.1\,F$,
$E(t) = 80\sin(t)\ V$

a) Find the steady-state charge on the capacitor.
b) Find the steady-state current in the circuit.

$$Lq'' + Rq' + \frac{1}{C}q = E$$
$$2q'' + 5q' + 10q = 80\sin t$$
$$q'' + \frac{5}{2}q' + 5q = 40\sin t$$

a) (1) $\underline{q_c}$: $q'' + \frac{5}{2}q' + 5q = 0$

$$m^2 + \frac{5}{2}m + 5 = 0$$

$$m = \frac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - 20}}{2} = -1.25 \pm 1.854i$$
$$\alpha \pm \beta i$$

$$\underline{q_c(t) = C_1 e^{-1.25t}\cos(1.854t) + C_2 e^{-1.25t}\sin(1.854t)}$$

(2) $\underline{q_p}$: (table) $q_p = A\sin t + B\cos t$
(no absorption)

$$q' = A\cos t - B\sin t$$
$$q'' = -A\sin t - B\cos t$$
$$q'' + \frac{5}{2}q' + 5q = 40\sin t$$
$$[-A\sin t - B\cos t] + \frac{5}{2}[A\cos t - B\sin t]$$
$$+5[A\sin t + B\cos t] = 40\sin t$$
$$(-A - \tfrac{5}{2}B + 5A)\sin t + (-B + \tfrac{5}{2}A + 5B)\cos t = 40\sin t$$
$$(4A - \tfrac{5}{2}B)\sin t + (\tfrac{5}{2}A + 4B)\cos t = (40)\sin t$$

System: $\begin{cases} 4A - \frac{5}{2}B = 40 \\ \frac{5}{2}A + 4B = 0 \end{cases}$

$$\left[\begin{array}{cc|c} 4 & -5/2 & 40 \\ 5/2 & 4 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 640/89 \\ 0 & 1 & -400/89 \end{array}\right] \begin{array}{l} = A \\ = B \end{array}$$

$$q_p = \frac{640}{89}\cos t - \frac{400}{89}\sin t$$

so $q_c(t) = q_c + q_p$

$$q_c(t) = C_1 e^{-1.25t}\cos(1.854t) + C_2 e^{-1.25t}\sin(1.854t) + \frac{640}{89}\cos t - \frac{400}{89}\sin t$$

(3)
$$q_c(t) = \underbrace{C_1 e^{-1.25t}\cos(1.854t) + C_2 e^{-1.25t}\sin(1.854t)}_{\text{transient terms}} + \underbrace{\frac{640}{89}\cos t - \frac{400}{89}\sin t}_{\text{Steady-state terms}}$$

(these die out due to the $e^{-1.25t}$)

So steady-state charge:

$$\boxed{q_{ss}(t) = \frac{640}{89}\cos t - \frac{400}{89}\sin t}$$

b) current $= i = \frac{dq}{dt}$ (of the steady-state charge)

$$\boxed{i_{ss}(t) = -\frac{640}{89}\sin t - \frac{400}{89}\cos t}$$