DiffEq - Ch 4 - Extra Practice

4.1

#1b. The given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

$y = C_1 e^{4x} + C_2 e^{-x}, \quad (-\infty, \infty);$

$y'' - 3y' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 2$

#3b. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = 5, \quad f_2(x) = \cos^2 x, \quad f_3(x) = \sin^2 x$

#2b. The given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

$y = C_1 + C_2 \cos x + C_3 \sin x, \quad (-\infty, \infty);$

$y''' + y' = 0, \quad y(\pi) = 0, \quad y'(\pi) = 2, \quad y''(\pi) = -1$

#4b. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = x, \quad f_2(x) = x - 1, \quad f_3(x) = x + 3$

#5b. Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

$f_1(x) = e^{-3x}, \quad f_2(x) = e^{4x}$

#6b. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$4y'' - 4y' + y = 0; \quad e^{\frac{1}{2}x}, xe^{\frac{1}{2}x}, \ (-\infty, \infty)$

#7b. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$x^2y'' + xy' + y = 0; \quad \cos(\ln x), \sin(\ln x), \ (0,\infty)$

#8b. Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Then form the general solution.

$x^3y''' + 6x^2y'' + 4xy' - 4y = 0; \quad x, x^{-2}, x^{-2}\ln x, \ (0,\infty)$

4.2

#1b. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$y'' + 2y' + y = 0; \quad y_1 = xe^{-x}$

#2b. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$y'' + 9y = 0; \quad y_1 = \sin(3x)$

#3b. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$6y'' + y' - y = 0; \quad y_1 = e^{\left(\frac{1}{3}x\right)}$

#4b. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$x^2 y'' + 2xy' - 6y = 0; \quad y_1 = x^2$

#5b. The indicated function is a solution of the given differential equation (no need to verify). Use reduction of order to find a second solution.

$$\left(1-2x-x^2\right)y''+2\left(1+x\right)y'-2y=0; \qquad y_1=x+1$$

4.3

#1b. Find the general solution of the differential equation.
$y'' - 36y' = 0$

#2b. Find the general solution of the differential equation.
$y'' - 10y' + 25y = 0$

#3b. Find the general solution of the differential equation.
$y'' + 4y' - y = 0$

#4b. Find the general solution of the differential equation.
$3y'' + y = 0$

#5b. Find the general solution of the differential equation.

$2y'' + 2y' + y = 0$

#7b. Find the general solution of the differential equation.

$y''' + 3y'' - 4y' - 12y = 0$

#6b. Find the general solution of the differential equation.

$3y'' + 2y' + y = 0$

#8b. Find the general solution of the differential equation.

$y''' - 6y'' + 12y' - 8y = 0$

#9b. Solve the initial-value problem.

$4\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 3y = 0, \quad y(0) = 1, \; y'(0) = 5$

#10b. Solve the initial-value problem.

$y'' - 4y' + 5y = 0, \quad y(0) = 2, \; y'(0) = 1$

#11b. Solve the initial-value problem.

$y''' + 2y'' - 5y' - 6y = 0, \quad y(0) = y'(0) = 0, \; y''(0) = 1$

4.4

#1b. Solve the differential equation using the method of undermined coefficients.
$y'' + 9y = 15$

#2b. Solve the differential equation using the method of undermined coefficients.
$y'' + y' - 6y = 2x$

#3b. Solve the differential equation using the method of undermined coefficients.
$4y'' - 4y' - 3y = \cos 2x$

#4b. Solve the differential equation using the method of undermined coefficients.
$y'' - 8y' + 20y = 100x^2 - 26xe^x$

#5b. Solve the differential equation using the method of undermined coefficients.

$y'' + 4y = 3\sin 2x$

#6b. Solve the differential equation using the method of undermined coefficients.

$y'' + 4y = 2x\cos x$

#7b. Solve the initial-value problem.

$y'' + 4y' + 5y = 35e^{-4x}, \quad y(0) = -3, \quad y'(0) = 1$

4.6 (we skip 4.5)

#1b. Solve the differential equation using the method of variation of parameters.
$y'' + y = \tan x$

#2b. Solve the differential equation using the method of variation of parameters.
$y'' + y = \sec\theta \tan\theta$

#3b. Solve the differential equation using the method of variation of parameters.

$y'' + y = \sec^2 x$

#4b. Solve the differential equation using the method of variation of parameters.

$y'' - 9y = \dfrac{9x}{e^{3x}}$

#5b. Solve the differential equation using the method of variation of parameters.

$$y'' - 2y' + y = \frac{e^x}{1+x^2}$$

#6b. Solve the differential equation using the method of variation of parameters.

$y'' + 2y' + y = e^{-t} \ln(t)$

#7b. Solve the initial value problem using variation of parameters.

$$2y'' + y' - y = x + 1$$

4.7

#1b. Solve the differential equation.
$xy'' - 3y' = 0$

#3b. Solve the differential equation.
$x^2 y'' + 3xy' - 4y = 0$

#2b. Solve the differential equation.
$4x^2 y'' + 4xy' - y = 0$

#4b. Solve the differential equation by variation of parameters.

$2x^2 y'' + 5xy' + y = x^2 - x$

#5b. Use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients, then solve.

$x^2 y'' - 9xy' + 25y = 0$

#6b. Use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients, then solve.

$x^2 y'' - 4xy' + 6y = \ln\left(x^2\right)$