

## DiffEq - Ch 3 - Extra Practice

### 3.1

#1b. The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time  $t$ . After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present.

What was the initial number of bacteria?

#2b. The population of a town grows at a rate proportional to the population present at time  $t$ . The initial population of 500 increases by 15% in 10 years.

(a) What will the population be in 30 years?

(b) How fast is the population growing at  $t = 30$  years?

#3b. Initially 5 grams of a radioactive substance was present. After 8 hours the mass had decreased by 4%. If the rate of decay is proportional to the amount of the substance present at time  $t$ , find the amount remaining after 10 hours.

#5b. When interest is compounded continuously, the amount of money increases at a rate proportional to the amount  $A$  present at time  $t$ , that is  $\frac{dA}{dt} = rA$ , where  $r$  is the annual rate of interest.

(a) Find the amount of money accrued at the end of 10 years when \$10,000 is deposited in an investment account drawing 8% annual interest compounded continuously.

(b) In how many years will the initial sum deposited have doubled?

#4b. Determine the half-life of the radioactive substance described in Problem #3b

#6b. A thermometer is taken from an inside room to the outside, where the air temperature is  $5^{\circ}\text{F}$ . After 1 minute the thermometer reads  $55^{\circ}\text{F}$ , and after 5 minutes it reads  $30^{\circ}\text{F}$ . What was the initial temperature of the inside room?

#7b. A thermometer reading  $70^{\circ}\text{F}$ , is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads  $110^{\circ}\text{F}$  after 30 seconds and  $145^{\circ}\text{F}$  after 1 minute. How hot is the oven?

#8b. A 40-Volt electromotive force is applied to an  $LR$  series circuit in which the inductance is 0.3 Henry and the resistance is 150 Ohms.

(a) Find the equation for current as a function of time  $i(t)$  if  $i(0) = 0$ .

(b) Determine the current at  $t \rightarrow \infty$ .

#9b. A 200-Volt electromotive force is applied to an  $RC$  series circuit in which the resistance is 1000 Ohms and the capacitance is  $5 \cdot 10^{-6}$  Farad.

(a) Find the charge on the capacitor as a function of time  $q(t)$  if  $i(0) = 0.4$  Amps.

(b) Determine the charge and current at  $t = 0.005$  s.

(c) Determine the current at  $t \rightarrow \infty$ .

#10b. Suppose a small cannonball weighing 22 pounds is shot vertically upward with an initial velocity  $v_0 = 400 \text{ ft/s}$ . The answer to the question “How high does the cannonball go?” depends upon whether we take air resistance into account or not.

(a) Suppose air resistance is ignored. If the positive direction is upward, then a model for the state of the cannonball is given by:

$$\frac{d^2s}{dt^2} = -g$$

Since  $\frac{ds}{dt} = v(t)$  the last differential equation is the same as:

$$\frac{dv}{dt} = -g$$

where we take  $g = 32 \text{ ft/s}^2$ .

Find the velocity of the cannonball at time  $t$ .

(b) Use the result obtained in part (a) to determine the height  $s(t)$  of the cannonball measured from ground level.

(c) Find the maximum height attained by the cannonball.

#11b. Repeat problem #10b, but this time, assume that air resistance is proportional to instantaneous velocity. Use the following differential equation:

$$m \frac{dv}{dt} = mg - kv$$

...and assume  $k = 0.005$ .

- (a) Find the velocity of the cannonball at time  $t$ .
- (b) Use the result obtained in part (a) to determine the height  $s(t)$  of the cannonball measured from ground level.
- (c) Find the maximum height attained by the cannonball.

#12b. A skydiver weighs 125 pounds, and her parachute and equipment combined weight another 35 pounds. After exiting from a plane at an altitude of 15,000 ft, she waits 15 seconds and opens her parachute. Assuming that air resistance is proportional to instantaneous velocity, then velocity and time are related by this differential equation:

$$m \frac{dv}{dt} = mg - kv$$

(positive direction is downward). Assume  $k = 0.5$  during free fall and  $k = 10$  after the parachute is opened, and assume that her initial velocity on leaving the plane is zero.

What is her velocity 20 seconds after leaving the plane?

### 3.2

#1b. If a constant number  $h$  of fish are harvested from a fishery per unit time, then a model for the population  $P(t)$  of the fishery at time  $t$  is given by:

$$\frac{dP}{dt} = P(a - bP) - h, \quad P(0) = P_0$$

where  $a$ ,  $b$ ,  $h$ , and  $P_0$  are positive constants.

Suppose  $a = 5$ ,  $b = 2$ , and  $h = 3$ .

(a) Since the DE is autonomous, use the phase portrait concept to sketch representative solution curves corresponding to the cases

$$P_0 > \frac{3}{2}, \quad 1 < P_0 < \frac{3}{2}, \quad \text{and} \quad 0 < P_0 < 1.$$

Determine the long-term behavior of the population in each case.

(b) Solve the differential equation initial-value problem. Then verify the results of your phase portrait in part (a) by graphing the solution with an initial condition taken from each of the three intervals given.

(c) Use the information in parts (a) and (b) to determine whether the fishery population becomes extinct in finite time. If so, find that time.