DiffEq - Ch 1-2 - Extra Practice

1.1

State the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear.

#1b. $x\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

#2b. $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$

#3b. $\frac{d^2R}{dt^2} = -\frac{k}{R^2}$

#4b. $\ddot{x} - \left(1 - \frac{\dot{x}^2}{3}\right)\dot{x} + x = 0$

#5b. Determine whether
$udv + \left(v + uv - ue^u\right)du = 0$

(i) is linear in v?

(ii) is linear is u?

#6b. Verify that $y = e^{3x}\cos(2x)$ is an explicit solution of $y'' - 6y' + 13y = 0$
(don't worry about interval of definition).

#7b. Verify that $y = \dfrac{1}{4 - x^2}$ is an explicit solution of $y' = 2xy^2$

Then give at least one interval *I* of definition for this solutions.

#8b. Verify that $-2x^2 y + y^2 = 1$ is an implicit solution of $2xy\, dx + \left(x^2 - y\right) dy = 0$

Then find at least one explicit solution $y = \phi\left(x\right)$, and graph this solution. Finally, give an interval *I* of definition for your solution.

#9b. Find values of m so that the function $y = x^m$ is a solution of $x^2 y'' - 7xy' + 15y = 0$

#10b. Verify that the pair of functions…

$$x = \cos 2t + \sin 2t + \frac{1}{5}e^t$$

$$y = -\cos 2t - \sin 2t - \frac{1}{5}e^t$$

is a solution of the system of differential equations…

$$\begin{cases} \dfrac{d^2x}{dt^2} = 4y + e^t \\ \dfrac{d^2y}{dt^2} = 4x - e^t \end{cases}$$

1.2

#1b. $y = \dfrac{1}{1+C_1 e^{-x}}$ is a one-parameter family of solutions of the first-order DE $y' = y - y^2$. Find a solution of the first-order Initial Value Problem given initial condition $y(-1) = 2$.

#2b. $y = \dfrac{1}{x^2 + c}$ is a one-parameter family of solutions of the first-order DE $y' + 2xy^2 = 0$. Find a solution of the first-order Initial Value Problem given initial condition $y(0) = 1$. Also, give the largest interval *I* over which the solution is defined.

#3b. $x = C_1 \cos t + C_2 \sin t$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$. Find a solution of the second-order Initial Value Problem given initial conditions $x\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}, \quad x'\left(\dfrac{\pi}{6}\right) = 0$.

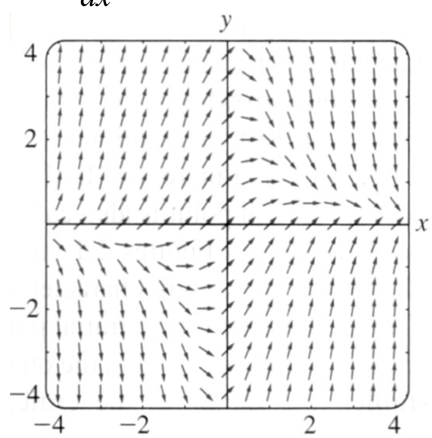
#4b. $y = C_1e^x + C_2e^{-x}$ is a two-parameter family of solutions of the second-order DE $y'' - y = 0$. Find a solution of the second-order Initial Value Problem given initial conditions $y(-1) = 5, \quad y'(-1) = -5$.

#5b. Determine a region of the *xy*-plane for which the given differential equation would have a unique solution whose graph passes through a point (x_0, y_0) in the region. $x\dfrac{dy}{dx} = y$

#6b. Determine whether the differential equation $y' = \sqrt{y^2 - 9}$ possesses a unique solutions through the point $(2, -3)$.

2.1

#1b. A direction field is given for the differential equation $\frac{dy}{dx} = 1 - xy$:



Without using a calculator, sketch an approximate solution curve on the direction field that passes through each of the indicated points.

(a) $y(0) = 0$ (b) $y(-1) = 0$

(c) $y(2) = 2$ (d) $y(0) = -4$

#2b. For the differential equation $\frac{dy}{dx} = x^2 + y^2$, sketch a few isoclines $(f(x, y) = c)$ for $c = \frac{1}{4}$, $c = 1$, $c = \frac{9}{4}$, $c = 4$. Then, construct a direction field by drawing lineal elements with the appropriate slope to match each isocline. Finally, use this rough direction field to sketch an approximate solution curve with the initial condition $y(0) = 1$.

#3b. For the autonomous, first-order differential equation $\frac{dy}{dx} = y^2(4 - y^2)$, find the critical points and phase portrait. Then, classify each critical point as asymptotically stable, unstable, or semi-stable. Finally, sketch typical solution curves in the regions in the xy-plan determined by the graphs of the equilibrium solutions.

Extra #4b. A population model is given by $\frac{dP}{dt} = kP - h$, where h and k are positive constants. For what initial values $P(0) = P_0$ does this model predict that the population will go extinct?

2.2

#1b. Solve by separation of variables:

$$\frac{dy}{dx} = (x+1)^2$$

#2b. Solve by separation of variables:

$$dy - (y-1)^2\,dx = 0$$

#3b. Solve by separation of variables:

$$\frac{dy}{dx} + 2xy^2 = 0$$

#4b. Solve by separation of variables:

$$e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

#5b. Solve by separation of variables:

$$\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2$$

#6b. Solve by separation of variables:
$\sin 3x \, dx + 2y \cos^3(3x) \, dy = 0$

#7c. Find an explicit solution of the given initial-value problem:

$\sqrt{1-y^2}\, dx - \sqrt{1-x^2}\, dy = 0, \quad y(0) = \frac{\sqrt{3}}{2}$

#7b. Find an explicit solution of the given initial-value problem: $x^2 \frac{dy}{dx} = y - xy, \quad y(-1) = -1$

2.3

#1b. Find the general solutions of the differential equation $\frac{dy}{dx}+y=e^{3x}$. Then, give the largest interval *I* over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#2b. Find the general solutions of the differential equation $x^2y'+x(x+2)y=e^x$. Then, give the largest interval *I* over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#3b. Find the general solutions of the differential equation $\frac{dP}{dt}+2tP=P+4t-2$. Then, give the largest interval *I* over which the general solution is defined. Finally, determine whether there are any transient terms in the general solution.

#4b. Solve the initial-value problem:

$$y\frac{dx}{dy} - x = 2y^2, \quad y(1) = 5.$$

#5b. Solve the initial-value problem:

$$\frac{dT}{dt} = k(T - T_m), \quad T(0) = T_0.$$

$[k, T_m, \textit{ and } T_0 \textit{ are constants}]$

2.4

#1b. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(5x+4y)\,dx+(4x-8y^3)\,dy=0$$

#2b. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(\sin y-y\sin x)\,dx+(\cos x+x\cos y-y)\,dy=0$$

#3b. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$(y\ln y-e^{-xy})\,dx+\left(\frac{1}{y}+x\ln y\right)dy=0$$

#4b. Determine whether the given differential equation is exact. If it is exact, then solve it.

$$\frac{dy}{dx}=\frac{-x^3-y^3}{3xy^2}$$

#5b. Solve the initial-value problem:
$(4y+2t-5)\,dt+(6y+4t-1)\,dy=0, \quad y(-1)=2$

#6b. Solve the given differential equation by first finding an integrating factor, then using it to convert the differential equation to exact form:
$(6xy)\,dx+(4y+9x^2)\,dy=0$

2.5

#1b. Solve the Bernoulli form differential equation by using an appropriate substitution: $\frac{dy}{dx} - y = e^x y^2$

#2b. Solve the Bernoulli form differential equation by using an appropriate substitution:
$x\frac{dy}{dx} - (1+x)y = xy^2$

#3b. Solve the differential equation by using an appropriate substitution: $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$

#4b. Solve the differential equation by using an appropriate substitution: $\frac{dy}{dx} = \frac{1 - x - y}{x + y}$

2.6

#1b. Use Euler's method to obtain a four-decimal approximation of $y(1.5)$ if $y(1)=1.4$ and $y'=(x-y)^2$. Use x-increments of 0.1.

#2. Use Euler's method to obtain a four-decimal approximation of $y(1.5)$ if $y(1)=1$ and $y'=xy^2-\frac{y}{x}$. Use x-increments of 0.05.