Formulas for DiffEq Chapter 8 Test

Homogeneous systems…

Finding eigenvalues: $\left|\vec{A}-\lambda\vec{I}\right|=0$

Finding eigenvector for an eigenvalue: $\left(\vec{A}-\lambda\vec{I}\right)\vec{K}=\vec{0}$

Distinct real eigenvalues: $\vec{X}_C=C_1\begin{bmatrix}k_{11}\\k_{21}\end{bmatrix}e^{\lambda_1 t}+C_2\begin{bmatrix}k_{12}\\k_{22}\end{bmatrix}e^{\lambda_2 t}$

Repeated real eigenvalues: find 2nd eigenvalue using $\left(\vec{A}-\lambda\vec{I}\right)\vec{P}=\vec{K}$

$$\vec{X}_C=C_1\begin{bmatrix}k_1\\k_2\end{bmatrix}e^{\lambda t}+C_2\left(\begin{bmatrix}k_1\\k_2\end{bmatrix}te^{\lambda t}+\begin{bmatrix}p_1\\p_2\end{bmatrix}e^{\lambda t}\right)$$

Complex conjugate eigenvalues:

use positive version $\lambda=\alpha+\beta i$ to find eigenvector $\vec{K}=\begin{bmatrix}a+bi\\c+di\end{bmatrix}$

$$\vec{B}_1=\operatorname{Re}\vec{K}=\begin{bmatrix}a\\c\end{bmatrix}\qquad \vec{B}_2=\operatorname{Im}\vec{K}=\begin{bmatrix}b\\d\end{bmatrix}$$

$$\vec{X}_C=C_1\left(\vec{B}_1\cos\beta t-\vec{B}_2\sin\beta t\right)e^{\alpha t}+C_2\left(\vec{B}_2\cos\beta t+\vec{B}_1\sin\beta t\right)e^{\alpha t}$$

Non-Homogeneous systems…

$$\vec{X}'=\vec{A}\vec{X}+\vec{F}$$

<u>Method of Variation of Parameters</u>: $\vec{\Phi}$ = fundamental matrix (from $\vec{X}_C$) $\quad \vec{X}_P=\vec{\Phi}\int\vec{\Phi}^{-1}\vec{F}\,dt$

$$\vec{\Phi}^{-1}=\frac{1}{\det\vec{\Phi}}\vec{\Phi}^{T}$$

$$\vec{\Phi}^{T}=transpose=for\ 2x2\ reverse\ elements\ on\ diagonal,\ negate\ everything\ else$$

<u>Solving with initial condition</u>: $\vec{X}=\vec{\Phi}(t)\vec{\Phi}^{-1}(t_0)\vec{X}_0+\vec{\Phi}\int_{t_0}^{t}\vec{\Phi}^{-1}(s)\vec{F}(s)ds$

Method of Undetermined Coefficients:

$$\vec{X}_p = \begin{bmatrix} \textit{function from table to match forms of terms of } F \\ \textit{function from table to match forms of terms of } F \end{bmatrix}$$

(must be the same form for all rows – so selected terms must cover all terms in all rows of F)

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

Then take derivative, plug into system of DEs, and solve for constants.

(Note: you need to multiply by extra *t*s if terms match any terms in X_C)