

Formulas/Tables for DiffEq Chapter 7 Test

Derivatives...

$$\mathcal{L}\{0\} = 0$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{y'(t)\} = sY(s) - y(0)$$

$$\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

$$\mathcal{L}\{y'''(t)\} = s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at} f(t)$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as} F(s)$$

$$\mathcal{L}^{-1}\{e^{-as} F(s)\} = f(t-a)u(t-a)$$

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{g(t)u(t-a)\} = e^{-as} \mathcal{L}\{g(t+a)\}$$

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. t	$\frac{1}{s^2}$
3. t^n	$\frac{n!}{s^{n+1}}$, n a positive integer
4. $t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$
5. $t^{1/2}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
6. t^α	$\frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}$, $\alpha > -1$
7. $\sin kt$	$\frac{k}{s^2 + k^2}$
8. $\cos kt$	$\frac{s}{s^2 + k^2}$
9. $\sin^2 kt$	$\frac{2k^2}{s(s^2 + 4k^2)}$
10. $\cos^2 kt$	$\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
11. e^{at}	$\frac{1}{s - a}$
12. $\sinh kt$	$\frac{k}{s^2 - k^2}$
13. $\cosh kt$	$\frac{s}{s^2 - k^2}$
14. $\sinh^2 kt$	$\frac{2k^2}{s(s^2 - 4k^2)}$
15. $\cosh^2 kt$	$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
16. te^{at}	$\frac{1}{(s - a)^2}$
17. $t^n e^{at}$	$\frac{n!}{(s - a)^{n+1}}$, n a positive integer
18. $e^{at} \sin kt$	$\frac{k}{(s - a)^2 + k^2}$
19. $e^{at} \cos kt$	$\frac{s - a}{(s - a)^2 + k^2}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
20. $e^{at} \sinh kt$	$\frac{k}{(s-a)^2 - k^2}$
21. $e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2 - k^2}$
22. $t \sin kt$	$\frac{2ks}{(s^2 + k^2)^2}$
23. $t \cos kt$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$
24. $\sin kt + kt \cos kt$	$\frac{2ks^2}{(s^2 + k^2)^2}$
25. $\sin kt - kt \cos kt$	$\frac{2k^3}{(s^2 + k^2)^2}$
26. $t \sinh kt$	$\frac{2ks}{(s^2 - k^2)^2}$
27. $t \cosh kt$	$\frac{s^2 + k^2}{(s^2 - k^2)^2}$
28. $\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s-a)(s-b)}$
29. $\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s-a)(s-b)}$
30. $1 - \cos kt$	$\frac{k^2}{s(s^2 + k^2)}$
31. $kt - \sin kt$	$\frac{k^3}{s^2(s^2 + k^2)}$
32. $\frac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$
33. $\frac{\cos bt - \cos at}{a^2 - b^2}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
34. $\sin kt \sinh kt$	$\frac{2k^2s}{s^4 + 4k^4}$
35. $\sin kt \cosh kt$	$\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
36. $\cos kt \sinh kt$	$\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
37. $\cos kt \cosh kt$	$\frac{s^3}{s^4 + 4k^4}$
38. $J_0(kt)$	$\frac{1}{\sqrt{s^2 + k^2}}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
39. $\frac{e^{bt} - e^{at}}{t}$	$\ln \frac{s-a}{s-b}$
40. $\frac{2(1 - \cos kt)}{t}$	$\ln \frac{s^2 + k^2}{s^2}$
41. $\frac{2(1 - \cosh kt)}{t}$	$\ln \frac{s^2 - k^2}{s^2}$
42. $\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$
43. $\frac{\sin at \cos bt}{t}$	$\frac{1}{2} \arctan \frac{a+b}{s} + \frac{1}{2} \arctan \frac{a-b}{s}$
44. $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$
45. $\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
46. $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
47. $2\sqrt{\frac{t}{\pi}} e^{-a^2/4t} - a \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s\sqrt{s}}$
48. $e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$
49. $-e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{be^{-a\sqrt{s}}}{s(\sqrt{s} + b)}$
50. $e^{at} f(t)$	$F(s-a)$
51. $\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
52. $f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$
53. $g(t)\mathcal{U}(t-a)$	$e^{-as}\mathcal{L}\{g(t+a)\}$
54. $f^{(n)}(t)$	$s^n F(s) - s^{(n-1)}f(0) - \dots - f^{(n-1)}(0)$
55. $t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
56. $\int_0^t f(\tau)g(t-\tau) d\tau$	$F(s)G(s)$
57. $\delta(t)$	1
58. $\delta(t-t_0)$	e^{-st_0}