Differential Equations: Formulas for Ch5 Test

Spring – Mass Systems

$F_{spring} = ks \qquad k = spring\ constant, \quad s = displacement\ of\ object\ under\ own\ weight$

$F_{weight} = mg \qquad m = \dfrac{F_{weight}}{g} \qquad g = 32\ ft/s^2,\ 9.8\ m/s^2\ \ (use\ these\ values) \qquad so \quad mg = ks$

$mx'' + \beta x' + kx = f(t) \qquad m = mass, \quad \beta = damping\ constant,$

$k = spring\ constant, \quad f(t) = forcing\ \ function$

$$x'' + \frac{\beta}{m}x' + \frac{k}{m}x = \frac{f(t)}{m}$$

$$x'' + 2\lambda x' + \omega^2 x = \frac{f(t)}{m} \qquad 2\lambda = \frac{\beta}{m} \qquad \omega^2 = \frac{k}{m} \qquad \omega = 2\pi f \qquad period\ T = \frac{1}{f}$$

$\lambda^2 - \omega^2 > 0$: Distinct Real Roots (*overdamped*): $x(t) = C_1 e^{\left(-\lambda + \sqrt{\lambda^2 - \omega^2}\right)t} + C_2 e^{\left(-\lambda - \sqrt{\lambda^2 - \omega^2}\right)t}$

$\lambda^2 - \omega^2 = 0$: One Repeated Root (*critically damped*): $x(t) = C_1 e^{-\lambda t} + C_2 t e^{-\lambda t}$

$\lambda^2 - \omega^2 < 0$: Complex Conjugate Roots (*underdamped*): $x(t) = C_1 e^{-\lambda t}\cos\left(\sqrt{\omega^2 - \lambda^2}\,t\right) + C_x e^{-\lambda t}\sin\left(\sqrt{\omega^2 - \lambda^2}\,t\right)$

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

DON'T FORGET TO CHECK FOR ABSORPTION

Series LRC Electrical Circuits

$$Lq'' + Rq' + \frac{1}{C}q = E(t)$$

$L = inductance\ (in\ Henrys)$

$q = charge\ (in\ Coulombs)$

$R = resistance\ (in\ Ohms)$

$C = capacitance\ (in\ Farads)$

$i = \frac{dq}{dt} = current\ (in\ Amperes)$

$E = voltage\ (in\ Volts)$