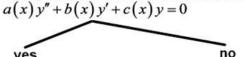
2nd-order linear, have one solution and need another?



Higher-Order DE procedures for solving homogeneous DEs

Reduction of Order

homogeneous with constant coefficients? ay'' + by' + cy = 0

$$y_2(x) = u(x)y_1(x)$$

(take derivatives and substitute w=u', solve resulting 1st-order DE, then integrate to get u).

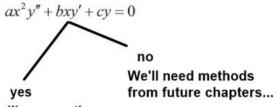
yes use auxiliary equation

$am^2 + bm + c = 0$

single root: $y = C_1 e^{mx}$ repeated roots: $y = C_1 e^{mx} + C_2 x e^{mx} + C_3 x^2 e^{mx}$...

complex pairs: $y = C_1 e^{\alpha x} \cos \beta x + C_2 e^{\alpha x} \sin \beta x$

2nd-order, Cauchy-Euler form?



use auxiliary equation $am^2 + (b-a)m + c = 0$

distinct roots: $y = C_1 x^{m_1} + C_2 x^{m_2}$ repeated roots: $y = C_1 x^m + C_2 x^m \ln x$

complex pairs: $y = C_1 x^{\alpha} \cos(\beta \ln x) + C_2 x^{\alpha} \sin(\beta \ln x)$

Check if solutions are Linearly Independent (form a fundamental solution set) using Wronskian:

$$W(f_1(x), f_2(x)...f_n(x)) = \begin{vmatrix} f_1 & f_2 & ... & f_n \\ f_1' & f_2' & ... & f_n' \\ ... & ... & ... & ... \\ f_1^{(n-1)} & f_2^{(n-1)} & ... & f_n^{(n-1)} \end{vmatrix}$$
 Linearly Independent if Wronskian is non-zero:

Linear non-homogeneous DE with constant coefficients?

$$ay'' + by' + cy = g(x)$$

(can be higher order too)

yes **Undetermined Coefficients** use table to find form ...

| g(x) | Form of y_p |
|----------------------|---|
| 1. 1 (any constant) | Α . |
| 2. $5x + 7$ | Ax + B |
| 3. $3x^2 - 2$ | $Ax^2 + Bx + C$ |
| 4. $x^3 - x + 1$ | $Ax^3 + Bx^2 + Cx + E$ |
| 5. sin 4x | $A \cos 4x + B \sin 4x$ |
| 6. cos 4x | $A\cos 4x + B\sin 4x$ |
| 7. e ^{5x} | Ae ^{5x} |
| 8. $(9x-2)e^{5x}$ | $(Ax + B)e^{5x}$ |
| 9. x^2e^{5x} | $(Ax^2 + Bx + C)e^{5x}$ |
| 10. $e^{3x} \sin 4x$ | $Ae^{3x}\cos 4x + Be^{3x}\sin 4x$ |
| 11. $5x^2 \sin 4x$ | $(Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x$ |
| 12. xe3x cos 4x | $(Ax+B)e^{3x}\cos 4x + (Cx+E)e^{3x}\sin 4x$ |

if a yp term matches a yc term (use smallest *n* that isn't absorbed) = 'absorbed' so multiply term by xn

...take derivatives, plug into DE and solve for constants, then: $y = y_C + y_P$

no Variation of Parameters

assume terms have same form as homogeneous solution multiplied by u functions:

$$y_C = C_1 y_1 + C_2 y_2$$

$$y_P = u_1(x) y_1 + u_2(x) y_2$$
Divide by leading term to get
$$f(x) = \frac{g(x)}{a_n(x)}$$

then find u function derivatives with Cramer's rule:

$$u'_{1} = \frac{\begin{vmatrix} 0 & y_{2} \\ f(x) & y'_{2} \end{vmatrix}}{\begin{vmatrix} y_{1} & y_{2} \\ y'_{1} & y'_{2} \end{vmatrix}}, \quad u'_{2} = \frac{\begin{vmatrix} y_{1} & 0 \\ y'_{1} & f(x) \end{vmatrix}}{\begin{vmatrix} y_{1} & y_{2} \\ y'_{1} & y'_{2} \end{vmatrix}}$$

then integrate to find u functions, and solution:

$$u_{1} = \int u'_{1} dx, \qquad u_{2} = \int u'_{2} dx$$

$$y_{p} = u_{1}y_{1} + u_{2}y_{2}$$

$$y = y_{C} + y_{p}$$