# Differential Equations: Formulas for Ch3 Test

## *Unrestricted Population Growth*

$$\frac{dP}{dt} = kP \quad solution: \ P(t) = P_0 e^{kt}$$

## *Continuously Compounded Interest*

$$\frac{dA}{dt} = kA \quad solution: \ A(t) = Pe^{rt}$$

$(P = Principal, \ r = annual\ interest\ rate)$

## *Radioactive Decay*

$$\frac{dQ}{dt} = kQ \quad solution: \ Q(t) = Q_0 e^{kt}$$

$(halflife = time\ for\ quantity\ to\ be\ cut\ in\ half)$

$\begin{pmatrix} carbon\ dating.... \\ halflife\ of\ C-14 = 5600\ years \end{pmatrix}$

## *Newton's Law of Cooling / Warming*

$$\frac{dT}{dt} = k(T - T_m) \quad solution: \ T(t) = T_m + Ce^{kt}$$

## *Falling Masses*

$(assuming\ positive\ direction\ is\ \underline{\underline{down}})$

$air-resistance\ proportion\ to\ velocity...$

$$m\frac{dv}{dt} = mg - kv$$

$air-resistance\ proportion\ to\ (velocity)^2 ...$

$$m\frac{dv}{dt} = mg - kv^2$$

$m = mass\ (kg\ or\ slugs)$

$g = gravity\ constant\ (9.81\ m/s^2\ or\ 32\ ft/s^2)$

$v = velocity$

## *Growth limited by environment*

$Standard\ logistic\ model:$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right) \qquad Solution: P(t) = \frac{L}{1 + Ce^{-kt}}$$

$L = carrying\ capacity,$

$k\ a\ constant\ unique\ to\ each\ environment$

## *Electrical Circuits*

### *LR series circuit*

$$L\frac{di}{dt} + Ri = E(t) \quad solution: \ i(t) = \frac{E}{R} + Ce^{-\left(\frac{R}{L}\right)t}$$

$L = inductance\ (in\ Henrys)$

$R = resistance\ (in\ Ohms)$

$E = voltage\ (in\ Volts)$

### *RC series circuit*

$$Ri + \frac{1}{C}q = E(t) \qquad i = \frac{dq}{dt}$$

$$R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$

$$solution: \ q(t) = EC + C_1 e^{-\left(\frac{1}{RC}\right)t}$$

$$i(t) = \frac{dq}{dt} = -\left(\frac{1}{RC}\right)C_1 e^{-\left(\frac{1}{RC}\right)t}$$

$C = capacitance\ (in\ Farads)$

$R = resistance\ (in\ Ohms)$

$E = voltage\ (in\ Volts)$

$[C_1\ is\ the\ integration\ constant]$