Review of AP Calc BC topics needed for Honors Calc3/DiffEq…

Self-assessment quiz – try these problems and check answers. If you need to review use the review videos in Schoology 'announcement' area below the weekly schedule.

Finding Derivatives

#1. $\frac{d}{dx}\left[x^7\right] = \boxed{7x^6}$

(shortcut)

#2. $\frac{d}{dx}\left[e^x\right] = \boxed{e^x}$

(shortcut)

#3. $\frac{d}{dx}[\sin x] = \boxed{\cos x}$

(shortcut)

#4. $\frac{d}{dx}[\tan x] = \boxed{\sec^2 x}$

(shortcut)

#5. $\frac{d}{dx}\left[\cos\left(x^3\right)\right] = \boxed{-\sin(x^3)\cdot 3x^2}$

(Chain Rule)

#6. $\frac{d}{dx}\left[e^{x^4}\right] = \boxed{e^{(x^4)}\cdot 4x^3}$

(Chain Rule

#7. $\frac{d}{dx}\left[\underline{x^2}\,\underline{\ln x}\right] =$

$x^2\frac{d}{dx}[\ln x] + \ln x\frac{d}{dx}[x^2]$

$x^2\cdot\frac{1}{x} + \ln x(2x)$

$\boxed{x + 2x\ln x}$

or

$\boxed{x(1+2\ln x)}$

(Product Rule)

#8. $\frac{d}{dx}\left[\frac{\sec x}{x^3}\right] =$

$\frac{x^3\frac{d}{dx}[\sec x] - \sec x\frac{d}{dx}[x^3]}{[x^3]^2}$

$\frac{x^3\sec x\tan x - \sec x(3x^2)}{x^6}$

$\frac{x^2\sec x[x\tan x - 3]}{x^6}$

(Quotient Rule)

$\boxed{\frac{\sec x[x\tan x - 3]}{x^4}}$

Evaluating Integrals

#9. $\int \sin x \, dx = \boxed{-\cos x + C}$

(shortcut)

#10. $\int x^3 \, dx = \boxed{\frac{x^4}{4} + c}$

(shortcut)

#11. $\int e^x \, dx = \boxed{e^x + c}$

(shortcut)

#12. $\int \frac{1}{1+x^2} \, dx = \boxed{\arctan(x) + C}$

(shortcut)

#13. $\int \frac{1}{x} \, dx = \boxed{\ln|x| + c}$

(shortcut)

#14. $\int \frac{1}{2x-1} \, dx =$

$u = 2x-1$

$\frac{du}{dx} = 2$

$du = 2dx$

$dx = \frac{1}{2}du$

$\int \frac{1}{u}\left(\frac{1}{2}du\right)$

$\frac{1}{2}\int \frac{1}{u} du$

$\frac{1}{2}\ln|u| + c$

$\boxed{\frac{1}{2}\ln|2x-1| + C}$

(u-substitution)

#15. $\int 3x^2 e^{x^3} \, dx =$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$\int e^u du$

$e^u + c$

$\boxed{e^{x^3} + c}$

(u-substitution)

#16. $\int x\sqrt{3x^2-4} \, dx =$

$u = 3x^2 - 4$

$\frac{du}{dx} = 6x$

$du = 6x dx$

$x dx = \frac{1}{6}du$

$\int \sqrt{u}\left(\frac{1}{6}du\right)$

$\frac{1}{6}\int u^{1/2} du$

$\frac{1}{6} \frac{u^{3/2}}{(3/2)} + c$

$\frac{1}{6} \cdot \frac{2}{3} u^{3/2} + c$

$\boxed{\frac{1}{9}(3x^2-4)^{3/2} + c}$

(u-substitution)

#17. $\int x\ln(x)\,dx =$ $u = \ln x$ $dv = x\,dx$

$\frac{du}{dx} = \frac{1}{x}$ $\int dv = \int x\,dx$

$uv - \int v\,du$ $du = \frac{1}{x}dx$ $v = \frac{1}{2}x^2$

$(\ln x)(\frac{1}{2}x^2) - \int(\frac{1}{2}x^2)\frac{1}{x}dx$

$\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$

$\frac{1}{2}x^2\ln x - \frac{1}{2}(\frac{1}{2}x^2) + C$

$\boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}$

(integration by parts)

#18. $\int x\cos(x)\,dx =$ $u = x$ $dv = \cos x\,dx$

$\frac{du}{dx} = 1$ $\int dv = \int \cos x\,dx$

$du = dx$ $v = \sin x$

$uv - \int v\,du$

$(x)(\sin x) - \int(\sin x)dx$

$x\sin x - (-\cos x) + C$

$\boxed{x\sin x + \cos x + C}$

(integration by parts)

#19. $\int \frac{1}{x^2+3x-10}dx =$ try factoring denominator: $x^2+3x-10$

$(x-2)(x+5)$

m	A
-10	3
$(-2)(5)$	3 ✓

$\int \frac{1}{(x-2)(x+5)}dx$ (partial fraction expansion)

$\frac{1}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5}$

$\frac{1}{(x-2)(x+5)} = \frac{A(x+5)}{(x-2)(x+5)} + \frac{B(x-2)}{(x-2)(x+5)}$ (now, just numerators)

$A(x+5) + B(x-2) = 1$

$Ax + 5A + Bx - 2B = 1$

$(A+B)x + (5A-2B) = (0)x + (1)$

a system... $\begin{cases} A+B=0 \\ 5A-2B=1 \end{cases}$ $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 5 & -2 & 1 \end{array}\right]$

rref

$\left[\begin{array}{cc|c} 1 & 0 & 1/7 \\ 0 & 1 & -1/7 \end{array}\right]$ $\begin{array}{l} =A \\ =B \end{array}$

$\frac{1}{7}\int \frac{1}{x-2}dx - \frac{1}{7}\int \frac{1}{x+5}dx$

(u-substitutions)

$u = x-2$ $u = x+5$

$du = dx$ $du = dx$

$\frac{1}{7}\int \frac{1}{u}dx - \frac{1}{7}\int \frac{1}{u}dx$

$\boxed{\frac{1}{7}\ln|x-2| - \frac{1}{7}\ln|x+5| + C}$

or

$\frac{1}{7}[\ln|x-2| - \ln|x+5|] + C$

$\boxed{\frac{1}{7}\ln\left|\frac{x-2}{x+5}\right| + C}$

(if denominator is factorable, factor, then partial fractions)

#20. $\int \frac{1}{x^2+4x+11}\,dx =$

if denominator doesn't factor, complete the square..

$x^2+4x+4+11-4$

$(x+2)^2+7$

$\int \frac{1}{(x+2)^2+7}\,dx$

try for arctan form: $\int \frac{1}{x^2+1}\,dx = \arctan x + C$

u-substitution: $u = x+2$

$du = dx$

$\int \frac{1}{u^2+7}\,du$

$\arctan(u)+C$

$\boxed{\arctan(x+2)+C}$

#21. $\int_3^4 \frac{1}{x-2}\,dx =$

(u-substitution) $u = x-2$

$du = dx$

$\int \frac{1}{x-2}\,dx = \int \frac{1}{u}\,du$

$\ln|u| = \ln|x-2|$

(now, Fund. Theorem of Calc.)

$\left[\ln|x-2|\right]_3^4$

$\left(\ln|4-2|\right)-\left[\ln|3-2|\right]$

$\ln(2)-\cancel{\ln(1)}^{0}$

$\boxed{\ln(2)}$

#22. $\int_1^4 e^{2x}\,dx =$

(u-substitution) $u = 2x$

$\frac{du}{dx} = 2$, $du = 2dx$

$dx = \frac{1}{2}du$

$\int e^{2x}dx = \int e^u \frac{1}{2}du$

$\frac{1}{2}e^u = \frac{1}{2}e^{2x}$

(now, Fund. Theorem of Calc.)

$\left[\frac{1}{2}e^{2x}\right]_1^4$

$\frac{1}{2}e^{2(4)} - \frac{1}{2}e^{2(1)}$

$\boxed{\begin{array}{c}\frac{1}{2}e^8 - \frac{1}{2}e^2 \\ \text{or} \\ \frac{1}{2}(e^8-e^2)\end{array}}$