Calc III - Ch 16 - Required Practice **Name: _____**

Ch16 Test Review (for test day 1)

#1. Find $div\ \vec{F}$ and $curl\ \vec{F}$

(a) $\vec{F}(x,y,z) = \langle x^2yz,\ \ xy^2z,\ \ xyz^2 \rangle$

(b) $\vec{F}(x,y,z) = \langle 1,\ \ x+yz,\ \ xy-\sqrt{z} \rangle$

#2. Determine whether or not the field is conservative. If it is, find the potential function such that $\nabla f = \vec{F}$.

(a) $\vec{F}(x, y) = \langle e^x \cos y, e^x \sin y \rangle$

(b) $\vec{F}(x, y) = \langle e^x \sin y, e^x \cos y \rangle$

(c) $\vec{F}(x, y) = \langle 3x^2 + 2y^2, \quad 4xy + 3 \rangle$

#3. Set up the integral to find the surface area (do not evaluate the integral).

(a) the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

(b) the part of the plane with vector equation $\vec{r}(u,v) = \langle 1+v,\ u-2v,\ 3-5u+v \rangle$ that is given by $0 \le u \le 1,\quad 0 \le v \le 1.$

#3 (continued). Set up the integral to find the surface area (do not evaluate the integral).

(c) the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

(d) the part of the plane $z = 4 - y$ that lies inside the cylinder $x^2 + y^2 = 4$.

#4. Set up (but do not evaluate) the line integral.

(a) $\int_C y^3 \, ds$ if C is defined by:

$x = t^3, \; y = t \;\; for \;\; 0 \le t \le 2$.

(b) $\int_C xy \, ds$ if C is defined by:

$x = t^2, \; y = 2t \;\; for \;\; 0 \le t \le 1$.

#5. Set up (**and evaluate**) the line integral. (hint: check to see if the field is conservative)

(a) $\int_C \vec{F} \bullet \vec{dr}$ for $\vec{F} = \langle x^2,\ y^2 \rangle$ along curve C

defined by the arc of the parabola $y = 2x^2$ from $(-1,2)$ to $(2,8)$.

(b) $\int_C \vec{F} \bullet \vec{dr}$ for $\vec{F} = \langle xy^2,\ x^2 y \rangle$ along curve C

defined by $\vec{r}(t) = \left\langle t + \sin\left(\frac{1}{2}\pi t\right),\quad t + \cos\left(\frac{1}{2}\pi t\right) \right\rangle$

$0 \le t \le 1$

#6. Set up (but do not evaluate) the line integral.
(hint: use Green's Theorem)

(a) $\int_C (xy)\,dx + (x^2y^3)\,dy$ along C which traces the path around a triangle from $(0,0)$ to $(1,0)$ to $(1,2)$ and back to $(0,0)$.

(b) $\int_C (xy^2)\,dx + (2x^2y)\,dy$ along C which traces the path around a triangle from $(0,0)$ to $(2,2)$ to $(2,4)$ and back to $(0,0)$.

#7. Find the equation of the tangent plane to the parametric surface.

(a) $x = u + v,\ y = 3u^2,\ z = u - v$ at $(2,3,0)$.

(b) $x = u^2,\ y = v^2,\ z = uv$ at $u = 1,\ v = 1$.

#8. Set up (but do not evaluate) the surface integral.

(a) $\iint_S (z)\, dS$ where S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

(b) $\iint_S (xz)\, dS$ where S is the part of the paraboloid $z = 16 - x^2 - y^2$ that lies above the plane $z = 7$.

Ch16 Test Review (for test day 2)

#9. Set up (but do not evaluate) the integral to evaluate the flux of $\vec{F}$ over S

(a) $\vec{F}(x,y,z) = \langle xy,\ \ yz,\ \ zx \rangle$

S is the part of the paraboloid

$z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1,\ 0 \le y \le 1$ and has upward orientation.

(b) $\vec{F}(x,y,z) = \langle x^2,\ \ xy,\ \ z \rangle$

S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$ with upward orientation.

#10. Using the Divergence Theorem, write the **triple-integral** (but do not evaluate) which is equivalent to the surface integral $\iint_S \vec{F} \bullet d\vec{S}$ which calculates the flux of $\vec{F}$ across *S*.

(a) $\vec{F} = \langle x^2, xy, z \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$.

(b) $\vec{F} = \langle x^2, -x^2yz, 2xy^2 \rangle$ and S is the cylinder $x^2 + y^2 = 4$ between the planes $z = x + 4$ and $z = 0$.

#11. Using Stokes' Theorem, write the **single-integral** (but do not evaluate) which calculates

$$\iint_S \left(curl\, \vec{F} \right) \bullet d\, \vec{S}$$

(a) $\vec{F} = \left\langle x^2 z^2,\ \ y^2 z^2,\ \ xyz \right\rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that inside the cylinder $x^2 + y^2 = 4$ and is oriented upward.

(b) $\vec{F} = \left\langle 2x^2,\ \ -2y^2,\ \ x^2 z \right\rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 13$ that lies above the plane $z = 3$ and is oriented upward.

#12. Using the Divergence Theorem, write (but do not evaluate) the **triple-integral** which is equivalent to the surface integral $\iint_S \vec{F} \bullet \vec{dS}$ which calculates the flux of $\vec{F}$ across S if $\vec{F} = \langle 3x^2, \ xyz, \ z^3 \rangle$ and S is the surface of a sphere of radius 9 centered at the origin.

#13. Using Stokes' Theorem, write the **double-integral** (but do not evaluate) which is equivalent to the line integral $\int_C \vec{F} \bullet d\vec{r}$ which sums the contributions of the given field along the given path.

Hints: You can assume the path is oriented such that the normal vector to the enclosed surface is in the 'positive' direction.

Remember: you can choose any surface that is bounded by the curve of the given surface – you don't have to use the original surface.

(a) $\vec{F} = \langle x^2 z^2,\ x^2,\ xyz \rangle$ along the path which is the intersection of the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$.

(b) $\vec{F} = \langle x + y^2,\ y + z^2,\ z + x^2 \rangle$ along the path which is the intersection of the sphere $x^2 + y^2 + z^2 = 13$ and the plane $z = 3$.