Calc III - Ch 16 - Required Practice **Name: ______________________________**

16.1 and 16.2 day 1

#1. Sketch the vector field for $\vec{F}(x,y)=\langle 1,\ x\rangle$

#2. Evaluate the line integral, where C is the given curve: $\int_C y^3\,ds \quad C: x=t^3,\ y=t,\ 0\le t\le 2$

#3. Evaluate the line integral, where C is the given curve:

$\int_C xy^4\,ds \quad C \text{ is the right half of circle } x^2+y^2=16$

#4. Evaluate the line integral, where C is the given curve: $\int_C xy\,dx + (x-y)\,dy$ where C consists of line segments from $(0,0)$ to $(2,0)$ and from $(2,0)$ to $(3,2)$.

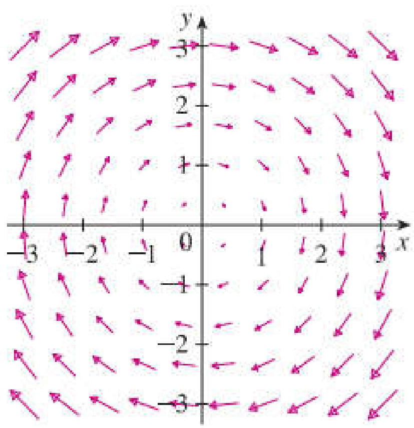
#5. Evaluate the line integral, where C is the given curve: $\int_C xe^{yz}\,ds$ where C is the line segment from $(0,0,0)$ to $(1,2,3)$.

16.2 day 2

#1. Let $\vec{F}$ be the vector field shown in the figure.

(i) If C_1 is the vertical line segment from $(-3,\ -3)$ to $(-3,\ 3)$, determine whether $\int_{C_1} \vec{F} \bullet d\vec{r}$ is positive, negative, or zero.

(ii) If C_2 is the counterclockwise-oriented circle with radius 3 and center at the origin, determine whether $\int_{C_2} \vec{F} \bullet d\vec{r}$ is positive, negative, or zero.



#2. Evaluate the line integral $\int_{C_1} \vec{F} \bullet d\vec{r}$ where C is given by the vector function $\vec{r}(t)$

$\vec{F}(x, y) = \langle xy,\ 3y^2 \rangle$

$\vec{r}(t) = \langle 11t^4,\ t^3 \rangle \quad 0 \le t \le 1$

#3. Find the work done by the force field $\vec{F}(x, y) = \langle x \sin y,\ y \rangle$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

#4. Show that a constant force field does zero work on a particle that moves once uniformly around the circle $x^2 + y^2 = 1$.

16.3

#1. Determine whether or not $\vec{F}$ is a conservative vector field. If it is, find a function *f* such that $\vec{F} = \nabla f$.

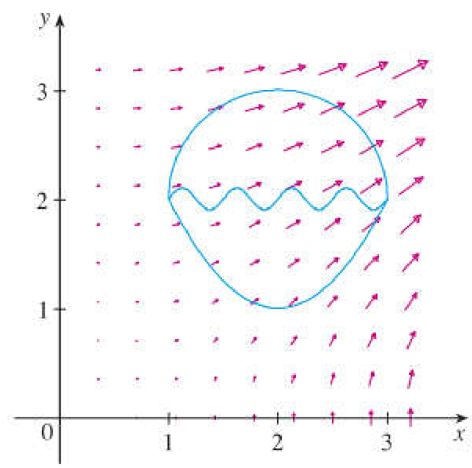
(i) $\vec{F}(x, y) = \langle 2x - 3y,\ -3x + 4y - 8 \rangle$

(ii) $\vec{F}(x, y) = \langle ye^x + \sin y,\ e^x + x\cos y \rangle$

#2. The figure shows the vector field $\vec{F}(x, y) = \langle 2xy,\ x^2 \rangle$ and three curves that start at (1,2) and end at (3,2).

(i) Explain why $\int_C \vec{F} \bullet d\vec{r}$ has the same value for all three curves.

(ii) What is this common value?



#3. Find a function *f* such that $\vec{F} = \nabla f$ and use it to evaluate $\int_C \vec{F} \bullet d\vec{r}$ along the given curve *C*.

$\vec{F}(x, y) = \langle xy^2,\ x^2 y \rangle$

$C:\ \vec{r}(t) = \left\langle t + \sin\left(\frac{\pi}{2}t\right),\ t + \cos\left(\frac{\pi}{2}t\right) \right\rangle \quad 0 \le t \le 1$

#4. Show that the line integral is independent of path and evaluate the integral.

$\int_C \tan y\, dx + x \sec^2 y\, dy$

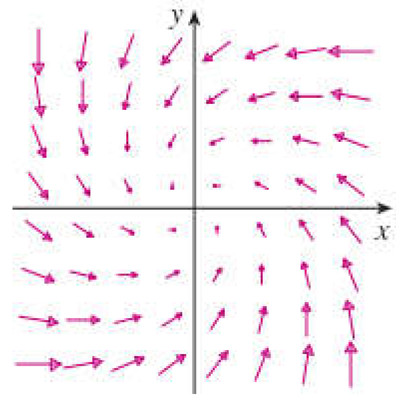
C is any path from $(1,0)$ to $\left(2, \frac{\pi}{4}\right)$

#5. Find the work done by the force field $\vec{F}$ in moving an object from P to Q.

$\vec{F}(x,y)=\left\langle 2y^{3/2},\ 3x\sqrt{y}\right\rangle$

$P(1,1),\quad Q(2,4)$

#6. Is the vector field shown in the figure conservative? Explain.



16.4

#1. Evaluate the line integral (i) directly and (ii) using Green's Theorem.

$$\oint_C (x-y)\,dx + (x+y)\,dy$$

C is the circle with center at the origin, radius 2.

#2. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\oint_C xy^2\,dx + 2x^2y\,dy$$

C is the triangle with vertices (0,0), (2,2) and (2,4).

#3. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\oint_C y^3 dx - x^3\, dy$$

C is the circle $x^2 + y^2 = 4$.

#4. Use Green's Theorem to evaluate $\int_C \vec{F} \bullet d\,\vec{r}$.

(Check the orientation of the curve before applying the theorem)

$$\vec{F}(x, y) = \left\langle \sqrt{x} + y^3,\ \ x^2 + \sqrt{y} \right\rangle$$

C consists of the arc of the curve $y = \sin x$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$.

#5. A particle starts at the point $\left(-2,0\right)$, moves along the *x*-axis to $\left(2,0\right)$, and then along the semicircle $y=\sqrt{4-x^2}$ to the starting point. Use Green's theorem to find the work done on this particle by the force field $\vec{F}\left(x,y\right)=\left\langle x,\ \ x^3+3xy^2\right\rangle$.

16.5

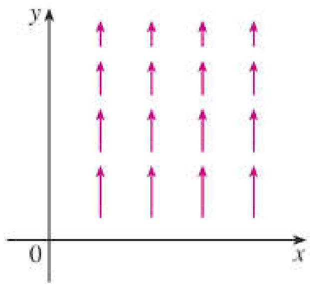
#1. Find (i) the curl and (ii) the divergence of the vector field $\vec{F}(x,y,z)=\langle xyz,\ 0,\ -x^2y\rangle$

#2. Find (i) the curl and (ii) the divergence of the vector field $\vec{F}(x,y,z)=\langle \ln x,\ \ln(xy),\ \ln(xyz)\rangle$

#3. The vector field $\vec{F}$ is shown in the xy-plane and looks the same in all other horizontal planes (its z-component is zero).

(i) Is $div\,\vec{F}$ positive, negative, or zero? Explain.

(ii) Determine whether $curl\,\vec{F} = \vec{0}$. If not, in which direction does $curl\,\vec{F}$ point?



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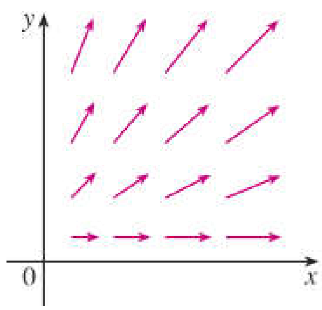

#5. Let f be a scalar field and $\vec{F}$ a vector field. State whether each expression is meaningful. If not, explain why. If so, state whether it is a scalar field or a vector field.

(i) $curl\,\vec{F}$

(ii) $div\,\vec{F}$

(iii) $\nabla\,\vec{F}$

(iv) $div(\nabla f)$

(v) $curl\left(curl\,\vec{F}\right)$

(vi) $(\nabla f)\,x\left(div\,\vec{F}\right)$

16.6 day 1

#1. Determine whether the points P and Q lie on the given surface.

$\vec{r}(u,v)=\langle 2u+3v,\ 1+5u-v,\ 2+u+v\rangle$

$P(7,10,4),\ \ Q(5,22,5)$

#2. Identify the surface with the given vector equation.

$\vec{r}(u,v)=\langle u+v,\ 3-v,\ 1+4u+5v\rangle$

#3. Find a parametric representation for the surface: the plane that passes through the point $(1, 2, -3)$ and contains the vectors $\langle 1, 1, -1 \rangle$ and $\langle 1, -1, 1 \rangle$.

#4. Find a parametric representation for the surface: the lower half of the ellipsoid $2x^2 + 4y^2 + z^2 = 1$

#5. Find a parametric representation for the surface: the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

16.6 day 2

#1. Find an equation of the tangent plane to the given parametric surface at the specified point.

$\vec{r}(u,v) = \langle u+v,\ 3u^2,\ u-v \rangle$

$(2,3,0)$

#2. Find the area of the surface: the part of the plane $3x+2y+z=6$ that lies in the first octant.

#3. Find the area of the surface: the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

#4. Find the area of the surface: the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

16.7 day 1

#1. Evaluate the surface integral $\iint_S x^2 yz \, dS$

S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $0 \le x \le 3, \quad 0 \le y \le 2$.

#2. Evaluate the surface integral $\iint_S yz\, dS$

S is the surface with parametric equations

$x = u^2, \quad y = u\sin v, \quad z = u\cos v$

$0 \le u \le 1, \quad 0 \le v \le \dfrac{\pi}{2}.$

#3. Evaluate the surface integral $\iint_S \left(x^2 z + y^2 z\right) dS$

S is the hemisphere $x^2 + y^2 + z^2 = 4, \quad z \geq 0$.

16.7 day 2

#1. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across S):

$\vec{F}(x, y, z) = \langle xy,\ yz,\ zx \rangle$

S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1,\quad 0 \le y \le 1$, and has upward orientation.

#2. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across *S*):

$\vec{F}(x, y, z) = \langle x, \ -z, \ y \rangle$

S is the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant, with orientation toward the origin.

#3. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across *S*):

$\vec{F}(x, y, z) = \langle 0,\ y,\ -z \rangle$

S consists of the paraboloid $y = x^2 + z^2,\quad 0 \le y \le 1$ and the disk $x^2 + z^2 \le 1,\quad y = 1$ with upward orientation.

#4. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across *S*):

$\vec{F}\,(x, y, z) = \langle x,\ 2y,\ 3z \rangle$

S is the cube with vertices $(\pm 1,\ \pm 1,\ \pm 1)$.

16.8

#1. Using Stokes' Theorem, write out and evaluate the single-integral which is equivalent to the surface integral which calculates $\iint_S \left(curl\, \vec{F} \right) \bullet dS$

where

$\vec{F}\,(x, y, z) = \left\langle x^2 z^2,\ y^2 z^2,\ xyz \right\rangle$

S is the part of the paraboloid $z = x^2 + y^2$ that lies insides the cylinder $x^2 + y^2 = 4$, oriented upward.

#2. Using Stokes' Theorem, write out and evaluate the double-integral which is equivalent to the line integral $\int_C \vec{F} \bullet d\vec{r}$ which sums the contributions of the field $\vec{F}$ along path C

$\vec{F}(x, y, z) = \left\langle x + y^2,\ y + z^2,\ z + x^2 \right\rangle$

C is the triangle with vertices (1,0,0), (0,1,0), and (0,0,1).

#3. Verify that Stokes' Theorem is true for the given vector field $\vec{F}$ and surface S by writing out and evaluating integrals for both sides of the Stokes' Theorem equation.

$\vec{F}(x,y,z) = \langle y^2,\ x,\ z^2 \rangle$

S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward.

Double-integral side….

#3 (continued). Verify that Stokes' Theorem is true for the given vector field $\vec{F}$ and surface S by writing out and evaluating integrals for both sides of the Stokes' Theorem equation.

Single-integral side….

$\vec{F}(x, y, z) = \langle y^2,\ x,\ z^2 \rangle$

S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward.

16.9

#1 Verify that the Divergence Theorem is true for the given vector field $\vec{F}$ on the region E by writing out and evaluating integrals for both sides of the Divergence Theorem equation.

$\vec{F}(x, y, z) = \langle xy,\ yz,\ zx \rangle$

E is the solid cylinder $x^2 + y^2 \le 1,\quad 0 \le z \le 1$.

Double-integral side….

#1(continued) Verify that the Divergence Theorem is true for the given vector field $\vec{F}$ on the region E by writing out and evaluating integrals for both sides of the Divergence Theorem equation.

$\vec{F}(x, y, z) = \langle xy,\ yz,\ zx \rangle$

E is the solid cylinder $x^2 + y^2 \leq 1, \quad 0 \leq z \leq 1$.

<u>Triple-integral side</u>….

#2. Using the Divergence Theorem, write out and evaluate the triple-integral which is equivalent to the surface integral $\iint_S \vec{F} \bullet dS$ which calculates the flux of $\vec{F}$ across S if

$\vec{F}(x, y, z) = \langle e^x \sin y,\ e^x \cos y,\ yz^2 \rangle$

S is the surface of the box bounded by the planes $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,$ and $z = 2$.

#3. Using the Divergence Theorem, write out and evaluate the triple-integral which is equivalent to the surface integral $\iint_S \vec{F} \bullet dS$ which calculates the flux of $\vec{F}$ across S if

$\vec{F}(x,y,z) = \langle \cos z + xy^2,\ xe^{-z},\ \sin y + x^2 z \rangle$

S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

#4. A vector field $\vec{F}$ is shown. Determine whether is positive or negative at P_1 and P_2 .