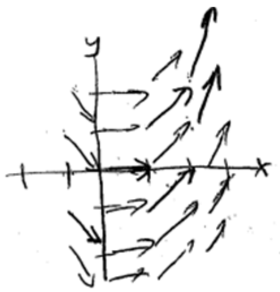
Calc III - Ch 16 - Required Practice **ANSWERS ONLY**

16.1 and 16.2 day 1

#1.



#2. $\frac{1}{54}\left(145\sqrt{145}-1\right)$

#3. $\frac{8192}{5}$

#4. $\frac{17}{3}$

#5. $\frac{\sqrt{14}}{12}\left(e^{6}-1\right)$

16.2 day 2

#1. (i) positive (ii) negative

#2. 45

#3. $\frac{1}{2}\left(15-\cos 4-\cos 1\right)$

#4. 0

16.3

#1. (i) $f\left(x,y\right)=x^{2}-3xy+2y^{2}-8y+C$
(ii) $f\left(x,y\right)=ye^{x}+x\sin y+C$

#2. (i) The field is conservative so line integral value is independent of path.
(ii) 16

#3. 2

16.3 (continued)

#4. 2

#5. 30

#6. This field is not conservative.

16.4

#1. (i) 8π (ii) 8π

#2. 12

#3. -24π

#4. $\frac{4}{3}-2\pi$

#5. 12π

16.5

#1. (i) $\left\langle -x^{2},3xy,-xz\right\rangle$ (ii) yz

#2. (i) $\left\langle \frac{1}{y},-\frac{1}{x},\frac{1}{x}\right\rangle$ (ii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

#3. (i) negative (ii) zero

#4. (i) positive (ii) zero

#5. (i) vector
(ii) scalar
(iii) not meaningful
(iv) scalar
(v) vector
(vi) not meaningful

16.6 day 1

#1. P is not on the surface, Q is on the surface

#2. This is a plane (specifically $-4x+y+z=4$)

#3. $\vec{r}(u,v)=\langle u,\ v,\ -1-v\rangle$

$D=\{(u,v)\mid -\infty<u<\infty,-\infty<v<\infty\}$

#4. $\vec{r}(u,v)=\left\langle u,\ v,\ -\sqrt{1-2u^2-4v^2}\right\rangle$

$D=\{(u,v)\mid 2u^2+4v^2\le 1\}$

#5. $\vec{r}(u,v)=\left\langle u,\ v,\ \sqrt{u^2+v^2}\right\rangle$

can also write using *x,y*:

$\vec{r}(x,y)=\left\langle x,\ y,\ \sqrt{x^2+y^2}\right\rangle$

$D=\{(x,y)\mid x^2+y^2\le 2\}$

--- alternatively, could use spherical coordinates ---

$\vec{r}(\phi,\theta)=\langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi\rangle$

$D=\left\{(\phi,\theta)\mid 0\le\phi\le\frac{\pi}{4}, 0\le\theta\le 2\pi\right\}$

16.6 day 2

#1. $3x-y+3z=3$

#2. $3\sqrt{14}$

#3. $\frac{2\pi}{3}\left(2\sqrt{2}-1\right)$

#4. $\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$

16.7 day 1

#1. $171\sqrt{14}$

#2. $\frac{5}{48}\sqrt{5}+\frac{1}{240}$

#3. 64π

16.7 day 2

#1. $\frac{857}{180}$

#2. $-\frac{4\pi}{3}$

#3. 2

#4. 48

16.8

#1. 0

#2. -1

#3. π

16.9

#1. $\frac{\pi}{2}$

#2. 2

#3. $\frac{32\pi}{3}$

#4. Negative at P_1, Positive at P_2

Ch16 Test Review (for test day 1)

#1.

(a) $6xyz \qquad \left\langle xz^2 - xy^2,\ -\left(yz^2 - x^2y\right),\ y^2z - x^2z \right\rangle$

(b) $z - \dfrac{1}{2\sqrt{z}} \qquad \langle x - y,\ -y,\ 1\rangle$

#2. (a) not conservative

(b) conservative, $f = e^x \sin y$

(c) conservative, $f = x^3 + 2xy^2 + 3y$

#3. (a) $\displaystyle\int_0^2 \int_0^{3-\frac{3}{2}x} (1)\sqrt{1+(-3)^2+(-2)^2}\ dy\, dx$

(b) $\displaystyle\int_0^1 \int_0^1 (1)\sqrt{107}\ du\, dv$

(c) $\displaystyle\int_0^{2\pi} \int_0^2 (1)\sqrt{1+4r^2}\ r\, dr\, d\theta$

(e) $\displaystyle\int_0^{2\pi} \int_0^2 (1)\sqrt{2}\ r\, dr\, d\theta$

#4. (a) $\displaystyle\int_0^2 t^3\sqrt{9t^4+1}\ dt$

(b) $\displaystyle\int_0^1 2t^3\sqrt{4t^2+4}\ dt$

#5. (a) $\dfrac{513}{3}$ (b) 2

#6. (a) $\displaystyle\int_0^1 \int_0^{2x} \left(2xy^3 - x\right) dy\, dx$

(b) $\displaystyle\int_0^2 \int_x^{2x} 2xy\ dy\, dx$

#7. (a) $-6x + 2y - 6z = -6$

(b) $-2x - 2y + 4z = 0$

#8. (a) $\displaystyle\int_0^{2\pi} \int_0^2 r^2\sqrt{1+4r^2}\ r\, dr\, d\theta$

(b) $\displaystyle\int_0^{2\pi} \int_0^3 (r\cos\theta)\left(16 - r^2\right)\sqrt{1+4r^2}\ r\, dr\, d\theta$

Ch16 Test Review (for test day 2)

#9. (a)

$$\int_0^1 \int_0^1 \left(2x^2y + 2y^2\left(4 - x^2 - y^2\right) + 4 - x^2 - y^2\right) dy\, dx$$

(b) $\displaystyle\int_0^{2\pi} \int_0^1 r^2\left(-2r\cos\theta + 1\right) r\, dr\, d\theta$

#10. (a) $\displaystyle\int_0^{2\pi} \int_0^1 \int_{r^2}^1 \left(3r\cos\theta + 1\right) r\, dz\, dr\, d\theta$

(b)

$$\int_0^{2\pi} \int_0^2 \int_0^{r\cos\theta+4} \left(2r\cos\theta - (r\cos\theta)^2 z\right) r\, dz\, dr\, d\theta$$

#11. (a) $\displaystyle\int_0^{2\pi} \left(-128\cos^2 t\sin t + 128\sin^2 t\cos t\right) dt$

(b) $\displaystyle\int_0^{2\pi} \left(-16\cos^2 t\sin t - 16\sin^2 t\cos t\right) dt$

#12.

$$\int_0^{2\pi} \int_0^{\pi} \int_0^3 6(\rho\sin\phi\cos\theta) + (\rho\sin\phi\cos\theta)(\rho\cos\phi) + 3(\rho\cos\phi)^3\ \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

#13. (a) $\displaystyle\int_0^{2\pi} \int_0^2 (2r\cos\theta)\, r\, dr\, d\theta$

(b) $\displaystyle\int_0^{2\pi} \int_0^2 (-2r\sin\theta)\, r\, dr\, d\theta$