Calc III - Ch 15 Part 2 - Required Practice **Name: ______________________________**

15.6

#1. Evaluate the given integral over the specified region using the three specified orders of integration.

$$\iiint_E xyz^2 \, dV$$

$$E = \{(x, y, z) \mid 0 \le x \le 1, \ -1 \le y \le 2, \ 0 \le z \le 3\}$$

(i) …integrating first with respect to y, then z, and then x.

(ii) …integrating first with respect to x, then y, and then z.

(iii) …integrating first with respect to z, then y, and then x.

#2. Evaluate the iterated integral

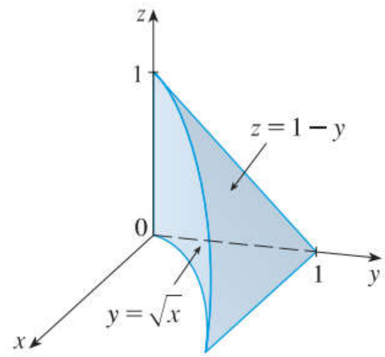
$$\int_0^1 \int_0^z \int_0^y z e^{-y^2} \, dx \, dy \, dz$$

#3. Evaluate the integral $\iiint_E 6xy \, dV$

where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

#4. Set up (but do not evaluate) a triple integral to find the volume of the solid bounded by the cylinder $y = x^2$
and the planes $z = 0$, $z = 4$, and $y = 9$.

#5. The figure shows the region of integration for the integral $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$.



Rewrite this integral (but do not evaluate) as an equivalent iterated integral in the specified integration orders….

(i) $\int_0^? \int_0^? \int_0^? f(x,y,z)\,dx\,dy\,dz$.

(ii) $\int_0^? \int_0^? \int_0^? f(x,y,z)\,dy\,dz\,dx$.

15.7

#1. Plot the point whose cylindrical coordinates are given:
$\left(2, \frac{\pi}{4}, 1\right)$

#3. Sketch the solid described by the inequalities:
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \quad 0 \le r \le 2, \quad 0 \le z \le 1$

#2. Sketch and describe in words the surface whose equation is given: $\theta = \frac{\pi}{4}$

#4. Sketch the solid whose volume is given by the integral: $\int_0^4 \int_0^{2\pi} \int_r^4 r\, dz\, d\theta\, dr$

#5. Evaluate $\iiint_E \left(x^3 + xy^2\right) dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

#6. Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

15.8

#1. Plot the point whose spherical coordinates are given and find the rectangular coordinates of the point:

(i) $(1,0,0)$ (ii) $\left(2,\frac{\pi}{3},\frac{\pi}{4}\right)$

#2. Sketch and describe in words the surface whose equation is given: $\phi=\frac{\pi}{3}$

#3. Identify the surface whose equation is given: $\rho=\sin\theta\sin\phi$

#4. Sketch the solid described by the given inequalities:

$\rho\le 2,\quad 0\le\phi\le\frac{\pi}{2},\quad 0\le\theta\le\frac{\pi}{2}$

#5. Write the equation in spherical coordinates:

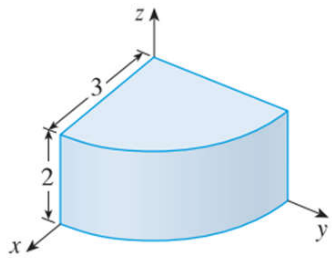
(i) $z^2 = x^2 + y^2$

(ii) $x^2 + z^2 = 9$

#6. Sketch the solid whose volume is given by the integral and evaluate the integral.

$$\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

#7. Set up the triple integral of an arbitrary continuous function *f(x,y,z)* in cylindrical or spherical coordinates over the solid shown.



#8. Evaluate using spherical coordinates $\iiint_B \left(x^2+y^2+z^2\right)^2 \, dV$, where B is the ball with center at the origin and radius 5.

#9. Find the volume of the part of the ball $\rho \le 4$ that lies between the cones $\phi = \dfrac{\pi}{6}$ and $\phi = \dfrac{\pi}{3}$.

#10. Find the volume of the solid that lies above the cone $\phi = \dfrac{\pi}{3}$ and below the sphere $\rho = 4\cos\phi$

#11. Evaluate the integral by changing to spherical coordinates: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,dz\,dy\,dx$

Ch 15 Part 2 Test Review

***NOTE*: On all test review problems, set up the integral including limits of integration but *do not evaluate*.**

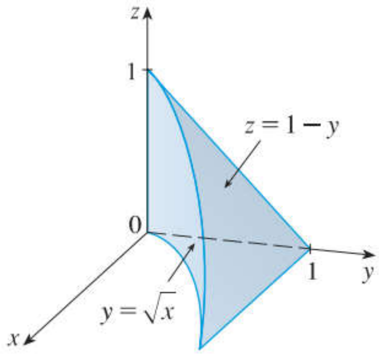
#1. $\iiint_T x^2 \, dV$, where T is the solid tetrahedron with vertices (0,0,0), (1,0,0), (0,1,0), and (0,0,1).

#2. $\iiint_T xyz \, dV$, where T is the solid tetrahedron with vertices (0,0,0), (1,0,0), (1,1,0), and (1,0,1).

#3. Express the integral $\iiint_E f(x, y, z)\, dV$, as an iterated integral in all six integration orderings, where E is the solid bounded by the given surface:

$$y = x^2, \quad z = 0, \quad y + 2z = 4$$

#4. The figure shows the region of integration for the integral $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$.



Rewrite this integral in the other five integration orders.

#5. Write five other iterated integrals that are equivalent to $\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z)\, dz\, dy\, dx$.

#6. Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

#7. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

#8. Find the volume of the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

#9. Evaluate $\iiint_E x^2 \, dV$, where E is bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$, and $y = \sqrt{16 - x^2 - z^2}$.

#10. Evaluate $\iiint_E xyz\, dV$, where E lies between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \frac{\pi}{3}$.

#11. Evaluate the integral by changing to spherical coordinates:

$$\int_0^9 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \left(x^2z + y^2z + z^3\right) dz\, dx\, dy$$

#12. Evaluate $\iiint_E x^2 y^2 \, dV$, where E is bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

#13. Evaluate $\iiint_E yz \, dV$, where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

#14. Find the volume of the solid tetrahedron with vertices (0,0,0), (0,0,1), (0,2,0), and (2,2,0).

#15. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$.

#16. Find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

#17. Convert the integral to spherical coordinates:

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\; dz\, dx\, dy.$$