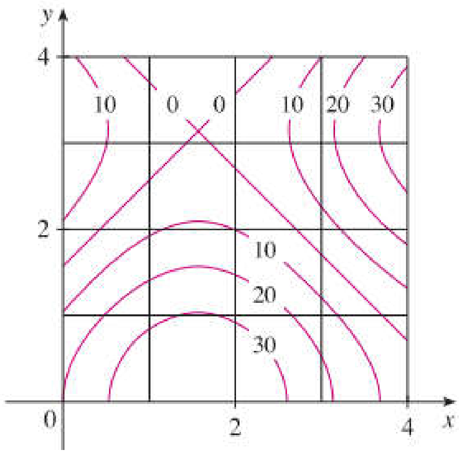
**Calc III - Ch 15 Part 1 - Required Practice** **Name: ______________________________**

**15.1 and 15.2**

#1. A contour map is shown for a function $f$ on the square $R: x=[0,4],\ y=[0,4]$.

(i) Use the Midpoint Rule with $m = n = 2$ to estimate the value of $\iint_R f(x,y)\,dA$.

(ii) Estimate the average value of $f$.




#2. Evaluate the double integral by first identifying it as the volume of a solid.

$\iint_R 3\,dA, \quad R=\{(x,y)\,|\,-2\le x\le 2,\ 1\le y\le 6\}$.

#3. Find $\int_0^5 f(x,y)\,dx$ and $\int_0^1 f(x,y)\,dy$

$f(x,y)=12x^2y^3$.

#4. Evaluate $\int_1^3\int_0^1 (1+4xy)\,dx\,dy$.

#5. Evaluate $\int_0^2 \int_0^{\pi/2} x \sin y \, dy \, dx$.

#6. Evaluate $\int_0^2 \int_0^1 (2x+y)^8 \, dx \, dy$.

#7. Evaluate the double integral:

$\iint_R (6x^2y^3 - 5y^4) \, dA, \quad R = \{(x,y) \mid 0 \le x \le 3, 0 \le y \le 1\}$

#8. Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^1 (4 - x - 2y) \, dx \, dy$

#9. Find the volume of the solid that lies under the plane $3x+2y+z=12$ and above the rectangle $R=\{(x,y)\,|\,0\le x\le 1,\ -2\le y\le 3\}$.

#10 (challenging). Find the volume of the solid that lies between the surface $z=2+x^2+(y-2)^2$ and the planes
$z=1,\ x=1,\ x=-1,\ y=0,$ and $y=4$.

**15.3**

#1. Evaluate $\int_0^1 \int_{2x}^2 (x-y)\, dy\, dx$ .

#2. Evaluate $\iint_D x \cos y \, dA$

$D$ is bounded by $y=0$, $y=x^2$, $x=1$

.

#3. Evaluate $\iint_D (2x-y)\, dA$

$D$ is bounded by the circle with center (0,0) and radius = 2.

#4. Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices (1,1), (4,1), and (1,2).

#5. Find the volume of the solid bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

#6. Find the volume of the solid bounded by the cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$, $y = 4$.

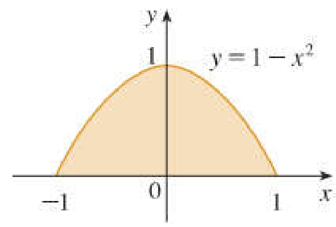
#7. Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^{1-x^2} (1-x)\, dy\, dx$

#8. Sketch the region of integration and change the order of integration (write the new integral but do not evaluate) $\int_0^4 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx$

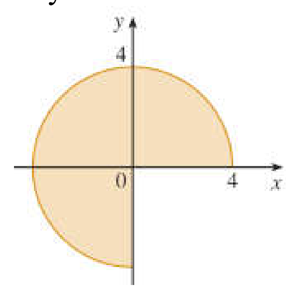
#9. Evaluate the integral by reversing the order of integration $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

**15.4**

#1. A region $R$ is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x,y)\, dA$ as an iterated integral, where $f$ is an arbitrary continuous function on $R$.




#2. A region $R$ is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x,y)\, dA$ as an iterated integral, where $f$ is an arbitrary continuous function on $R$.



#3. Sketch the region whose area is given by the integral and evaluate the integral: $\int_{\pi}^{2\pi} \int_{4}^{7} r \, dr \, d\theta$

#4. Evaluate the given integral by changing to polar coordinates: $\iint_D xy \, dA$ where $D$ is the disk with center (0,0) and radius 3.

#5. Evaluate the given integral by changing to polar coordinates: $\iint_D e^{-x^2-y^2} \, dA$ where $D$ is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the $y$-axis.

#6. Use a double integral to find the area of the region: one loop of the rose $r = \cos 3\theta$.

#7. Use polar coordinates to find the volume of the given solid: under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \le 4$.

#8. Use polar coordinates to find the volume of the given solid: enclosed by the hyperboloid $-x^2 - y^2 + z^2 = 1$ and the plane $z = 2$.

#9. Evaluate the iterated integral by converting to polar coordinates: $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\, dx\, dy$

**15.5**

#1. Find the mass and center of mass of the lamina that occupies the region $D$ and has the density function $\rho$.

$D \text{ is bounded by } y=\sqrt{x},\ \ y=0,\ \ \text{and } x=1;\ \ \rho(x,y)=x$

#2. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the *x*-axis.

#3. (i) Verify that

$$f(x,y)=\begin{cases} 4xy & 0\le x\le 1,\ 0\le y\le 1 \\ 0 & \textit{otherwise} \end{cases}$$

is a joint density function.

If $X$ and $Y$ are random variables whose joint density function is the function $f$ in part (i):

(ii) Find $P\left(X\ge\frac{1}{2}\right)$

(iii) Find $P\left(X\ge\frac{1}{2}\ \ AND\ \ Y\le\frac{1}{2}\right)$

(iv) Find the expected values of $X$ and $Y$.

**Ch 15 Part 1 Test Review**

#1. Evaluate $\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy$.

#3. Evaluate $\int_0^1 \int_{2x}^2 (x - y)\, dy\, dx$.

#2. Evaluate $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

#4. Evaluate $\int_0^{\pi/2} \int_0^{\cos\theta} e^{\sin\theta}\, dr\, d\theta$.

#5. Set up the double-integral which evaluates the following, including limits of integration (but do not evaluate the integral). $\iint\limits_D \sqrt{9x^2+9y^2}\, dA$

where $D$ is the region in the third quadrant bounded by the $y=\sqrt{3}x$, $x=0$, and $x^2+y^2=16$.

#6. Set up the double-integral which evaluates the following, including limits of integration (but do not evaluate the integral). $\iint\limits_D \cos\left(e^{2x-y}\right) dA$ where

$D$ is the triangular region with vertices $(1,1)$, $(4,1)$, and $(1,2)$.

#7. Set up the double-integral which evaluates the following, including limits of integration (but do not evaluate the integral).

The volume of the solid enclosed by the paraboloid $z=x^2+3y^2$ and the planes $x=0$, $y=1$, $y=x$, and $z=0$.

#8. Set up the double-integral which evaluates the following, including limits of integration (but do not evaluate the integral).

The volume of the solid formed by the region above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

#9. Set up the double-integral which evaluates the following, including limits of integration (but do not evaluate the integral).

The volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

For #10, #11, and #12: Find the mass and center of mass for the following lamina. (Set up the integrals including limits of integration, but do not evaluate the integrals. For center of mass, show the formula for computing it using symbols from your earlier work).

#10. The triangular region with vertices (0,0), (4,0), and (0,2) if density is proportional to three times the distance a point is from the y-axis.

#11. The boundary of the lamina consists of the semicircles $y = \sqrt{1 - x^2}$ and $y = \sqrt{4 - x^2}$ together with the portions of the *x*-axis that join them, if the density at any point is proportional to its distance from the origin.

#12. The part of the disk $x^2 + y^2 \leq 1$ in the first quadrant, if the density at any point is proportional to twice the distance from the *x*-axis.