Calc III - Ch 14 Part 2 - Required Practice

Name: ______________________________

14.6

#1. Find the directional derivative of f at the given point in the direction indicated by the angle θ

$f(x,y) = ye^{-x}, \quad (0,4), \quad \theta = \frac{2\pi}{3}$

#2. (i) Find the gradient of f.
(ii) Evaluate the gradient at the point P.
(iii) Find the rate of change of f at P in the direction of the vector $\vec{u}$.

$f(x,y) = \sin(2x+3y), \; P(-6,4), \; \vec{u} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

#3. Find the directional derivative of the function at the given point in the direction of the vector $\vec{v}$

$f(x,y) = 1 + 2x\sqrt{y}, \quad (3,4), \quad \vec{v} = \langle 4, -3 \rangle$

#4. Find the directional derivative of the function at the given point in the direction of the vector $\vec{v}$

$f(x,y,z) = xe^{y} + ye^{z} + ze^{x}, \quad (0,0,0), \quad \vec{v} = \langle 5, 1, -2 \rangle$

#5. Find the directional derivative of $f(x,y)=\sqrt{xy}$ at $P(2,8)$ in the direction of $Q(5,4)$.

#6. Find the maximum rate of change of *f* at the given point and the direction in which it occurs.

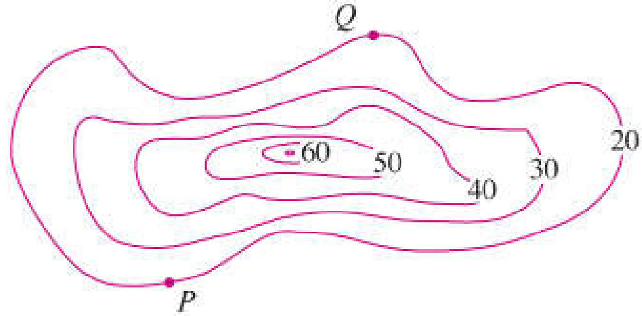
$f(x,y)=\frac{y^2}{x}, \quad (2,4)$

#7. Find all points at which the direction of fastest change of the function $f(x,y)=x^2+y^2-2x-4y$ is $\langle 1,1\rangle$.

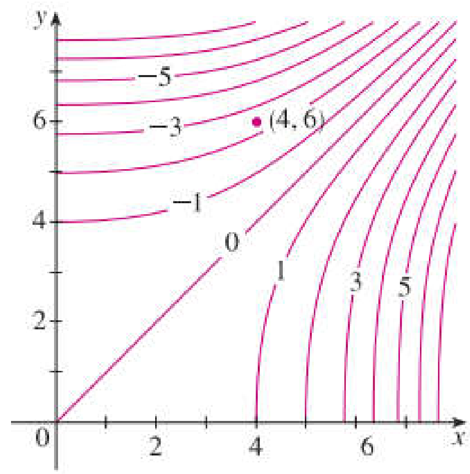
#8. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where *x, y,* and *z* are measured in meters, and you are standing at a point with coordinates (60, 40, 966). The positive *x*-axis points east and the positive *y*-axis points north.

(i) If you walk due south, will you start to ascend or decent? At what rate?
(ii) If you walk northwest, will you start to ascend or descend? At what rate?
(iii) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

#9. For the given contour map draw the curve of steepest ascent starting at P.



#10. Sketch the gradient vector $\nabla f(4,6)$ for the function *f* whose level curves are shown.



#11. Find an equation of the tangent plane to the given surface at the specified point.

$2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10, \quad (3,3,5)$

14.7

#1. Suppose (0,2) is a critical point of a function g with continuous second derivatives. What can you say about g at (0,2)?

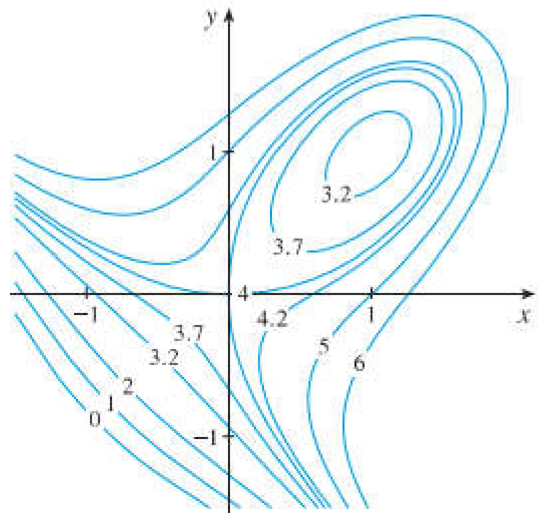
(i) $g_{xx}(0,2) = -1, \quad g_{xy}(0,2) = 6, \quad g_{yy}(0,2) = 1$

(ii) $g_{xx}(0,2) = -1, \quad g_{xy}(0,2) = 2, \quad g_{yy}(0,2) = -8$

(iii) $g_{xx}(0,2) = 4, \quad g_{xy}(0,2) = 6, \quad g_{yy}(0,2) = 9$

#2. Use the level curves in the figure to predict the location of the critical points of f and whether f has a saddle point or local maximum or minimum at each critical point. Then use the Second Derivatives Test to confirm your predictions.

$f(x, y) = 4 + x^3 + y^3 - 3xy$



#3. Find the local maximum and minimum values and saddle point(s) of the function.

$f(x,y)=9-2x+4y-x^2-4y^2$

#4. Find the absolute maximum and minimum values of *f* on the set *D*.

$f(x,y)=1+4x-5y$

D is the closed triangular region with vertices (0,0), (2,0), and (0,3).

#5. Find the shortest distance from the point $(2,1,-1)$ to the plane $x+y-z=1$.

#6. Find three positive numbers whose sum is 100 and whose product is a maximum.

#7. A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of cardboard used.

14.8

#1. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

$f(x, y) = x^2 + y^2; \quad xy = 1$

#2. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

$f(x, y, z) = 2x + 6y + 10z; \quad x^2 + y^2 + z^2 = 35$

#3. Consider the problem of maximizing the function $f(x, y) = 2x + 3y$ subject to the constraint $\sqrt{x} + \sqrt{y} = 5$.

(i) Try using Lagrange multipliers to solve the problem.

(ii) Does $f(25,0)$ give a larger value than the one in part (i)?

(iii) Solve the problem by graphing the constraint equation and several level curves of f.

(iv) Explain why the method of Lagrange multipliers fails to solve the problem.

(v) What is the significance of $f(9.4)$?