Calc III - Ch 14 Part 1 - Required Practice **Name: ______________________________**

14.1

#1. The *temperature-humidity index I* is the perceived air temperature when the actual temperature is *t* and the relative humidity is *h*, so we can write $I = f(T, h)$. The following table to values of *I* is an excerpt from a table compiled by the National Oceanic & Atmospheric Administration:

Relative humidity (%)

Actual temperature (°F): T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

(i) What is the value of $f(95, 70)$? What is its meaning?

(ii) For what value of *h* is $f(95, h) = 100$?

(iii) For what value of *T* is $f(T, 50) = 88$?

(iv) What are the meanings of the functions $I = f(80, h)$ and $I = f(100, h)$? Compare the behavior of these two functions of *h*.

#2. Let $f(x, y) = x^2 e^{3xy}$.

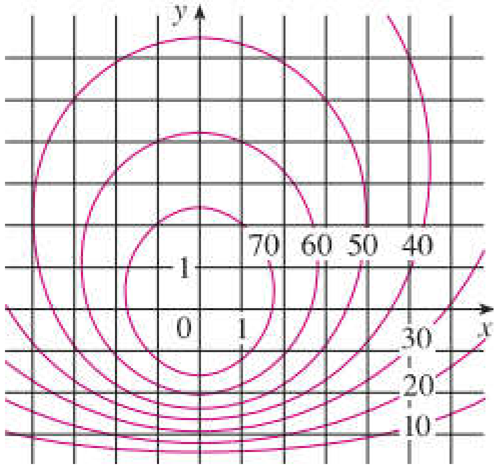
(i) Evaluate $f(2, 0)$.

(ii) Find the domain of *f*.

(iii) Find the range of *f*.

#3. Sketch the graph of the function $f(x, y) = 10 - 4x - 5y$.

(14.1) #4. A contour map for a function *f* is shown. Use it to estimate the values of $f(-3,3)$ and $f(3,-2)$. What can you say about the shape of the graph?



#5. Draw a contour map of the function showing several level curves: $f(x,y)=ye^x$.

#6. Describe the level surfaces of the function $f(x,y,z)=x+3y+5z$.

14.2

#1. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$.

#2. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}$.

#3. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$.

14.3

#1. The *wind-chill index W* is the perceived temperature when the actual temperature is *t* and the wind speed is *v*, so we can write $W = f(T, v)$. The following table provide values:

Wind speed (km/h)

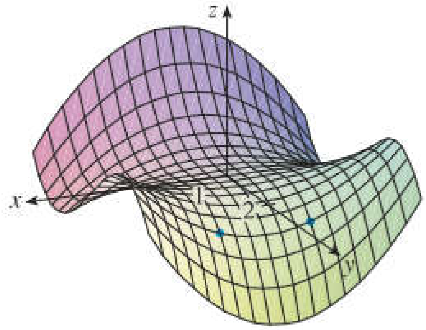
Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

(i) Estimate the values of $f_T(-15, 30)$ *and* $f_v(-15, 30)$. What are the practice interpretations of these values?

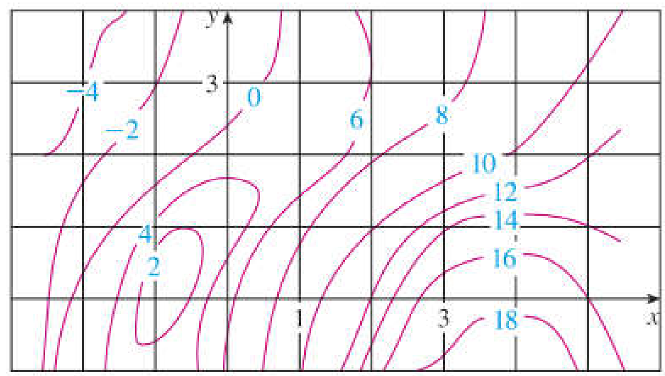
(ii) In general, what can you say about the signs of $\frac{\partial W}{\partial T}$ and $\frac{\partial W}{\partial v}$?

(iii) What appear to be the value of the following limit? $\lim_{v\to\infty} \frac{\partial W}{\partial v}$

#2. Determine the signs of the partial derivatives $f_x(1,2)$, $f_y(1,2)$, and $f_{xx}(1,2)$ for the function *f* whose graph is shown.



#3. A contour map is given for a function *f*. Use it to estimate $f_x(2,1)$ and $f_y(2,1)$



#4. Find both first partial derivatives of the function $f(x, y) = y^5 - 3xy$

(14.3) #5. Find both first partial derivatives of the function $w = \frac{e^v}{u + v^2}$

#6. Find both first partial derivatives of the function $f(x, y) = \int_y^x \cos(t^2)\, dt$

#7. Find both first partial derivatives of the function $w = \ln(x + 2y + 3z)$

#8. Find $f_y(2,1,-1)$ for $f(x, y, z) = \frac{y}{x + y + z}$

#9. Verify that the conclusion of Clariaut's Theorem hold (that $u_{xy} = u_{yx}$) $u = x\sin(x + 2y)$

14.4

#1. Find an equation of the tangent plane to the given surface at the specified point
$z = 4x^2 - y^2 + 2y, \quad (-1, 2, 4)$.

#2. Find an equation of the tangent plane to the given surface at the specified point
$z = y \ln x, \quad (1, 4, 0)$.

#3. Explain why the function is differentiable at the given point. Then find the linearization *L(x,y)* of the function at that point.
$f(x, y) = x^3 y^4, \quad (1, 1)$.

#4. Find the differential of the function.
$z = x^3 \ln(y^2)$.

(14.4) #5. Verify the linear approximation at (0,0).

$$\frac{2x+3}{4y+1} \approx 3+2x-12y .$$

#6. If $z = 5x^2 + y^2$ and (x, y) changes from $(1,2)$ to $(1.05, 2.1)$, compare the values of Δz and dz .

14.5

#1. Use the Chain Rule to find $\frac{dz}{dt}$

$z = x^2 + y^2 + xy, \quad x = \sin t, \quad y = e^t$.

#2. Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

$z = x^2 y^3, \quad x = s\cos t, \quad y = s\sin t$.

#3. Use a tree diagram to write out the Chain Rule for the given case. Assume all functions are differentiable.

$u = f(x, y), \quad x = x(r, s, t), \quad y = y(r, s, t)$.

#4. Use the Chain Rule to find the indicated partial derivatives

$z = x^2 + xy^3, \quad x = uv^2 + w^3, \quad y = u + ve^w$

$\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$ *when* $u = 2, \; v = 1, \; w = 0$.

#5. Use $\frac{dy}{dx} = -\frac{\left(\frac{\partial F}{\partial x}\right)}{\left(\frac{\partial F}{\partial y}\right)} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$.

$\sqrt{xy} = 1 + x^2 y$.

#6. The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohn's Law, $V = IR$ to find how the current I is changing at the moment when

$R = 400\Omega,\ I = 0.08A,\ \frac{dV}{dt} = -0.01 \, {}^{V}\!/_{s}$,

and $\frac{dR}{dt} = 0.03 \, {}^{\Omega}\!/_{s}$.

Ch 14 Part 1 Test Review

#1. Draw a contour map of the function showing several level curves: $f(x,y)=(y-2x)^2$.

#2. Draw a contour map of the function showing several level curves: $f(x,y)=x^3-y$.

#3. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}$.

#4. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$.

#5. Find both first partial derivatives of the function $f(x,y) = y^5 - 3xy$

#6. Find both first partial derivatives of the function $f(x,y) = x^4 y^3 + 8x^2 y$

#7. Find both first partial derivatives of the function $u = te^{\left(w/t\right)}$

#8. Find all the second partial derivatives of
$f(x, y) = x^3 y^5 + 2x^4 y$

#9. Find an equation of the tangent plane to the given surface at the specified point
$z = 4x^2 - y^2 + 2y, \quad (-1, \ 2, \ 4)$

#10. Find an equation of the tangent plane to the given surface at the specified point
$z = 3(x-1)^2 + 2(y+3)^2 + 7, \quad (2, \ -2, \ 12)$

#11. Find the linear approximation of the function $f(x, y) = \ln(x - 3y)$ at $(7, 2)$ and use it to approximate $f(6.9,\ 2.06)$.

#13. Find the differential of the function $v = y \cos xy$.

#12. Find the differential of the function $z = x^3 \ln(y^2)$.

#14. Use the Chain Rule to find $\dfrac{dz}{dt}$

$z = \sqrt{1 + x^2 + y^2}, \quad x = \ln t, \quad y = \cos t$.

#15. Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

$z = \sin\theta\cos\phi,\quad \theta = st^2,\quad \phi = s^2 t$.

#16. Use a tree diagram to write out the Chain Rule for the given case. Assume all functions are differentiable.

$u = f(x,y),\quad x = x(r,s,t),\quad y = y(r,s,t)$.

#17. Use a tree diagram to write out the Chain Rule for the given case. Assume all functions are differentiable.

$u = f(r,s,t),\quad r = r(x,y),\quad s = s(x,y),\quad t = t(x,y)$

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