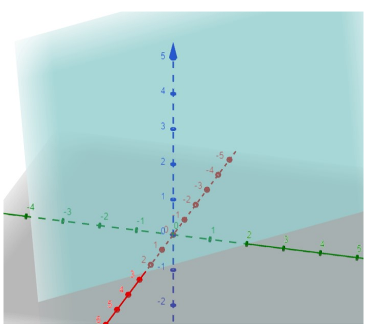
Calc III - Ch 14 – Part 1 - Required Practice

14.1

#1. (i) 124
When the actual temperature is 95°F and the relative humidity is 70%, the perceived air temperature is 124°F.
(ii) $\approx 39\%$
(iii) *I=f(80,h)* shows how perceived temperature varies as a function of humidity when actual temperature is 80°F. *I=f(100,h)* shows how perceived temperature varies as a function of humidity when actual temperature is 100°F. *f(100,h)* increases more rapidly as *h* increases than *f(80,h)*, meaning perceived temperature increases more quickly with humidity for hotter actual temperatures.

#2. (i) 4
(ii) $D:\{(x,y) \mid -\infty < x < \infty, -\infty < y < \infty\}$
(iii) $R:\{z \mid z \geq 0\}$

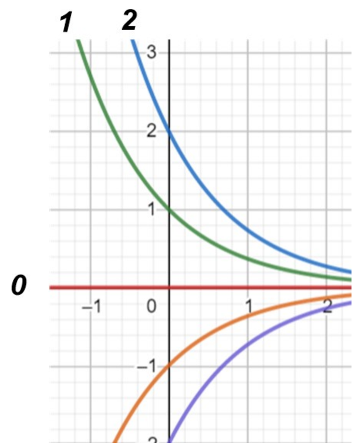
#3. $intercepts: (0,0,10),\ (0,2,0),\ \left(\frac{5}{2},0,0\right)$



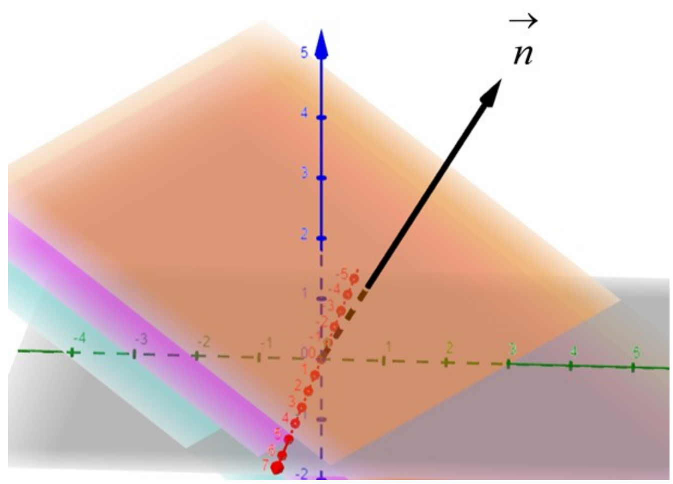
#4. $f(-3,3) \approx 56$
$f(3,-2) \approx 35$

ANSWERS ONLY

#5.



#6. Level surfaces are parallel planes all with normal $\vec{n} = \langle 1,3,5 \rangle$



14.2

#1. $\frac{2}{7}$

#2. DNE

#3. DNE .

14.3

#1.

(i) $f_T(-15,30)=1.3\ \dfrac{°C\ perceived}{°C\ actual}$

For every increase in actual temp of 1 ºC, perceived temp increases by 1.3 ºC, on average.

$f_V(-15,30)=-0.15\ \dfrac{°C\ perceived}{km\ per\ hr}$

For every increase in wind speed of 1 kph, perceived temp decreases by 0.15 ºC, on average.

(ii) $\dfrac{\partial W}{\partial T}>0,\ \dfrac{\partial W}{\partial v}<0$

(iii) As v increases, the amount of change in W is getting smaller, which suggests that $\lim\limits_{v\to\infty}\dfrac{\partial W}{\partial v}=0$.

#2. $f_x(1,2)>0,\quad f_y(1,2)<0,\quad f_{xx}(1,2)>0$

#3. $f_x(2,1)\approx 4,\quad f_y(2,1)\approx -2.5$

#4. $f_x=-3y,\quad f_y=5y^4-3x$

#5. $\dfrac{\partial W}{\partial u}=\dfrac{-e^v}{\left(u+v^2\right)^2},\quad \dfrac{\partial W}{\partial v}=\dfrac{\left(u+v^2\right)e^v-e^v(2v)}{\left(u+v^2\right)^2}$

#6. $f_x=\cos\left(x^2\right),\quad f_y=-\cos\left(y^2\right)$

#7. $W_x=\dfrac{1}{x+2y+3z}(1)$

$W_y=\dfrac{1}{x+2y+3z}(2)$

$W_z=\dfrac{1}{x+2y+3z}(3)$

#8. $\dfrac{1}{4}$

#9. $u_{xy}=u_{yx}=-2x\sin(x+2y)+2\cos(x+2y)$

14.4

#1.

$z-4=-8(x+1)-2(y-2)\quad or\quad 8x+2y+z=0$

#2. $z=4x-4$

#3. $L(x,y)=-6+3x+4y$

#4. $dz=3x^2\ln\left(y^2\right)dx+\dfrac{2x^3}{y}dy$,

#5.

$Show\ L(x,y)=3+2x-12y\ for\ f(x,y)=\dfrac{2x+3}{4y+1}$

#6. $\Delta z=0.9225,\quad dz=0.9$

14.5

#1. $\dfrac{dz}{dx}=(2x+y)(\cos t)+(2y+x)\left(e^t\right)$

#2. $\dfrac{\partial z}{\partial s}=\left(2xy^3\right)(\cos t)+\left(3x^2y^2\right)(\sin t)$

$\dfrac{\partial z}{\partial t}=\left(2xy^3\right)(-s\sin t)+\left(3x^2y^2\right)(s\cos t)$

#3. $\dfrac{\partial u}{\partial r}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial r}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial r}$

$\dfrac{\partial u}{\partial s}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial s}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial s}$

$\dfrac{\partial u}{\partial t}=\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t}+\dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial t}$

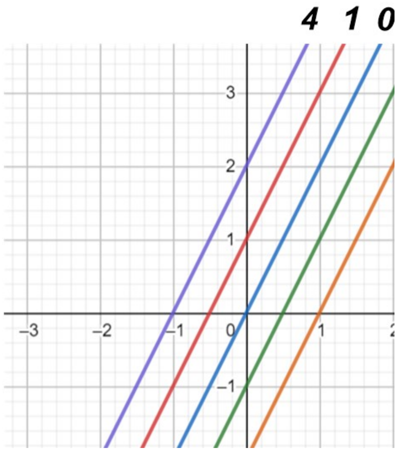
#4. $\dfrac{\partial z}{\partial u}=85,\quad \dfrac{\partial z}{\partial v}=178,\quad \dfrac{\partial z}{\partial w}=54$

#5. $\dfrac{dy}{dx}=\dfrac{4xy\sqrt{xy}-y}{x-2x^2\sqrt{xy}}$

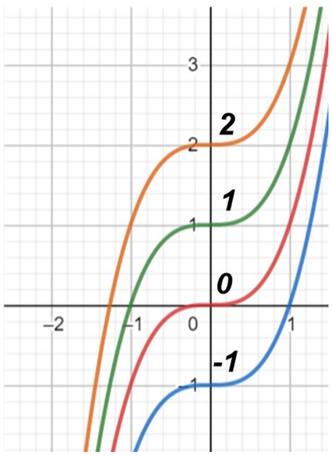
#6. $\dfrac{dI}{dt}=-0.031\,mA/s$

Ch14 Part 1 Test Review

#1.



#2.



#3. *DNE*

#4. *DNE*

#5. $f_x = -3y, \quad f_y = 5y^4 - 3x$

#6. $f_x = 4x^3y^3 + 16xy, \quad f_y = 3x^4y^2 + 8x^2$

#7. $\dfrac{\partial u}{\partial t} = \dfrac{-we^{w/t}}{t} + e^{w/t}, \quad \dfrac{\partial u}{\partial w} = e^{w/t}$

#8.
$f_{xx} = 6xy^5 + 24x^2y, \quad f_{xy} = f_{yx} = 15x^2y^4 + 8x^3, \quad f_{yy} = 20x^3y^3$

#9. $z - 4 = -8(x+1) - 2(y-2)$

#10. $z - 12 = 6(x-2) + 4(y+2)$

#11. -0.28

#12. $dz = \left(3x^2 \ln\left(y^2\right)\right)dx + \left(\dfrac{2x^3}{y}\right)dy$

#13.
$dv = \left(-y^3 \sin(xy)\right)dx + \left(-xy\sin(xy) + \cos(xy)\right)dy$

#14.
$$\frac{dz}{dt} = \left(\frac{x}{\sqrt{1+x^2+y^2}}\right)\left(\frac{1}{t}\right) + \left(\frac{y}{\sqrt{1+x^2+y^2}}\right)(-\sin t)$$

#15.
$$\frac{\partial z}{\partial s} = (\cos\theta\cos\phi)\left(t^2\right) + (-\sin\theta\sin\phi)(2st)$$
$$\frac{\partial z}{\partial t} = (\cos\theta\cos\phi)(2st) + (-\sin\theta\sin\phi)\left(s^2\right)$$

#16.
$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

#17.
$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$$
$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$