Calc III - Ch 13 - Required Practice

Name: ________________________________

13.1

#1. Find the domain of the vector function $\vec{r}(t)=\left\langle \sqrt{4-t^2},\ e^{-3t},\ \ln(t+1)\right\rangle$.

#2. Find the limit: $\lim_{t\to 0}\left\langle \frac{e^t-1}{t},\ \frac{\sqrt{1+t}-1}{t},\ \frac{3}{1+t}\right\rangle$.

#3. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases: $\vec{r}(t)=\langle \sin t,\ t\rangle$.

#4. Find a vector equation and parametric equations for the line segment that joins P to Q: $P(0,0,0)$, $Q(1,2,3)$.

#5. Sketch in 3D the parametric curve given by the parametric equations: $x=\sin t$, $y=t$, $z=\cos t$.

#6. At what points does the curve $\vec{r}(t)=\langle t,\ 0,\ 2t-t^2\rangle$ intersect the paraboloid $z=x^2+y^2$?

13.2

#1. For $\vec{r}(t)=\langle 1+t,\ \sqrt{t}\rangle$

(i) Sketch the plane curve.

(ii) Find $\vec{r}'(t)$

(iii) On your plane curve sketch, add sketches for $\vec{r}(1)$ and $\vec{r}'(1)$

#2. Find the derivative of $\vec{r}(t)=\langle t\sin t,\ \ t^2,\ t\cos 2t\rangle$

#3. Find the derivative of $\vec{r}(t) = \langle 1, \ -1, \ e^{4t} \rangle$

#5. If $\vec{r}(t) = \langle t, \ t^2, \ t^3 \rangle$, find $\vec{r}'(t), \vec{T}(1), \vec{r}''(t)$ and $\vec{r}'(t) x \vec{r}''(t)$

#4. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter *t*.

$\vec{r}(t) = \langle 4\sqrt{t}, \ t^2, \ t \rangle, \quad t = 1$

#6. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$x = 1 + 2\sqrt{t}, \quad y = t^3 - t, \quad z = t^3 + t; \quad (3, 0, 2)$

#7. The curves $\vec{r_1} = \langle t,\ t^2,\ t^3 \rangle$ and $\vec{r_2} = \langle \sin t,\ \sin 2t,\ t \rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.

#8. Evaluate the integral: $\int_0^1 \langle 16t^3,\ -9t^2,\ 25t^4 \rangle \, dt$

13.3

#1. Find the length of the curve $\vec{r}(t) = \langle 2\sin t,\ 5t,\ 2\cos t \rangle, \quad -10 \le t \le 10$

#2. Reparametrize the curve $\vec{r}(t) = \langle 2t,\ 1-3t,\ 5+4t \rangle$ with respect to arc length measured from the point where $t = 0$ in the direction of increasing t .

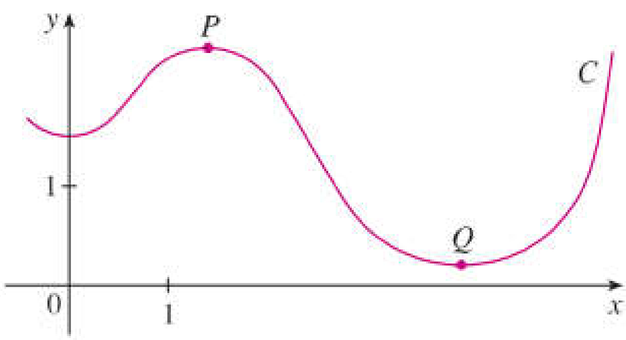
#3. Find the unit tangent vector $\vec{T}(t)$, the unit normal vector $\vec{N}(t)$, and the curvature κ for $\vec{r}(t) = \langle 2\sin t,\ 5t,\ 2\cos t \rangle$.

#4. Find the curvature κ for $\vec{r}(t) = \langle t^2,\ 0,\ t \rangle$.

#5. Find the curvature κ for $y = 2x - x^2$.

#7. Find the vectors $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{B}(t)$ for $\vec{r}(t) = \langle \cos t,\ \sin t,\ \ln(\cos t) \rangle$ at $(1,0,0)$.

#6. Given the curve:



(i) Is the curvature of the curve C shown in the figure greater at P or at Q? Explain.

(ii) Estimate the curvature at P and at Q by sketching the osculating circles at those points.

#8. Find equations of the normal plane and the osculating plane of the curve at the given point: .
$x = 2\sin 3t,\ \ y = t,\ \ z = 2\cos 3t$ at $(0, \pi, -2)$

13.4

#1. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of *t*.

$\vec{r}(t) = \left\langle -\frac{1}{2}t^2,\ t \right\rangle,\quad t = 2$

#2. Find the velocity, acceleration, and speed of a particle with the given position function.

$\vec{r}(t) = \left\langle t^2 + 1,\quad t^3,\quad t^2 - 1 \right\rangle$

#3. Find the position vector of a particle that has the given acceleration and the specified initial velocity and position:

$\vec{a}(t) = \langle 2t, \sin t, \cos 2t \rangle,\ \vec{v}(0) = \langle 1,0,0 \rangle,\ \vec{r}(0) = \langle 0,1,0 \rangle$

#4. The position function of a particle is given by $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum?

#5. A projectile is fired from the ground with an initial speed of 500 m/s and angle of elevation of 30°. Find (i) the range of the projectile, (ii) the maximum height reached, and (iii) the speed at impact.

#6. A ball is thrown at an angle of 45^o to the ground. If the ball land 90 m away, what was the initial speed of the ball?

#7. (very challenging – if you get a quadratic of the form $A\tan^2\theta + B\tan\theta + C = 0$ then you are on the right track ☺)A gun has muzzle speed of 150 m/s. Find two angles of elevation that can be used to hit a target 800 m away. (You may also need to know that $\sec^2\theta = \tan^2\theta + 1$).

Ch13 Test Review

#1. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases (always do this for vector function curves, whether it asks for it or not).
$\vec{r}(t) = \langle \sin t,\ t \rangle$.

#2. Sketch the curve with the given vector equation. $\vec{r}(t) = \langle t,\ \cos 2t,\ \sin 2t \rangle$.

#3. Find a vector equation and parametric equations for the line segment that joins P to Q..
$P(1,-1,2),\ \ Q(4,1,7)$.

#4. Find a vector equation and parametric equations for the line segment that joins P to Q..
$P(0,0,0),\ \ Q(1,2,3)$.

#5. Find a vector equation that represents the curve of intersection of the two surfaces
$x^2 + y^2 = 4$ and $z = xy$.

#6. Find a vector equation that represents the curve of intersection of the two surfaces $z = 4x^2 + y^2$ and $y = x^2$.

#7. Find the derivative of the vector function $\vec{r}(t) = \langle t\sin t,\ t^2,\ t\cos 2t \rangle$.

#8. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter *t*: $\vec{r}(t) = \langle 4\sqrt{t},\ t^2,\ t \rangle,\ \ t = 1$.

#9. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point: $x = 1 + 2\sqrt{t},\ y = t^3 - t,\ z = t^3 + t;\ \ (3,0,2)$.

#10. Find $\vec{r}(t)$ if $\vec{r}'(t) = \langle 2t,\ 3t^2,\ \sqrt{t} \rangle$ and $\vec{r}(1) = \langle 1,1,0 \rangle$.

#11. Find $\vec{r}(t)$ if $\vec{r}'(t) = \langle t,\ e^t,\ te^t \rangle$ and $\vec{r}(0) = \vec{i} + \vec{j} + \vec{k}$.

#12. Find the length of the curve $\vec{r}(t) = \left\langle 2t,\ t^2,\ \frac{1}{3}t^3 \right\rangle, \quad 0 \le t \le 1$.

#13. Find the length of the curve $\vec{r}(t) = \vec{i} + t^2\,\vec{j} + t^3\,\vec{k}, \quad 0 \le t \le 1$.

#14. Find the length of the curve (correct to four decimal places):
$\vec{r}(t) = \langle \sin t,\ \cos t,\ \tan t \rangle, \quad 0 \le t \le \frac{\pi}{4}.$

#15. Find the curvature κ of $\vec{r}(t) = \langle t,\ t^2,\ t^3 \rangle$ at the point $(1,1,1)$.

#16. Find the unit tangent vector $\vec{T}(t)$, unit normal vector $\vec{N}(t)$, and curvature κ for $\vec{r}(t) = \langle 2\sin t,\ 5t,\ 2\cos t \rangle$.

#17. Find the unit tangent vector $\vec{T}(t)$, unit normal vector $\vec{N}(t)$, and curvature κ for $\vec{r}(t)=\left\langle t,\ \frac{1}{2}t^2,\ t^2\right\rangle$.

#18. Find the equation of the normal plane of the curve at the given point
$x=t,\ \ y=t^2,\ \ z=t^3;\ (1,1,1)$.

#19. Find the velocity, acceleration, and speed of a particle with the given position function
$\vec{r}(t)=\left\langle t^2+1,\ t^3,\ t^2-1\right\rangle$.

#20. Find the velocity, acceleration, and speed of a particle with the given position function $\vec{r}(t) = \sqrt{2}t\,\vec{i} + e^{t}\,\vec{j} + e^{-t}\,\vec{k}$.

#21. A projectile is fired from a position 200 m above the ground with an initial speed of 500 m/s and angle of elevation of 30°. Find (i) the range of the projectile, (ii) the maximum height reached, and (iii) the speed at impact.

#22. A ball is thrown at an angle of 45^o to the ground. If the ball lands 90 m away, what was the initial speed of the ball?

#23. Find the curvature of the ellipse $x = 3\cos t, \quad y = 4\sin t$ at the points $(3,0)$ and $(0,4)$.

#24. An athlete puts a shot (throws an object) at an angle of 45° to the horizontal at an initial speed 43 ft/s. It leaves the athlete's hand 7 ft above the ground. (i) Where is the shot 2 seconds later? (ii) What is the maximum height of the object? (iii) Where does the object land?