Calc III - Ch 12 - Required Practice

Name: ______________________________

12.1

#1. On a single set of coordinate axes, sketch the points (0,5,2), (4,0,-1), (1,-1,2), and (2,4,6).

#2. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

#3. Find the lengths of the sides of the triangle *PQR*. Is it a right triangle? Is it an isosceles triangle? $P(3,-2,-3)$, $Q(7,0,1)$, $R(1,2,1)$.

#4. Find an equation of the sphere with center $(1,-4,3)$ and radius 5. What is the intersection of this sphere with the *xz*-plane?

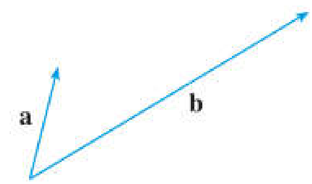
(12.1) #5. Show that the equation represents a sphere, and find its center and radius:
$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

#6. Find an equation of a sphere if one of its diameters has endpoints $(2,1,4)$ and $(4,3,10)$.

#7. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality: $x^2 + y^2 + z^2 \leq 3$.

12.2

#1. For the given vectors $\vec{a}$ and $\vec{b}$:



sketch each of the following:

$\underline{\vec{a} + \vec{b}}$ $\underline{\vec{a} - \vec{b}}$ $\underline{\vec{b} - 2\vec{a}}$

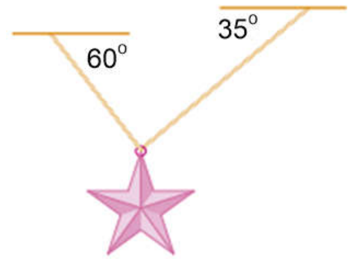
#2. Find the vector $\vec{AB}$ if $A(-1,3)$, $B(2,2)$.

Then sketch $\vec{AB}$ starting at the origin.

(12.2) #3. Find a unit vector that has the same direction as the given vector $9\vec{i}-\vec{j}+4\vec{k}$.

#4. A woman walks due west on the deck of a ship at 3 mi/hr. The ship is moving north at a speed of 22 mi/hr. Find the speed and direction of the woman relative to the surface of the water.

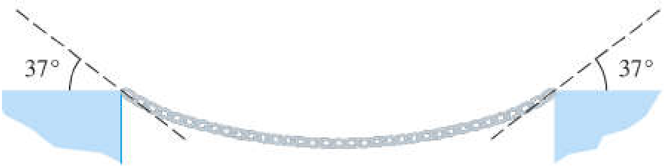
#5. An object weighting 10 lbs is suspended with two wires in the following arrangement:



Find the tensions in the two wires.

(12.2) #6.

The tension **T** at each end of the chain has magnitude 25 N. What is the weight of the chain?



12.3

#1. Which of the following expressions are meaningful and which are meaningless?

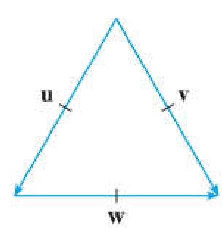
$\left(\vec{a} \bullet \vec{b}\right) \bullet \vec{c}$ $\quad\quad$ $\left(\vec{a} \bullet \vec{b}\right) \vec{c}$ $\quad\quad$ $\left|\vec{a}\right| \left(\vec{b} \bullet \vec{c}\right)$

#2. Find $\vec{a} \bullet \vec{b}$ *if* $\vec{a} = \left\langle -2, \frac{1}{3} \right\rangle, \ \vec{b} = \langle -5, 12 \rangle$

#3. Find $\vec{a} \bullet \vec{b}$ *if* $\vec{a} = \langle s, 2s, -s^2 \rangle, \ \vec{b} = \langle t, 3t, 2t^2 \rangle$

#4. Find $\vec{a} \bullet \vec{b}$ *if* $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}, \ \vec{b} = 5\vec{i} + 9\vec{k}$

#5. If $\vec{u}$ is a unit vector, find $\vec{u} \bullet \vec{v}$ and $\vec{u} \bullet \vec{w}$.



(12.3) #6. Find the angle between the vectors (exact and decimal): $\vec{a} = \langle -8, 6 \rangle$, $\vec{b} = \langle \sqrt{7}, 3 \rangle$

#7. Find the angle between the vectors (exact and decimal): $\vec{a} = \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$

#8. Determine whether the given vectors are orthogonal, parallel, or neither:

$\vec{a} = \langle -5, 3, 7 \rangle$, $\vec{b} = \langle 6, -8, 2 \rangle$

$\vec{a} = \langle 4, 6 \rangle$, $\vec{b} = \langle -3, 2 \rangle$

$\vec{a} = 2\vec{i} + 6\vec{j} - 4\vec{k}$, $\vec{b} = -3\vec{i} - 9\vec{j} + 6\vec{k}$

#9. For what values of b are the vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$ orthogonal?

#10. Find a unit vector that is orthogonal to both $\langle 1, 1, 0 \rangle$ and $\langle 1, 0, 1 \rangle$.

#11. A sled is pulled along a level path through snow by a rope. A 30-lb force acting at an angle of 40° above the horizontal moves the sled 80 ft. Find the work done by the force.

12.4

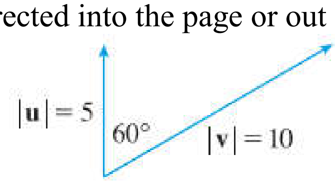
#1. Find the cross product $\vec{a} \times \vec{b}$ and verify that it is orthogonal to both $\vec{a}$ and $\vec{b}$.

$\vec{a} = \langle 1,1,-1 \rangle$, $\vec{b} = \langle 2,4,6 \rangle$

#2. Which of the following expressions are meaningful and which are meaningless? If meaningful, state whether the result is a vector or a scalar.

$\vec{a} \bullet \left(\vec{b} \times \vec{c} \right)$ $\qquad$ $\vec{a} \times \left(\vec{b} \bullet \vec{c} \right)$ $\qquad$ $\vec{a} \times \left(\vec{b} \times \vec{c} \right)$

#3. Find $\left| \vec{u} \times \vec{v} \right|$ and determine whether $\vec{u} \times \vec{v}$ is directed into the page or out of the page.



#4. If $\vec{a} = \langle 1,2,1 \rangle$ and $\vec{b} = \langle 0,1,3 \rangle$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

#5. Show that $\vec{0} \times \vec{a} = \vec{0} = \vec{a} \times \vec{0}$ for any vector $\vec{a}$ in $\mathbb{R}^3$.

(12.4) #6. Find the area of the parallelogram with vertices: $A(-2,1)$, $B(0,4)$, $C(4,2)$, and $D(2,-1)$.

#7. (i) Find a nonzero vector orthogonal to the plane through the given points:
$P(1,0,0), Q(0,2,0), R(0,0,3)$.
(ii) Find the area of triangle PQR.

#8. Find the volume of the parallelepiped with adjacent edges, PQ, PR, and PS.
$P(2,0,-1)$, $Q(4,1,0)$, $R(3,-1,1)$, $S(2,-2,2)$.

#9. Use the scalar triple product to verify that the vectors $\vec{u} = \langle 1,5,-2 \rangle$, $\vec{v} = \langle 3,-1,0 \rangle$, $\vec{w} = \langle 5,9,-4 \rangle$ are coplanar.

12.5

#1. Find a vector equation and parametric equations for the line though the point $(2, 2.4, 3.5)$ and parallel to the vector $\langle 3, 2, -1 \rangle$.

#2. Find parametric equations and symmetric equations for the line though the points $(1, 3, 2)$ and $(-4, 3, 0)$.

#3. Find parametric equations and symmetric equations for the line though $(2, 1, 0)$ and perpendicular to both $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$.

#4. Is the line through $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?

#5. Find a vector equation for the line segment from $(2, -1, 4)$ to $(4, 6, 1)$.

(12.5) #6. Find an equation of the plane through the point $(1,-1,1)$ and with normal vector $\langle 1,1,-1\rangle$.

#7. Find an equation of the plane through the points $(0,0,0)$, $(2,-4,6)$, and $(5,1,3)$

#8. Find an equation of the plane that passes through the line of intersection of the planes $x-z=1$ and $y+2z=3$ and is perpendicular to the plane $x+y-2z=1$.

(12.5) #9. Find an equation of the plane that passes through the point $(-1,2,1)$ and contains the line of intersection of the planes

$x+y-z=2$ and $2x-y+3z=1$

(12.5) #10. Where does the line through $(1,0,1)$ and $(4,-2,2)$ intersect the plane $x+y+z=6$?

#11. Determine whether the plane are parallel, perpendicular, or neither. If neither, find the angle between them.
$x+4y-3z=1, \quad -3x+6y+7z=0$

#12. Find an equation for the plane consisting of all points that are equidistant from the points $(1,0,-2)$ and $(3,4,0)$.

12.6 day 1 – problems done in class

For each problem, complete the square to put the equation in standard form, then sketch the 2D conic section…

#Example 1. $x^2 - 6x - 8y - 7 = 0$

#Example 2. $4x - y^2 - 2y - 9 = 0$

#Example 3. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$

#Example 4. $2x^2 + 2y^2 + 12x - 16y + 40 = 0$

#Example 5. $16x^2 - 4y^2 + 32x + 16y - 64 = 0$

#Example 6. $9x^2 - 4y^2 - 18x - 16y + 29 = 0$

12.6

#1. Describe and sketch the surface: $y^2 + 4z^2 = 4$.

#2. Describe and sketch the surface: $x - y^2 = 0$.

#3. Describe and sketch the surface: $z = \cos x$.

#4. Draw at least three traces in each of the coordinate planes and identify the surface for $x = y^2 + 4z^2$.

(12.6) #5. Put the equation in standard form, then name and sketch the surface:
$x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$.

#7. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the y-axis.

#6. Sketch the region bounded by the surfaces
$z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \le z \le 2$.

Ch12 Test Review

#1. Find the lengths of the sides of the triangle *PQR*. Is it a right triangle? Is it an isosceles triangle? $P(3,-2,-3)$, $Q(7,0,1)$, $R(1,2,1)$.

#2. Determine whether the points lie on a straight line.

(i) $A(2,4,2)$, $B(3,7,-2)$, $C(1,3,3)$.

(ii) $D(0,-5,5)$, $E(1,-2,4)$, $F(3,4,2)$.

#3. For $\vec{a}=\langle 5,-12\rangle$, $\vec{b}=\langle -3,-6\rangle$, find:

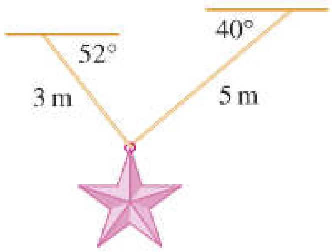
(i) $\vec{a}+\vec{b}$

(ii) $2\vec{a}+3\vec{b}$

(iii) $\left|\vec{a}-\vec{b}\right|$

#4. Find a unit vector that has the direction as $\langle -4,2,4\rangle$.

#5. Rope 3m and 5m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fasted at different heights, make angles of 52^o and 40^o with the horizontal. Find the tension in each wire.



#6. A clothesline is tied between two poles, 8m apart. The line is quite taut and has negligible sag. When a wet shirt with a mass ot 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the tension in each half of the clothesline.

#7. Find the angle between the vectors (in exact and decimal form). $\vec{a} = \langle 4,0,2 \rangle$, $\vec{b} = \langle 2,-1,0 \rangle$.

#9. If $\vec{a} = \langle 1,2,1 \rangle$ and $\vec{b} = \langle 0,1,3 \rangle$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

#8. For what values of b are the vectors $\langle -6,b,2 \rangle$ and $\langle b,b^2,b \rangle$ orthogonal? .

#10. Find two unit vectors orthogonal to both $\langle 1,-1,1 \rangle$ and $\langle 0,4,4 \rangle$.

#11. Find a vector equation, parametric equations, and symmetric equations for the line through the points $(6,1,-3)$ and $(2,4,5)$.

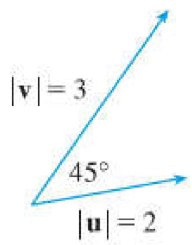
#12. Find a vector equation for the line of intersection of the planes $x+y+z=1$ and $x+z=0$.

#13. Find an equation of the plane through the point $(-2,8,10)$ and perpendicular to the line $x=1+t,\ y=2t,\ z=4-3t$.

#14. Find an equation of the plane that contains the line $x=3+2t,\ y=t,\ z=8-t$ and is parallel to the plane $2x+4y+8z=17$.

#15. Draw at least two traces for each coordinate plane for $4x^2 - 16y^2 + z^2 = 16$. What kind of solid is this? What is its main axis? Sketch the solid in $\mathbb{R}^3$.

#16. If $\vec{u}$ and $\vec{v}$ are the vectors shown in the figure:



(i) Find $\vec{u} \bullet \vec{v}$.

(ii) Find $\left|\vec{u}\ x\ \vec{v}\right|$.

(iii) Is $\vec{u}\ x\ \vec{v}$ directed into the page or out of the page?

#17. For $\vec{a} = \vec{i} + \vec{j} - 2\vec{k},\ and\ \vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$
Find each of the following:

(i) $2\vec{a} + 3\vec{b}$

(ii) $\left|\vec{b}\right|$

(iii) $\vec{a} \bullet \vec{b}$

(iv) $\vec{a}\ x\ \vec{b}$