Calc III - Ch 16 - Extra Practice

16.1 and 16.2 day 1

#1b. Sketch the vector field for

$\vec{F}(x,y) = \langle x - y,\ x \rangle$

#2b. Evaluate the line integral, where C is the given curve: $\int_C xy\, ds \quad C: x = t^2,\ y = 2t,\ 0 \le t \le 1$

#3b. Evaluate the line integral, where C is the given curve: $\int_C x \sin y\, ds$ where C is the line segment from $(0,3)$ to $(4,6)$.

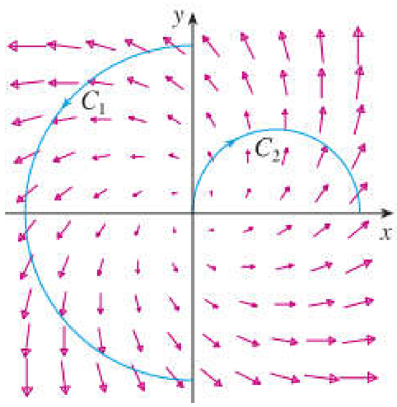
#4b. Evaluate the line integral, where C is the given curve: $\int_C (x + yz)\, dx + 2x\, dy + xyz\, dz$ where C consists of line segments from $(1,0,1)$ to $(2,3,1)$ and from $(2,3,1)$ to $(2,5,2)$.

#5b. Evaluate the line integral, where C is the given curve: $\int_C (2x + 9z)\, ds$

$C: x = t,\ \ y = t^2,\ \ z = t^3,\ \ 0 \le t \le 1$

16.2 day 2

#1b. The figure shows a vector field $\vec{F}$ and two curves C_1 and C_2. Are the line integrals of $\vec{F}$ over C_1 and C_2 $\left(\int_C \vec{F} \cdot d\vec{r}\right)$ positive, negative, or zero. Explain.



#2b. Evaluate the line integral $\int_{C_1} \vec{F} \cdot d\vec{r}$ where C is given by the vector function $\vec{r}(t)$

$\vec{F}(x,y,z) = \langle \sin x,\ \cos y,\ xz \rangle$

$\vec{r}(t) = \langle t^3,\ -t^2,\ t \rangle \quad 0 \le t \le 1$

#3b. Find the work done by the force field $\vec{F}(x,y)=\langle y,\ -x\rangle$ on a particle that moves along the curve $y=2x^3$ from $(1,2)$ to $(3,54)$.

#4b. Show that a constant force field does zero work on a particle that moves once uniformly around the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$.

16.3

#1b. Determine whether or not $\vec{F}$ is a conservative vector field. If it is, find a function *f* such that $\vec{F} = \nabla f$.

(i) $\vec{F}(x, y) = \langle e^x \sin y,\ e^x \cos y \rangle$

(ii) $\vec{F}(x, y) = \left\langle \ln y + 2xy^3,\ 3x^2y^2 + \dfrac{x}{y} \right\rangle$

#2b. (hints) Check to see if the field is conservative. If it is, then the value computed for a line integral depends only upon the endpoints (independent of path), and you can also use the function *f* as the antiderivative to compute the value of the line path integral.

#3b. Find a function f such that $\vec{F} = \nabla f$ and use it to evaluate $\int_C \vec{F} \bullet d\,\vec{r}$ along the given curve C.

$\vec{F}(x, y, z) = \langle yz,\ xz,\ xy + 2z \rangle$

C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$

#4b. Show that the line integral is independent of path and evaluate the integral.

$\int_C \left(1 - ye^{-x}\right) dx + e^{-x}\, dy$

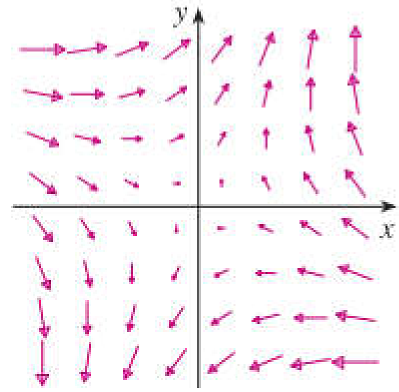
C is any path from $(0, 1)$ to $(1, 2)$

#5b. Find the work done by the force field $\vec{F}$ in moving an object from P to Q.

$\vec{F}(x,y)=\langle e^{-y},\ -xe^{-y}\rangle$

$P(0,1),\quad Q(2,0)$

#6b. Is the vector field shown in the figure conservative? Explain.



16.4

#1b. Evaluate the line integral (i) directly and (ii) using Green's Theorem.

$$\oint_C xy\,dx + x^2y^3\,dy$$

C is the triangle with vertices (0,0), (1,0) and (1,2).

#1c. Evaluate the line integral

$$\oint_C x\,dx + y\,dy$$

C is the triangle with vertices (0,0), (1,0) and (1,2).

#2b. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\oint_C \left(y+e^{\sqrt{x}}\right)dx+\left(2x+\cos y^2\right)dy$$

C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$.

#3b. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\oint_C y^3 dx - x^2\, dy$$

C is the top half of the circle $x^2+y^2=9$.

#4b. Use Green's Theorem to evaluate $\int_C \vec{F} \bullet d\vec{r}$.

(Check the orientation of the curve before applying the theorem)

$\vec{F}(x, y) = \langle e^x + x^2 y,\ e^y - xy^2 \rangle$

C is the circle $x^2 + y^2 = 25$ oriented clockwise.

#5b. Use Green's Theorem to find the work done by the force $\vec{F}(x, y) = \langle x(x+y),\ \ xy^2 \rangle$ in moving a particle from the origin along the x-axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin along the y-axis.

16.5

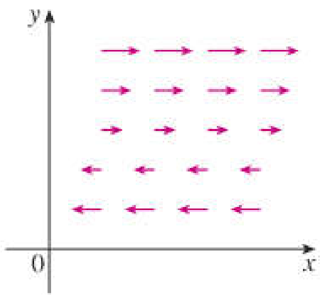
#1b. Find (i) the curl and (ii) the divergence of the vector field $\vec{F}(x,y,z)=\left\langle 1,\ x+yz,\ xy-\sqrt{z}\right\rangle$

#2b. Find (i) the curl and (ii) the divergence of the vector field $\vec{F}(x,y,z)=\dfrac{1}{\sqrt{x^2+y^2+z^2}}\langle x,\ y,\ z\rangle$

#3b. The vector field $\vec{F}$ is shown in the xy-plane and looks the same in all other horizontal planes (its z-component is zero).

(i) Is $div\,\vec{F}$ positive, negative, or zero? Explain.

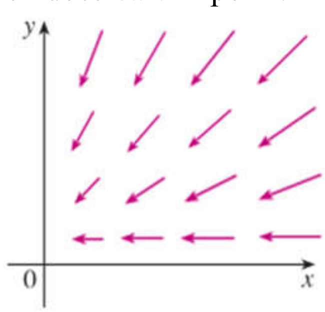
(ii) Determine whether $curl\,\vec{F} = \vec{0}$. If not, in which direction does $curl\,\vec{F}$ point?



#4b. The vector field $\vec{F}$ is shown in the xy-plane and looks the same in all other horizontal planes (its z-component is zero).

(i) Is $div\,\vec{F}$ positive, negative, or zero? Explain.

(ii) Determine whether $curl\,\vec{F} = \vec{0}$. If not, in which direction does $curl\,\vec{F}$ point?



#5b. Let f be a scalar field and $\vec{F}$ a vector field. State whether each expression is meaningful. If not, explain why. If so, state whether it is a scalar field or a vector field.

(i) $grad\ f$

(ii) $curl(grad\ f)$

(iii) $\nabla\left(div\,\vec{F}\right)$

(iv) $\nabla(div\ f)$

(v) $div\left(div\,\vec{F}\right)$

(vi) $div(curl(\nabla f))$

16.6 day 1

#1b. Determine whether the points P and Q lie on the given surface.

$\vec{r}(u,v) = \langle u+v,\ \ u^2 - v,\ \ u+v^2 \rangle$

$P(3,-1,5),\ \ Q(-1,3,4)$

#2b. Identify the surface with the given vector equation.

$\vec{r}(s,t) = \langle s,\ t,\ t^2 - s^2 \rangle$

#3b. Find a parametric representation for the surface: the plane that passes through the point $(2, 4, 6)$ and contains the vectors $\langle 2, 1, -1 \rangle$ and $\langle 1, -1, 2 \rangle$.

#4b. Find a parametric representation for the surface: the part of the hyperboloid $x^2 + y^2 - z^2 = 1$ that lies to the right of the *xz*-plane.

#5b. Find a parametric representation for the surface: the part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

16.6 day 2

#1b. Find an equation of the tangent plane to the given parametric surface at the specified point.

$\vec{r}(u,v) = \langle u^2,\ 2u\sin v,\ u\cos v \rangle$

$(1,0,1)$

#2b. Find the area of the surface: the part of the plane $2x+5y+z=10$ that lies inside the cylinder $x^2+y^2=9$.

#3b. Find the area of the surface: the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices (0,0), (0,1), and (2,1).

#4b. Find the area of the surface: the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

16.7 day 1

#1b. Evaluate the surface integral $\iint_S yz\, dS$

S is the part of the plane $x + y + z = 1$ that lies in the first octant.

#2b. Evaluate the surface integral $\iint_S x^2 z^2 \, dS$

S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.

#3b. Evaluate the surface integral $\iint\limits_S \left(z + x^2 y\right) dS$

S is the part of the cylinder $y^2 + z^2 = 1$ that lies between the planes $x = 0$ and $x = 3$ in the first octant.

16.7 day 2

#1b. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across S):

$\vec{F}(x, y, z) = \langle xze^y,\ -xze^y,\ z \rangle$

S is the part of the plane $x + y + z = 1$ in the first octant and has downward orientation.

#2b. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across S):

$\vec{F}(x, y, z) = \langle xy,\ 4x^2,\ yz \rangle$

S is the surface $z = xe^y$, $0 \le x \le 1,\ 0 \le y \le 1$ with upward orientation.

#3b. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across *S*):

$\vec{F}(x, y, z) = \langle xz,\ x,\ y \rangle$

S is the hemisphere $x^2 + y^2 + z^2 = 25, \quad y \geq 0$
oriented in the direction of the positive *y*-axis.

#4b. Evaluate the surface integral $\iint_S \vec{F} \bullet dS$

(find the flux of $\vec{F}$ across *S*):

$\vec{F}(x, y, z) = \langle y,\ z - y,\ x \rangle$

S is the surface of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

16.8

#1b. Using Stokes' Theorem, write out and evaluate the single-integral which is equivalent to the surface integral which calculates

$\iint_S \left(curl\, \vec{F} \right) \bullet dS$ where

$\vec{F}(x,y,z) = \langle xyz,\ xy,\ x^2yz \rangle$

S consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1,\ \pm 1,\ \pm 1)$ oriented outward.

#2b. Using Stokes' Theorem, write out and evaluate the double-integral which is equivalent to the line integral $\int_C \vec{F} \bullet d\vec{r}$ which sums the contributions of the field $\vec{F}$ along path C

$\vec{F}(x,y,z) = \langle yz,\ 2xz,\ e^{xy} \rangle$

C is the circle $x^2 + y^2 = 16, \quad z = 5$.

#3b. Verify that Stokes' Theorem is true for the given vector field $\vec{F}$ and surface S by writing out and evaluating integrals for both sides of the Stokes' Theorem equation.

$\vec{F}(x,y,z) = \langle y,\ z,\ x \rangle$

S is the hemisphere $x^2 + y^2 + z^2 = 1,\ \ y \geq 0$ oriented in the direction of the positive y-axis.

Double-integral side....

#3b. Verify that Stokes' Theorem is true for the given vector field $\vec{F}$ and surface S by writing out and evaluating integrals for both sides of the Stokes' Theorem equation.

$\vec{F}(x,y,z)=\langle y,\ z,\ x\rangle$

S is the hemisphere $x^2+y^2+z^2=1,\ \ y\geq 0$ oriented in the direction of the positive y-axis.

<u>Single-integral side</u>….

Extra #4. A particle moves along line segments from the origin to the points (1,0,0), (1,2,1), (0,2,1), and back to the origin under the influence of the force field:

$$\vec{F}\left(x,y,z\right)=\left\langle z^2,\ 2xy,\ 4y^2\right\rangle$$

Find the work done.

16.9

#1b Verify that the Divergence Theorem is true for the given vector field $\vec{F}$ on the region E by writing out and evaluating integrals for both sides of the Divergence Theorem equation.

$\vec{F}(x,y,z)=\langle x^2,\ xy,\ z\rangle$

E is the solid bounded by the paraboloid $z=4-x^2-y^2$ and the xy-plane.

Double-integral side….

#1b (continued) Verify that the Divergence Theorem is true for the given vector field $\vec{F}$ on the region E by writing out and evaluating integrals for both sides of the Divergence Theorem equation.

$\vec{F}(x,y,z)=\langle x^2,\ xy,\ z\rangle$

E is the solid bounded by the paraboloid $z=4-x^2-y^2$ and the *xy*-plane.

Triple-integral side….

#2b. Using the Divergence Theorem, write out and evaluate the triple-integral which is equivalent to the surface integral $\iint_S \vec{F} \bullet dS$ which calculates the flux of $\vec{F}$ across S if

$\vec{F}(x, y, z) = \langle 3xy^2,\ xe^z,\ z^3 \rangle$

S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

#3b. Using the Divergence Theorem, write out and evaluate the triple-integral which is equivalent to the surface integral $\iint_S \vec{F} \bullet dS$ which calculates the flux of $\vec{F}$ across S if

$\vec{F}(x,y,z) = \left\langle 4x^3z,\ 4y^3z,\ 3z^4 \right\rangle$

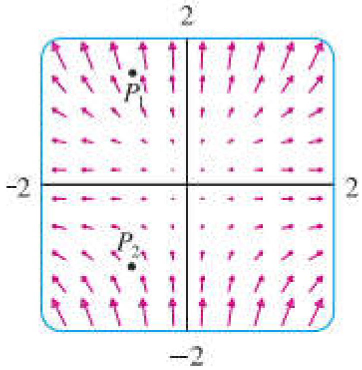
S is the sphere with radius R and center at the origin.

#3c. Using the Divergence Theorem, write out and evaluate the triple-integral which is equivalent to the surface integral $\iint_S \vec{F} \bullet dS$ which calculates the flux of $\vec{F}$ across S if

$\vec{F}(x, y, z) = \langle xy \sin z,\ \cos(xz),\ y \cos z \rangle$

S is the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

#4b. A vector field $\vec{F}$ is shown.



(i) Determine whether is positive or negative at P_1 and P_2 just from looking at the field picture.

(ii) Given the $\vec{F}(x, y) = \langle x, y^2 \rangle$ for this field, use the definition of divergence to verify your answers in part (i).