Calc III - Ch 15 Part 2 - Extra Practice

15.6

#1b. Evaluate the given integral over the specified region using the three specified orders of integration.

$$\iiint_E \left(xz - y^3\right) dV$$

$$E = \left\{(x, y, z) \mid -1 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 1\right\}$$

(i) …integrating first with respect to y, then z, and then x.

(ii) …integrating first with respect to x, then y, and then z.

(iii) …integrating first with respect to z, then y, and then x.

#2b. Evaluate the iterated integral

$$\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \sin y \, dy \, dz \, dx$$

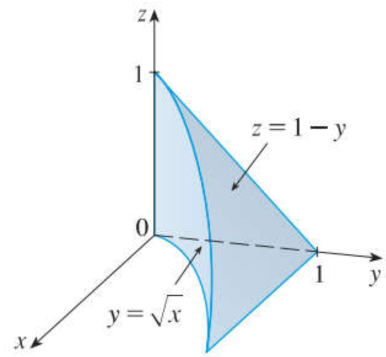
#3b. Evaluate the integral $\iiint_T xyz \, dV$

where T is the solid tetrahedrom with vertices (0,0,0), (1,0,0), (1,1,0), and (1,0,1).

#4b. Set up (but do not evaluate) a triple integral to find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

#4c. Set up (but do not evaluate) a triple integral to find the volume of the solid enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 16$.

#5b. The figure shows the region of integration for the integral $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\, dy\, dx$.



Rewrite this integral (but do not evaluate) as an equivalent iterated integral in the specified integration orders….

(i) $\int_0^? \int_0^? \int_0^? f(x,y,z)\, dz\, dx\, dy$.

(ii) $\int_0^? \int_0^? \int_0^? f(x,y,z)\, dy\, dx\, dz$.

Extra #6. Write five other iterated integrals that are equal to the given iterated integral:

$$\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy .$$

15.7

#1b. Plot the point whose cylindrical coordinates are given:

$$\left(4, -\frac{\pi}{3}, 5\right)$$

#3b. Sketch the solid described by the inequalities:

$$0 \le \theta \le \frac{\pi}{2}, \quad r \le z \le 2$$

#2b. Sketch and describe in words the surface whose equation is given: $r = 5$

#4b. Sketch the solid whose volume is given by the integral: $\int_0^{\pi/2}\int_0^{2}\int_0^{9-r^2} r\,dz\,dr\,d\theta$

#5b. Evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the region the lies inside the cylinder $x^2+y^2=16$, and between the planes $z=-5$ and $z=4$.

#6b. Find the volume of the solid that lies within both the cylinder $x^2+y^2=1$ and the sphere $x^2+y^2+z^2=4$.

15.8

#1b. Plot the point whose spherical coordinates are given and find the rectangular coordinates of the point:

(i) $\left(5, \pi, \frac{\pi}{2}\right)$ (ii) $\left(4, \frac{3\pi}{4}, \frac{\pi}{3}\right)$

#2b. Sketch and describe in words the surface whose equation is given: $\rho = 3$

#3b. Identify the surface whose equation is given:

$\rho^2\left(\sin^2\phi\sin^2\theta + \cos^2\phi\right) = 9$

#4b. Sketch the solid described by the given inequalities:

$2 \le \rho \le 3, \quad \frac{\pi}{2} \le \phi \le \pi$

#5b. Write the equation in spherical coordinates:

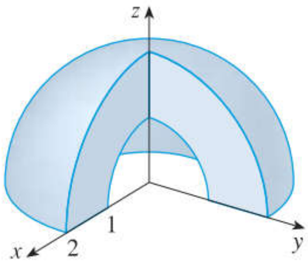
(i) $x^2 - 2x + y^2 + z^2 = 0$

(ii) $x + 2y + 3z = 1$

#6b. Sketch the solid whose volume is given by the integral and evaluate the integral.

$$\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

#7b. Set up the triple integral of an arbitrary continuous function *f(x,y,z)* in cylindrical or spherical coordinates over the solid shown.



#8b. Evaluate using spherical coordinates $\iiint_E z\,dV$, where E lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.

#9b. Evaluate using spherical coordinates $\iiint_E x^2 \, dV$, where E is bounded by the *xz*-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$ in the first octant.

#10b. Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

#11b. Evaluate the integral by changing to spherical coordinates:

$$\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} \left(x^2 z + y^2 z + z^3\right) dz\, dx\, dy$$