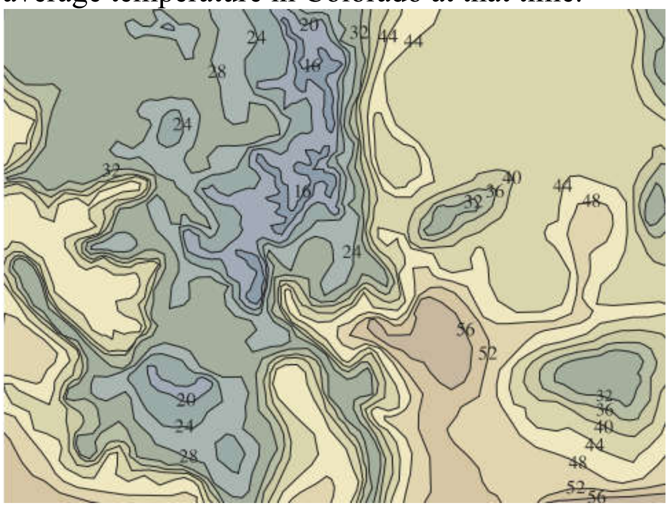
Calc III - Ch 15 Part 1 - Extra Practice

15.1 and 15.2

#1b. The contour map shows the temperature, in degrees Fahrenheit, at 4:00 PM on February, 26, 2007, in Colorado. (The state measures 388 mi east to west and 276 mi north to south.) Use the Midpoint Rule with $m = n = 2$ to estimate the average temperature in Colorado at that time.



#2b. Evaluate the double integral by first identifying it as the volume of a solid.

$$\iint_R (5-x)\, dA, \quad R = \{(x,y) \mid 0 \le x \le 5,\ 0 \le y \le 3\}.$$

#3b. Find $\int_0^5 f(x,y)\, dx$ and $\int_0^1 f(x,y)\, dy$

$f(x,y) = y + xe^y$.

#4b. Evaluate $\int_0^1 \int_1^2 \left(4x^3 - 9x^2 y^2\right) dy\, dx$.

#5b. Evaluate $\int_{\pi/6}^{\pi/2}\int_{-1}^{5} x\cos y \, dx \, dy$.

#6b. Evaluate $\int_{0}^{1}\int_{0}^{1} xy\sqrt{x^2+y^2} \; dy \, dx$.

#6c. Evaluate $\int_{0}^{2}\int_{0}^{\pi} r\sin^2\theta \, d\theta \, dr$.

#7b. Evaluate the double integral:

$$\iint_R \frac{xy^2}{x^2+1}\, dA, \quad R = \{(x,y) \mid 0 \le x \le 1, -3 \le y \le 3\}$$

#8b. Sketch the solid whose volume is given by the iterated integral $\int_{0}^{1}\int_{0}^{1}\left(2-x^2-y^2\right) dy \, dx$

#9b. Find the volume of the solid that lies under the elliptic paraboloid $\frac{x^2}{4}+\frac{y^2}{9}+z=1$ and above the rectangle $R=\{(x,y)\,|\,-1\le x\le 1,\ -2\le y\le 2\}$.

#10. (hints)
Use 3D graphing software to picture…the surface is an elliptical paraboloid which starts at z = 2 and extends towards more positive z.

That means the volume of the solid between this surface and z =1 has a height which is just the surface minus 1: height = $2+x^2+(y-2)^2-1$ so this is the integrand for the volume.

15.3

#1b. Evaluate $\int_0^{\pi/2}\int_0^{\cos\theta} e^{\sin\theta}\, dr\, d\theta$.

#2b. Evaluate $\iint_D \frac{y}{x^5+1}\, dA$

$D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le x^2\}$

#3b. Evaluate $\iint_D 2xy\, dA$

D is the triangular region with vertices (0,0), (1,2), and (0,3).

#4b. Find the volume of the solid under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$

#5b. Find the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$.

#6b. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y,\ x = 0,\ z = 0$ in the first octant.

#7b. Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^{1-x} (1 - x - y)\, dy\, dx$

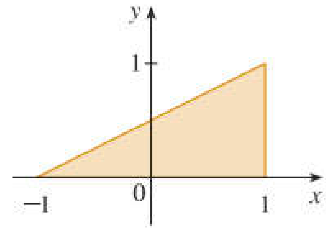
#8b. Sketch the region of integration and change the order of integration (write the new integral but do not evaluate) $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y)\, dx\, dy$

#8c. Sketch the region of integration and change the order of integration (write the new integral but do not evaluate) $\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx$

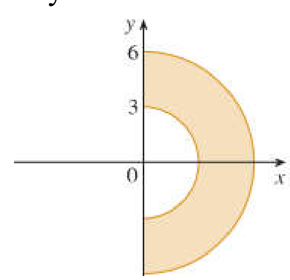
#9b. Evaluate the integral by reversing the order of integration $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

15.4

#1b. A region R is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x,y)\, dA$ as an iterated integral, where f is an arbitrary continuous function on R.



#2b. A region R is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x,y)\, dA$ as an iterated integral, where f is an arbitrary continuous function on R.



#3b. Sketch the region whose area is given by the integral and evaluate the integral: $\int_0^{\pi/2}\int_0^{4\cos\theta} r\,dr\,d\theta$

#5b. Evaluate the given integral by changing to polar coordinates: $\iint_D x\,dA$ where D is the region in the first quadrant the lies between the circles $x^2+y^2=4$ and $x^2+y^2=2x$.

#4b. Evaluate the given integral by changing to polar coordinates: $\iint_R \cos\left(x^2+y^2\right) dA$ where R is the region that lies above the x-axis between the circles $x^2+y^2=1$ and $x^2+y^2=4$.

#6b. Use a double integral to find the area of the region: the part of the rose $r = \sin 2\theta$ which has positive y.

#7b. Use polar coordinates to find the volume of the given solid: above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

#8b. Use polar coordinates to find the volume of the given solid: inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

#9b. Evaluate the iterated integral by converting to polar coordinates: $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin\left(x^2 + y^2\right) dy\, dx$

Extra #10. Use polar coordinates to find the volume of a sphere of radius *a*.

15.5

#1b. Find the mass and center of mass of the lamina that occupies the region D and has the density function ρ.

D is the triangular region with vertices (0,0), (2,1), (0,3); $\rho(x,y) = x + y$.

#2b. A lamina occupies the part of the disk $x^2 + y^2 \le 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to *square* of its distance from the *origin*.

#2c. A lamina occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. Find the center of mass if the density at any point is *inversely* proportional to its distance from the *origin*.

#3b. Xavier and Yolanda both has classes that end at noon and they agree to meet every day after class. They arrive at the coffee shop independently. Xavier's arrival time is X and Yolanda's arrival time is Y, where X and Y are measured in minutes after noon. The individual probability density functions are:

$$f_1(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad f_2(y) = \begin{cases} \frac{1}{50}y & 0 \leq y \leq 10 \\ 0 & \textit{otherwise} \end{cases}$$

(Xavier arrives sometime after noon and is more likely to arrive promptly than late. Yolanda always arrives by 12:10 PM and is more like to arrive late than promptly.)

After Yolanda arrives, she'll wait for up to half an hour for Xavier, but he won't wait for her.

Find the probability that they meet.