Calc III - Ch 14 Part 2 - Extra Practice

14.6

#1b.Find the directional derivative of *f* at the given point in the direction indicated by the angle θ

$$f(x,y)=x^2y^3-y^4,\quad (2,1),\quad \theta=\frac{\pi}{4}$$

#2b. (i) Find the gradient of *f*.
(ii) Evaluate the gradient at the point *P*.
(iii) Find the rate of change of *f* at *P* in the direction of the vector $\vec{u}$.

$$f(x,y,z)=xe^{2yz},\ P(3,0,2),\ \vec{u}=\left\langle \frac{2}{3},\ -\frac{2}{3},\ \frac{1}{3}\right\rangle$$

#3b. Find the directional derivative of the function at the given point in the direction of the vector $\vec{v}$

$$g(p,q)=p^4-p^2q^3,\quad (2,1),\quad \vec{v}=\langle 1,\ 3\rangle$$

#4b. Find the directional derivative of the function at the given point in the direction of the vector $\vec{v}$

$$g(x,y,z)=(x+2y+3z)^{3/2},\quad (1,1,2),\quad \vec{v}=\langle 0,\ 2,-1\rangle$$

#5b. Find the directional derivative of $f(x,y,z) = xy + yz + zx$ at $P(1,-1,3)$ in the direction of $Q(2,4,5)$.

#6b. Find the maximum rate of change of *f* at the given point and the direction in which it occurs.
$f(x,y) = \sin(xy), \quad (1,0)$

#6c. Find the maximum rate of change of *f* at the given point and the direction in which it occurs.
$f(x,y,z) = \sqrt{x^2 + y^2 + z^2}, \quad (3,6,-2)$

#7b. Find all points at which the direction of fastest change of the function $f(x,y) = x^2 + 3xy + y^2$ is $\langle 2,1 \rangle$.

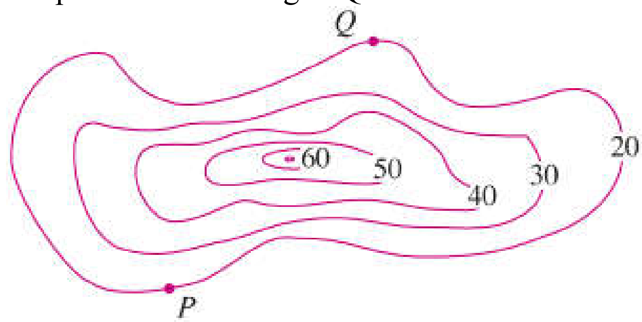
#8b. (hints)

(i) south = $\langle 0,-1\rangle$, find the directional derivative in this direction.

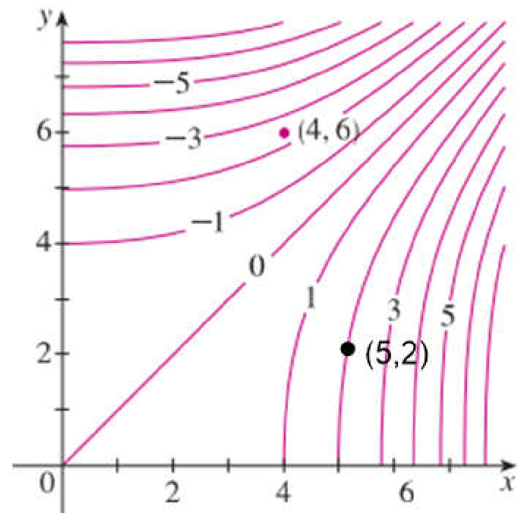
(ii) northwest = $\langle -1,1\rangle$, but remember to make this a unit vector before finding the directional derivative.

(iii) max in direction of gradient. Rate is the magnitude of the gradient. To find angle above horizontal remember that the gradient is a slope, so you can make a triangle with gradient up for every 1 unit horizontally.

#9b. For the given contour map draw the curve of steepest ascent starting at Q.



#10b. Sketch the gradient vector $\nabla f(5,2)$ for the function *f* whose level curves are shown.



#11b. Find an equation of the tangent plane to the given surface at the specified point.

$x^2 - 2y^2 + z^2 + yz = 2, \quad (2,1,-1)$

14.7

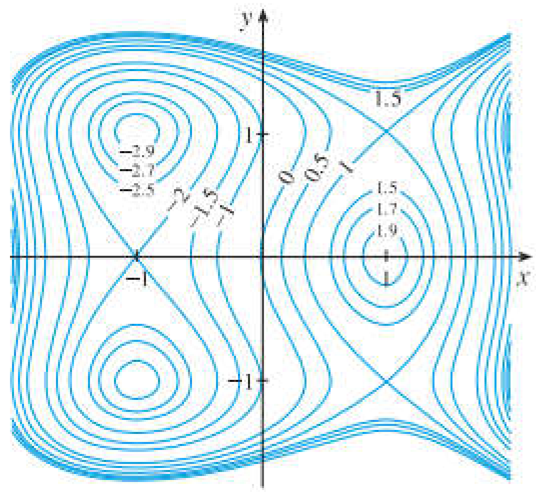
#1b. Suppose (1,1) is a critical point of a function *f* with continuous second derivatives. What can you say about *f* at (1,1)?

(i) $f_{xx}(1,1)=4, \quad f_{xy}(1,1)=1, \quad f_{yy}(1,1)=2$

(ii) $f_{xx}(1,1)=4, \quad f_{xy}(1,1)=3, \quad f_{yy}(1,1)=2$

#2b. Use the level curves in the figure to predict the location of the critical points of *f* and whether *f* has a saddle point or local maximum or minimum at each critical point. Then use the Second Derivatives Test to confirm your predictions.

$f(x,y)=3x-x^3-2y^2+y^4$



#3b. Find the local maximum and minimum values and saddle point(s) of the function.

$f(x,y)=x^4+y^4-4xy+2$

#4b. Find the absolute maximum and minimum values of *f* on the set *D*.

$f(x,y)=3+xy-x-2y$

D is the closed triangular region with vertices (1,0), (5,0), and (1,4).

#5b. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

#6b. Find three positive numbers whose sum is 12 and whose sum of squares is as small as possible.

#7b. Find the dimensions of the rectangular parallelepiped with faces parallel to the coordinate plane that can be inscribed in the ellipsoid $16x^2 + 4y^2 + 9z^2 = 144$ which will maximize the volume inside the parallelepiped.

14.8

#1b. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

$f(x, y) = x^2 y; \quad x^2 + 2y^2 = 6$

#2b. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

$f(x, y, z, t) = x + y + z + t; \quad x^2 + y^2 + z^2 + t^2 = 1$

#3b. Consider the problem of maximizing the function $f(x,y)=x$ subject to the constraint $y^2+x^4-x^3=0$

(i) Try using Lagrange multipliers to solve the problem.

(ii) Show that the minimum value is $f(0,0)=0$ but the Lagrange condition $\nabla f(0,0)=\lambda\nabla g(0,0)$ is not satisfied for any value of λ.

(iii) Explain why Lagrange multipliers fail to find the minimum value in this case.

Extra #4. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.