Calc III - Ch 14 Part 1 - Extra Practice

14.1

#1b. The *wave heights* h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v,t)$ are recorded in feet in the following table:

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

(i) What is the value of $f(40,15)$? What is its meaning?

(ii) What is the meaning of the function $h = f(30,t)$? Describe the behavior of this function.

(iii) What is the meaning of the function $h = f(v,30)$? Describe the behavior of this function.

#2b. Let $f(x,y) = \ln(x+y-1)$.

(i) Evaluate $f(1,1)$ and $f(e,1)$.

(ii) Find the domain of f.

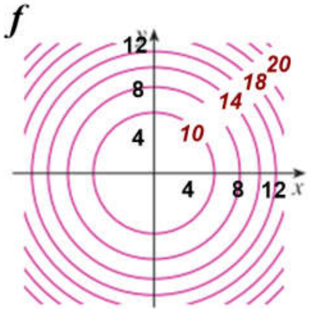
(iii) Find the range of f.

#3b. Sketch the graph of the function $f(x,y) = \sqrt{16 - x^2 - 16y^2}$.

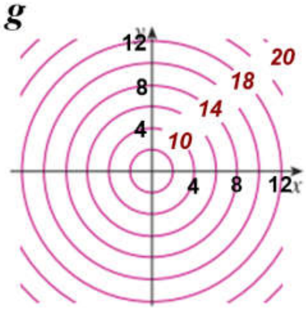
#3c. Sketch the graph of the function $f(x,y) = 3$.

#4b. Contour maps for function f and g are shown. Use these to estimate the values of $f(8,8)$ and $g(9,2)$. One of these is a cone and the other a paraboloid – which is which?

f



g



#5b. Draw a contour map of the function showing several level curves: $f(x,y)=x^3-y$.

#6b. Describe the level surfaces of the function $f(x,y,z)=x^2-y^2+z^2$.

14.2

#1b. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(1,2)} \left(5x^3 - x^2y^2\right)$.

#2b. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2 + y^2}$.

#3b. Find the limit, if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$.

14.3

#1b. The *wave heights h* in the open sea depend on the speed *v* of the wind and the length of time *t* that the wind has been blowing at that speed. Values of the function $h = f(v,t)$ are recorded in feet in the following table:

Duration (hours)

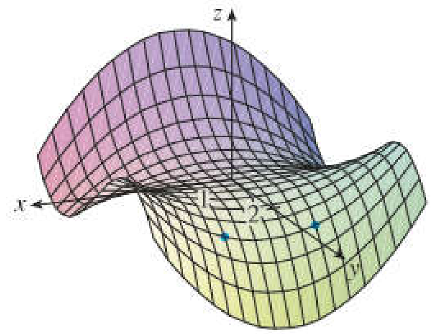
Wind speed (knots) $v \backslash t$	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

(i) Estimate the values of $f_t(40,15)$ *and* $f_v(40,15)$. What are the practice interpretations of these values?

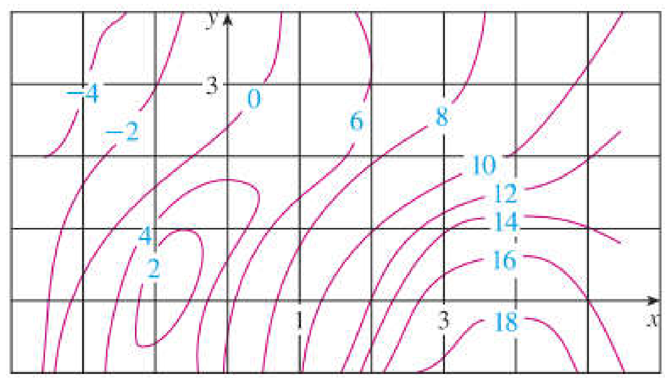
(ii) What are the meanings of the partial derivatives $\dfrac{\partial h}{\partial t}$ and $\dfrac{\partial h}{\partial v}$?

(iii) What appear to be the value of the following limit? $\lim_{t \to \infty} \dfrac{\partial h}{\partial t}$

#2b. Determine the signs of the partial derivatives $f_x(-1,2)$, $f_y(-1,2)$, and $f_{yy}(-1,2)$ for the function *f* whose graph is shown.



#3b. A contour map is given for a function *f*. Use it to estimate $f_x(-1,3)$ and $f_y(-1,3)$



#4b. Find both first partial derivatives of the function $f(x,y) = 2xy^2 - 3x$

#5b. Find both first partial derivatives of the function $z = x^y$

#8b. Find $f_z\left(0,0,\frac{\pi}{4}\right)$ for $f(x,y,z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$

#6b. Find both first partial derivatives of the function $f(x,y) = \int_x^y e^{t^2}\, dt$

#9b. Verify that the conclusion of Clariaut's Theorem hold (that $u_{xy} = u_{yx}$) $u = \ln\left(\sqrt{x^2 + y^2}\right)$

#7b. Find both first partial derivatives of the function $p = ze^{xyz}$

14.4

#1b. Find an equation of the tangent plane to the given surface at the specified point
$z = 3(x-1)^2 + 2(y+3)^2 + 7, \quad (2, -2, 12)$.

#2b. Find an equation of the tangent plane to the given surface at the specified point
$z = e^{x^2 - y^2}, \quad (1, -1, 1)$.

#3b. Explain why the function is differentiable at the given point. Then find the linearization *L(x,y)* of the function at that point.
$f(x, y) = \dfrac{x}{x+y}, \quad (2, 1)$.

#4b. Find the differential of the function.
$v = y \cos xy$.

#5b. Verify the linear approximation at (0,0).

$\sqrt{y+\cos^2 x} \approx 1+\frac{1}{2}y$.

#6b. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3,-1)$ to $(2.96,-0.95)$, compare the values of Δz and dz.

14.5

#1b. Use the Chain Rule to find $\frac{dz}{dt}$

$z=\sqrt{1+x^2+y^2}, \quad x=\ln t, \quad y=\cos t$.

#2b. Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

$z=\sin\theta\cos\phi, \quad \theta=st^2, \quad \phi=s^2t$.

#3b. Use a tree diagram to write out the Chain Rule for the given case. Assume all functions are differentiable.

$w=f(r,s,t), \quad r=r(x,y), \quad s=s(x,y), \quad t=t(x,y)$

.

#4b. Use the Chain Rule to find the indicated partial derivatives

$R=\ln\left(u^2+v^2+w^2\right), \quad u=x+2y, \quad v=2x-y, \quad w=2xy$

$\frac{\partial R}{\partial x}, \frac{\partial R}{\partial y}$ $\quad when\ x=y=1$

.

#5b. Use $\frac{dy}{dx} = -\frac{\left(\frac{\partial F}{\partial x}\right)}{\left(\frac{\partial F}{\partial y}\right)} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$.

$y^5 + x^2 y^3 = 1 + y e^{x^2}$.

#6b. The pressure of 1 mole of an ideal gas is increasing at a rate of 0.05 kPa/s and the temperature is increasing at a rate of 0.15 K/s. Use the equation $P = 8.31\frac{T}{V}$ to find the rate of change of the volume when the pressure is 20 kPa and the temperature is 320 K.

#Extra 7. If $z = f(x-y)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.